

Long-Range OASDI Projection Methodology

Intermediate Assumptions of the 2009 Trustees Report

September 2009

**Office of the Chief Actuary
Social Security Administration**

A. Flow Charts

Chart 1:

Overview of Long-Range OASDI Projection Methodology

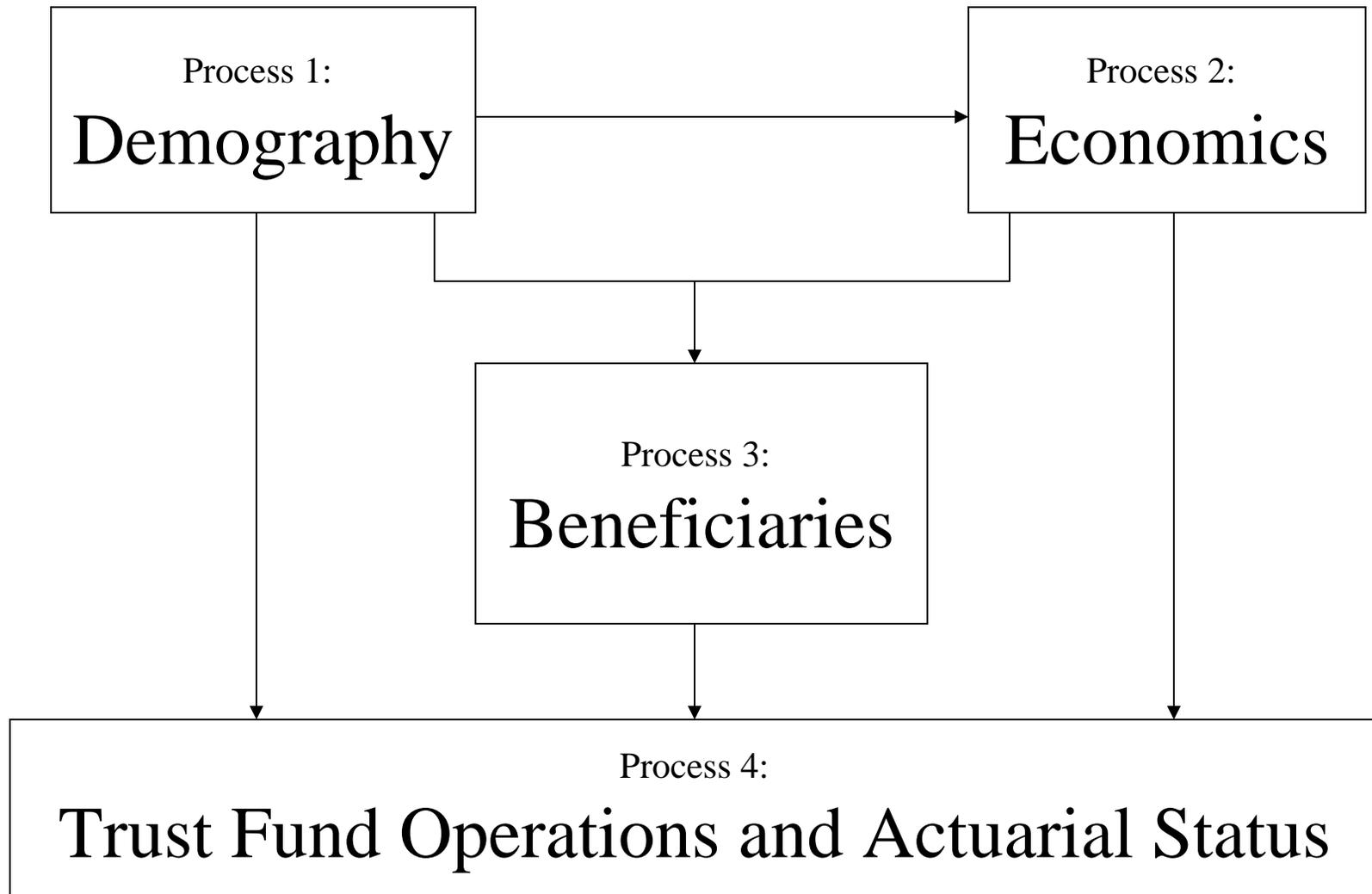
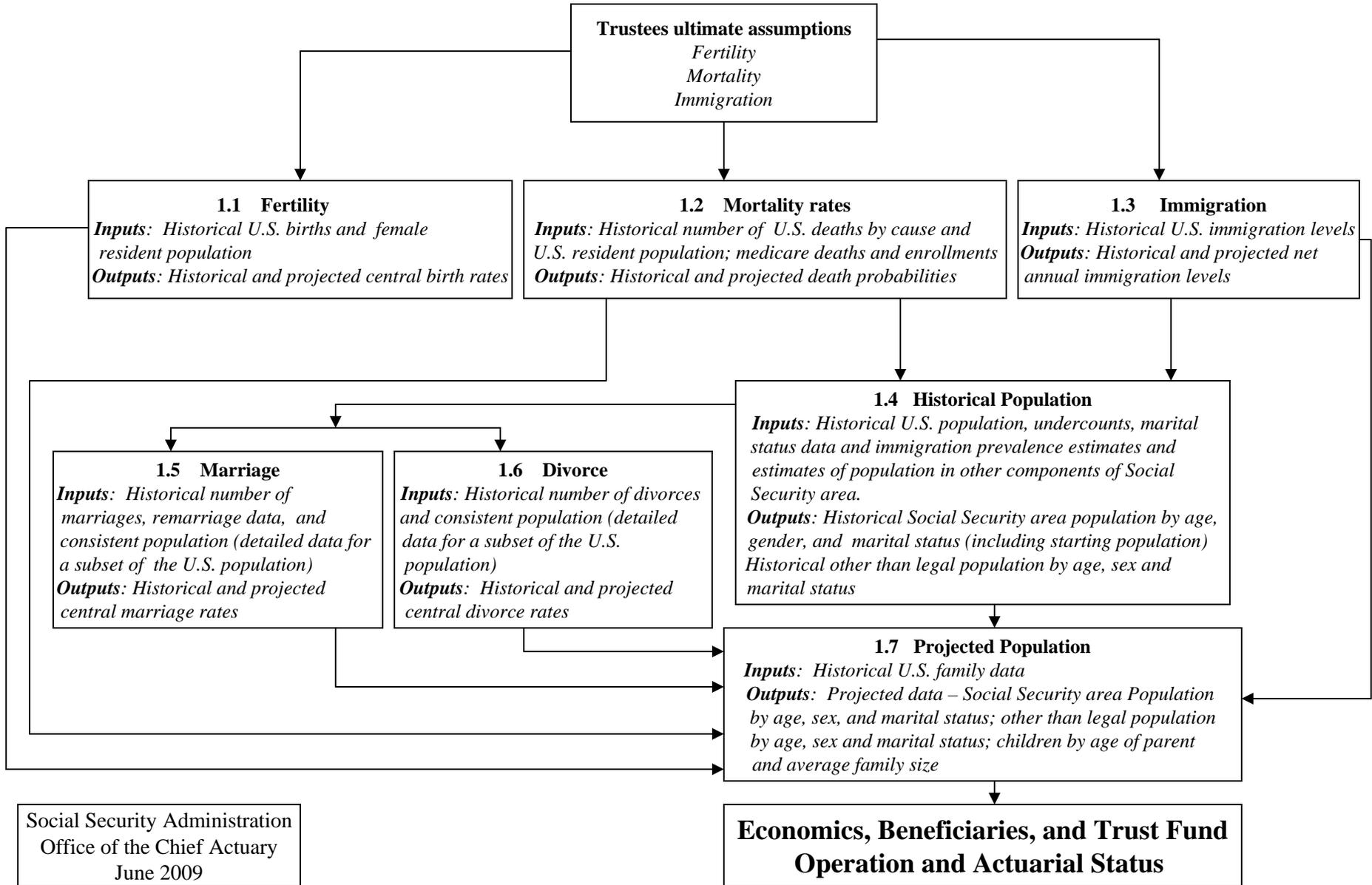


Chart 2: Demography – Process 1



Social Security Administration
Office of the Chief Actuary
June 2009

**Economics, Beneficiaries, and Trust Fund
Operation and Actuarial Status**

Chart 3: Economics – Process 2

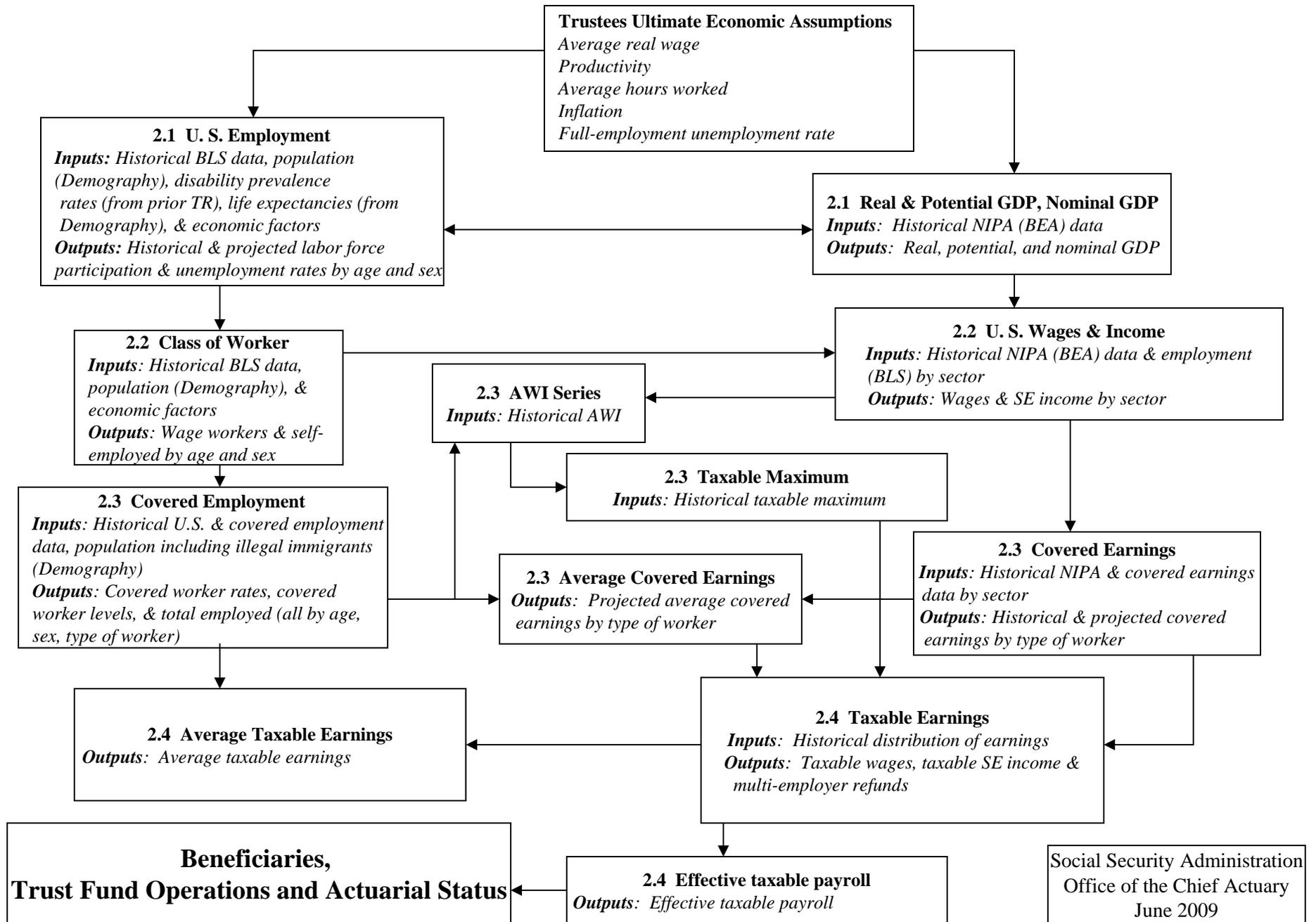
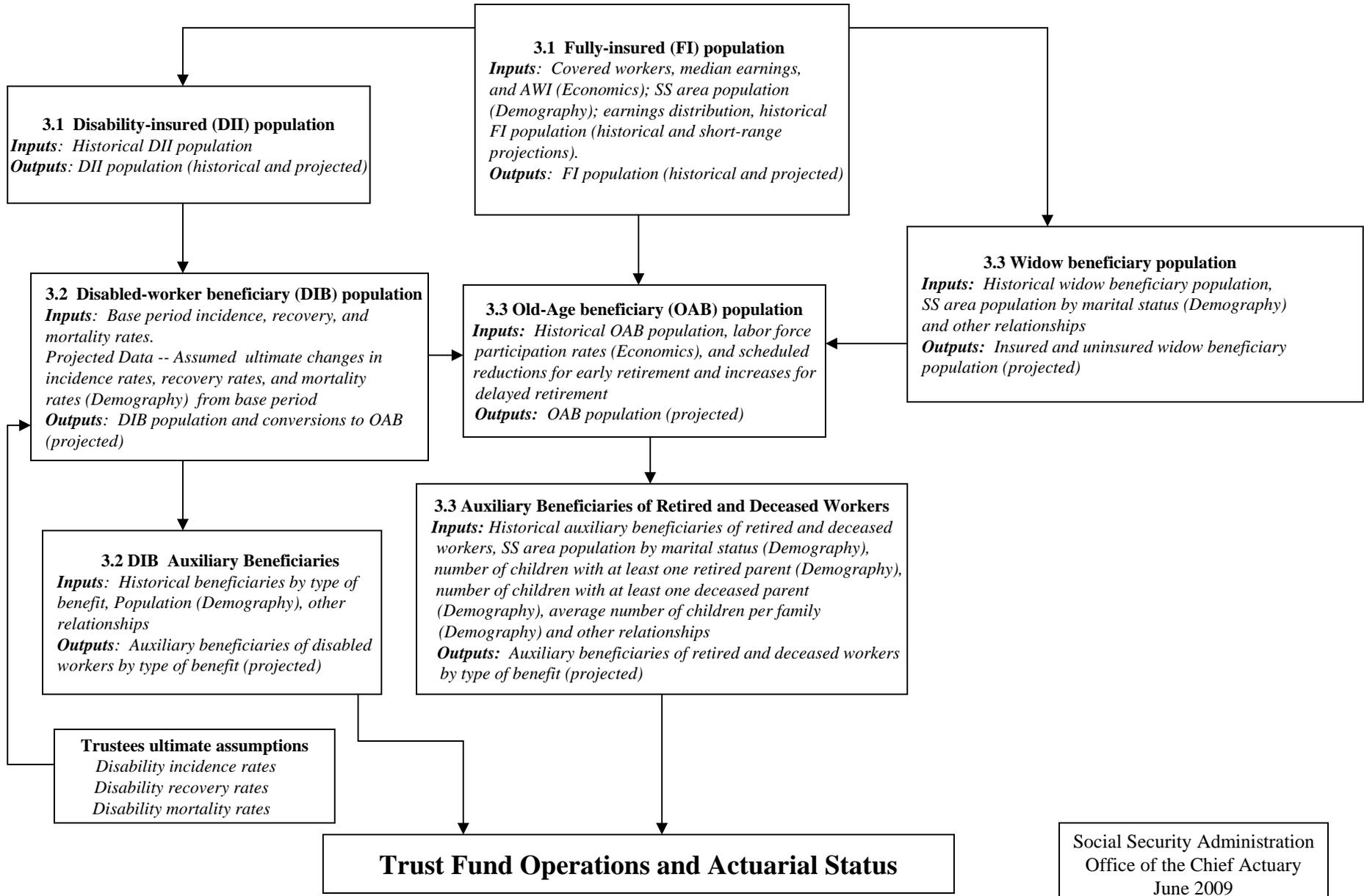
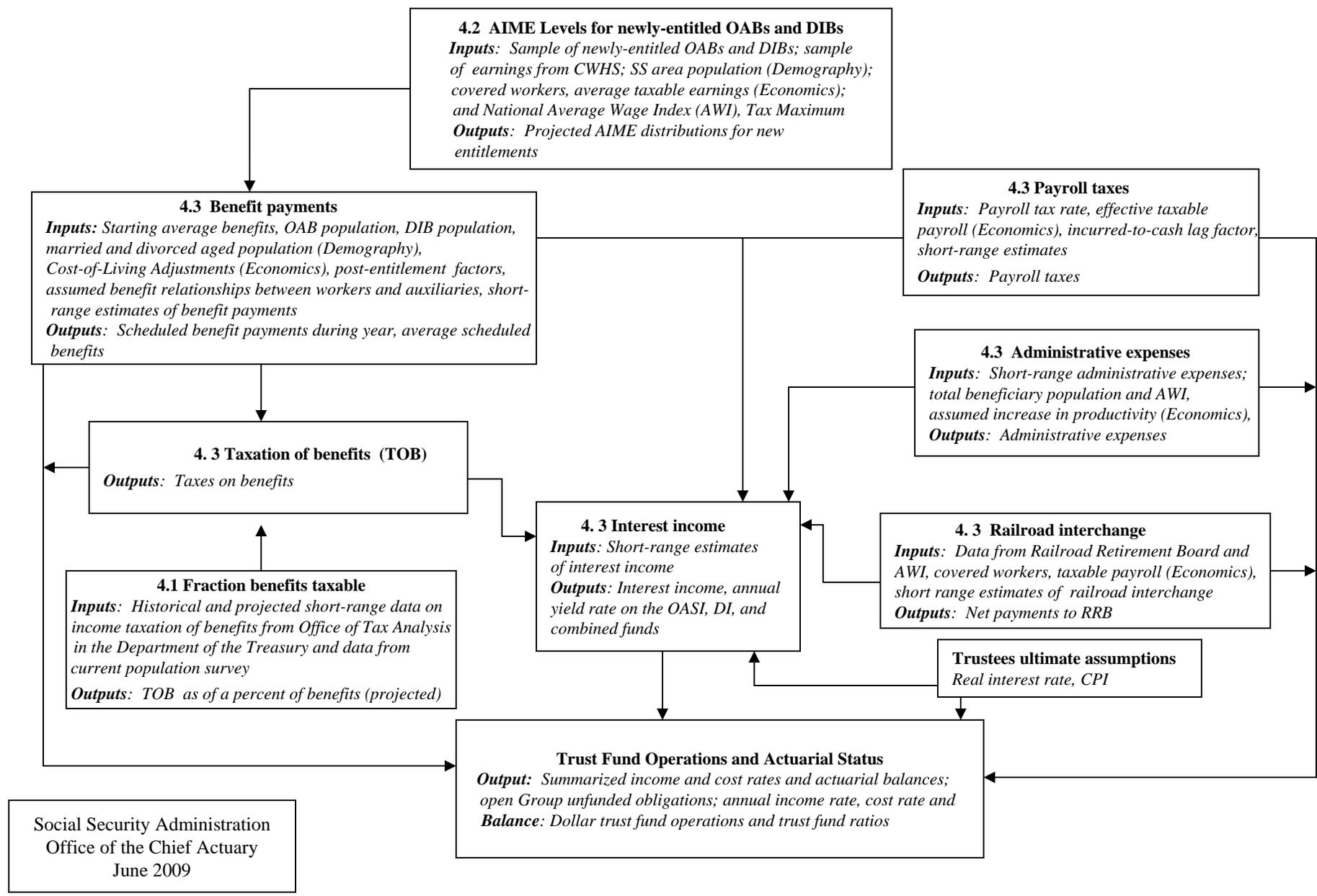


Chart 4: Beneficiaries – Process 3



Note: Insured widow refers to widow beneficiaries who are insured for OAIB benefits, but not receiving those benefits

Chart 5: Trust Fund Operations and Actuarial Status – Process 4



B. Process Descriptions

The long-range programs used to make projections for the annual Trustees Report are grouped into four major processes. These include Demography, Economics, Beneficiaries, and Trust Fund Operations and Actuarial Status. Each major process consists of a number of subprocesses. Each subprocess is described in terms of three elements:

- This overview attempts to provide a general description of the purpose of each subprocess. Key projected variables used in the subprocess are introduced. Some variables are represented as being dependent in an equation, where the dependent variable is *defined* in terms of one or more independent variables. Independent variables may include previously calculated dependent variables or data provided from outside the subprocess. Other key variables are referenced by “(·)” following the variable name. This indicates that the calculation of this variable can not easily be communicated by an equation and, thus, requires a more complex discussion.
- Input Data – Data used in the subprocess are described. These data include those from other subprocesses, ultimate long-range assumptions provided by the Board of Trustees of the OASDI Trust Funds, data from other offices of the Social Security Administration, and data from outside the Social Security Administration (e.g., estimates of the U.S. population). Data description includes data source and data detail (e.g., define age detail of data). In addition, an indication of how often additional data are expected to be received is included.
- Development of Output – The key variables are described in greater detail, including the level of disaggregation of the data.

Process 1:

Demography

1. Demography

The primary purpose of the Demography Process is to provide estimates of the projected Social Security area population¹ for each year of the 75-year projection period in the Trustees Report. For the 2009 report, the projection period covers the years 2009 through 2083. The Demography Process receives input data mainly from other government agencies and provides output data to the Economics, Beneficiaries, and Trust Fund Operations and Actuarial Status processes.

The Demography Process is composed of seven subprocesses: FERTILITY, MORTALITY, IMMIGRATION, HISTORICAL POPULATION, MARRIAGE, DIVORCE, and PROJECTED POPULATION. As a rough overview, FERTILITY projects birth rates by age of mother, MORTALITY projects probabilities of death by age and sex, and IMMIGRATION projects numbers of immigrants by age and sex. HISTORICAL POPULATION combines population estimates from different sources to obtain historical estimates of the Social Security area population by single year of age, sex and marital status. MARRIAGE projects marriage rates by age-of-wife crossed with age-of-husband and DIVORCE projects divorce rates by age-of-husband crossed with age-of-wife. PROJECTED POPULATION starts with the latest estimates of the Social Security area population from HISTORICAL POPULATION and projects the population by age, sex, and marital status using projected values from FERTILITY, MORTALITY, IMMIGRATION, MARRIAGE, and DIVORCE.

¹ The Social Security area population consists of all persons who are potentially eligible to either receive benefits under the Social Security program or who have the potential to work in covered employment. This population consists of residents of the U.S. and its territories, citizens living abroad, and beneficiaries living abroad.

1.1. FERTILITY

1.1.a. Overview

The National Center for Health Statistics (NCHS) collects data on annual numbers of births and the U.S. Census Bureau produces estimates of the resident population. Birth rates for historical years are calculated from these data by single year of age of mother. Age-specific birth rates (b_x^z) for a given year z are defined as the ratio of (1) births (B_x^z) during the year to mothers at the specified age x to (2) the midyear female population (P_x^z) at that age. The total fertility rate TFR^z summarizes the age-specific fertility rates for a given year. The total fertility rate for a given year z is defined as the sum of the age-specific birth rates for all ages x during the year z . It can be interpreted as the number of children born to a woman if she were to survive her childbearing years and experience the age-specific fertility rates of year z throughout her childbearing years.

The FERTILITY subprocess combines the historical values of b_x^z and TFR^z with an ultimate assumed future value of the TFR to develop projections of b_x^z . The primary equations of this subprocess are given below:

$$b_x^z = b_x^z(\cdot) \quad (1.1.1)$$

$$TFR^z = \sum_x b_x^z \quad (1.1.2)$$

1.1.b. Input Data

Trustees Assumptions -

Each year the Board of Trustees of the OASDI Trust Funds sets the ultimate assumed values for the TFR . The TFR reaches its ultimate value in the 25th year of the projection period. Under the intermediate assumptions underlying the 2009 Trustees Report, the ultimate TFR is 2.0 and it is assumed to be reached in 2033.

Other input data -

- From NCHS, annual numbers of births by age of mother² (10-14, 15, 16, 17, ..., 48, 49-54) for years 1980-2005. In general, NCHS provides an annual update including one additional year of birth data and the previous historical years are only updated if

² The ages provided include 10-14, 15, 16, 17, ..., 48, 49-54. Births at ages less than 14 are treated as having occurred at age 14 and ages reported to mothers older than 49 are treated as having occurred at age 49.

NCHS makes a historical revision to their data.

- From the U.S. Census Bureau, estimates of the July 1st female resident population by single year of age for ages 14-49 for 1980-2005. Each year, Census provides updated data for years after the most recent decennial census.
- NCHS historical birth rates, by single year of age of mother (14-49) for the period 1917-79. No updates of these data are needed.

1.1.c. *Development of Output*

Equation 1.1.1 - Age-specific birth rates

The FERTILITY subprocess produces the age-specific birth rates, by childbearing ages 14 through 49, for years 1941 through the 75-year projection period. For historical years prior to 1980, age-specific birth rates were obtained from NCHS. For years 1980 through the remaining historical period, age-specific birth rates are calculated as: $b_x^z = \frac{B_x^z}{P_x^z}$, using birth data from NCHS and estimates of the July 1st female resident population from the U.S. Census Bureau.

The age-specific birth rates are projected using a process that is consistent with both the observed trends in recent data and the ultimate assumed total fertility rate. This process consists of the following steps:

1. Averaged birth rates by age group³, designated as ${}_5b_x^z$, are calculated from the age-specific birth rates b_x^z for each year during the period 1980-2005.
2. To calculate the starting values of the projection process, the ${}_5b_x^z$ values from the last five years of historical data are averaged using weights of 5, 4, 3, 2 and 1 for years 2005, 2004, 2003, 2002, and 2001 respectively.
3. For each ${}_5b_x^z$, the slope of the least squares line is calculated based on a regression over the period 1980-2005.
4. For 2006, each of the seven starting values of ${}_5b_x^z$ (from Step 2) is projected forward by adding 100 percent of their respective slope (from Step3).

³ The average is calculated by giving each age in the group equal weight without regard to population. The age groups calculated are: 14-19, 20-24, 25-29, 30-34, 35-39, 40-44, and 45-49.

5. Then, a preliminary total fertility rate for 2006, TFR_p^{2006} , is calculated such that it is equal to 5 times the sum of each ${}_5b_x^{2006}$. For the age group 14-19, 6 times the sum is actually included since this age group contains one additional age.
6. For 2006, the preliminary TFR released by NCHS was 2.101 and thus the Trustees set TFR^{2006} equal to 2.101.
7. To ensure the assumed total fertility rate is achieved for 2006, each value of ${}_5b_x^{2006}$ (step 4) is now multiplied by the ratio of the assumed TFR^{2006} (step 6) and the respective value of TFR_p^{2006} (Step 5).
8. For 2007, each final ${}_5b_x^z$ for 2006 is projected forward by adding 96 percent of the respective slope (from Step 3). For subsequent projection years (2008-2033), an arithmetically decreasing portion of the slopes⁴ is added to the previous year's final values of ${}_5b_x^z$ to get preliminary values of ${}_5b_x^z$.
9. For years 2007 and later, a preliminary total fertility rate, TFR_p^z , is calculated from the preliminary values of ${}_5b_x^z$ in Step 8 and is calculated in the same manner as in Step 5.
10. Then, for each year, an adjustment is made so that the annual TFR^z is consistent with the Trustees' assumed ultimate level. TFR^{2007} is assumed to equal the level estimated for 2006 (2.101) and TFR^{2008} is assumed to equal the weighted average of every TFR^z of the last five years (2.084). For years after 2008, TFR^z is assumed to decrease linearly from TFR^{2008} until reaching the ultimate value in 2033.
11. To ensure the assumed total fertility rate is achieved, each value of ${}_5b_x^z$ (step 8) is multiplied by the ratio of the assumed TFR^z (step 10) and the respective value of TFR_p^z (Step 9).
12. The final step of the projection method disaggregates the adjusted ${}_5b_x^z$ into single age birth rates by multiplying the final ${}_5b_x^z$ values for each year (Steps 7 or 11) by the ratio of the single year b_x^z to the ${}_5b_x^z$ for each of the respective ages and age groups as calculated in the last year of complete historical data (Step 1).

⁴ Each year of the projection the slope is reduced by four percentage points.

1.2 MORTALITY

1.2.a. Overview

The NCHS collects data on annual numbers of deaths and the U.S. Census Bureau produces estimates of the U.S. resident population. Central death rates (${}_yM_x$) are defined as the ratio of (1) the annual number of deaths occurring during the year to persons between exact ages x and $x+y$ to (2) the mid-year population between exact ages x and $x+y$. For historical years prior to 1968, ${}_yM_x$, are calculated from the NCHS and Census data by sex. For historical years after 1968, the same data are used in the calculations for ages under 65 but data from the Centers for Medicare and Medicaid Services (CMS) are used for ages 65 and over. Based on death by cause data from NCHS, the ${}_yM_x$, are distributed by cause of death for years 1979 and later⁵.

Over the last century, death rates have decreased substantially. The historical improvement in mortality is quantified by calculating the average annual percentage reduction (${}_yAA_x$) in the central death rate. In order to project future ${}_yM_x$, the Board of Trustees of the OASDI Trust Funds determines the ultimate average annual percentage reduction that will be realized during the projection period (${}_yAA_x^u$).

The basic mortality outputs of the MORTALITY subprocess that are used in projecting the population are probabilities of death by age and sex (q_x). The probability that a person age x will die within one year (q_x) is calculated from the central death rates (the series of ${}_yM_x$).

Period life expectancy (e_x) is defined as the average number of years of life remaining for people who are age x and are assumed to experience the assumed probabilities of death throughout their lifetime. It is generated from the probabilities of death for a given year and is a summary statistic of overall mortality for that year.

Age-adjusted death rates (*ADR*) are also used to summarize the mortality experience of a single year, making different years comparable to each other. Age-adjusted death rates are a weighted average of the ${}_yM_x$, where the weights used are the numbers of people in the corresponding age groups of the standard population, the 2000 U.S. Census resident population (${}_ySP_x$). Thus, if the age-adjusted death rate for a particular year and sex is multiplied by the total 2000 U.S.

⁵ Data needed in order to project central death rates by cause of death were obtained from Vital Statistics tabulations for years since 1979. For the years 1979-1998, adjustments were made to the distribution of the numbers of deaths by cause. The adjustments were needed in order to reflect the revision in the cause of death coding that occurred in 1999, making the data for the years 1979-1998 more comparable with the coding used for the years 1999 and later. The adjustments were based on comparability ratios published by the National Center for Health Statistics.

Census resident population, the result gives the number of deaths that would have occurred in the 2000 U.S. Census resident population if the ${}_yM_x$ for that particular year and sex had been experienced. Age-sex-adjusted death rates (*ASDR*) are calculated to summarize death rates for both sexes combined and are calculated as a weighted average of the ${}_yM_x$, where each weight is the number of people in the corresponding age and sex group of the 2000 U.S. Census resident population.

MORTALITY projects annual ${}_yM_x$ which are then used to calculate the program's additional outputs. The equations for this subprocess, 1.2.1 through 1.2.6, are given below:

$${}_yM_x = {}_yM_x(\cdot) \quad (1.2.1)$$

$${}_yAA_x = {}_yAA_x(\cdot) \quad (1.2.2)$$

$$q_x = q_x(\cdot) \quad (1.2.3)$$

$$\overset{\circ}{e}_x = \overset{\circ}{e}_x(\cdot) \quad (1.2.4)$$

$$ADR_s^z = \frac{\sum_x {}_ySP_x \cdot {}_yM_{x,s}^z}{\sum_x {}_ySP_x} \quad (1.2.5)$$

$$ASDR^z = \frac{\sum_s \sum_x {}_ySP_{x,s} \cdot {}_yM_{x,s}^z}{\sum_s \sum_x {}_ySP_{x,s}} \quad (1.2.6)$$

where ${}_yM_{x,s}^z$ refers to the central death rate between exact age x and $x+y$ by sex in year z ; ${}_ySP_x$ denotes the number of people in the standard population (male and female combined) who are between exact age x and $x+y$; and ${}_ySP_{x,s}$ denotes the number of people by sex in the standard population who are between exact age x and $x+y$.

1.2.b. *Input Data*

Trustees Assumptions -

Each year the Board of Trustees of the OASDI Trust Funds sets the ultimate assumed values for the ${}_yAA_x$ by sex, age group⁶ and cause of death⁷. The average annual percentage reductions reach their ultimate value in the 25th year of the 75-year projection period. The

⁶ Age groups are: less than 15, 15-49, 50-64, 65-84, 85+

⁷ The seven causes of death are: Heart Disease, Cancer, Vascular Disease, Violence, Respiratory Disease, Diabetes Mellitus, and Other

ultimate rates of reduction by sex, age group, and cause of death can be found in Appendix 1.2-1.

NCHS Data -

- Annual numbers of registered deaths by sex and age group for the period 1900-1978. These data are not updated. Registered deaths refer to deaths in the Death Registration area. Since 1933, the Death Registration area has included all of the U.S.
- Annual numbers of deaths by sex, age group, and cause for the period 1979-2005. Each year, a new year of data is generally received. In addition, revised data are often available for years beginning with 1999 (1999 was the starting year of the latest international classification of diseases – ICD10).
- The monthly number of births by sex for years 1938-2005. These data are updated annually, when NCHS provides an additional year of data.
- The number of infant deaths by age sex and age group⁸ for years 1938-2005. These data are updated annually, when NCHS provides an additional year of data.
- Deaths for 1995 and 1996 by sex, 4 marital statuses, and 21 age groups. The age groups are generally 5-year age groups and are as follows: 0, 1-4, 5-9, 10-14,, 95+). These data are updated as resources are available.
- The population of states in the Death Registration area by age group⁹ and sex for years 1900-1939. These data are not updated.
- The number of registered deaths by sex and age groups (85-89, 90-94, and 95+) for the years 1900- 1967. These data are not updated.

U.S. Census Bureau Data -

- Estimates of the July 1 resident population by single year of age (0 through 100+) for years 1980-2005. Each year, Census provides an additional year of data and updated data for years after the most recent decennial census.

⁸ Age groups are: under 24 hours, 1-2 days, 3-6 days, 7 -27 days, 28 days-1 month, 2 months, 3 months, ..., 11 months

⁹ Age groups are: 0, 1-4, 5-14, 15-24, ..., 75-84, 85+

- From the Current Population Survey (CPS), the population by sex, marital status, and age group¹⁰ for the years 1995 and 1996. These data are updated as resources are available.
- The resident population by sex, marital status, and age group¹¹ as of as of July 1, 1995 and 1996. These data are updated when new NCHS death data by marital status are incorporated.
- The resident population by sex and age group¹² for years 1900-1939. These data are not updated.
- The resident population at ages 75-79 and 80-84, by sex, for years 1900-1940 (at ten year intervals). These data are not updated.
- The resident population by sex and age groups¹³ for 1940-2000. These data are not updated.

Centers for Medicare and Medicaid Services Data -

- Annual numbers of deaths by sex and single year of age (ages 65 and over) for the period 1968-2005. These data are updated annually, when CMS provides an additional year of data.
- Annual numbers of Medicare enrollments (who are insured for Social Security benefits) by sex and single year of age (ages 65 and over) for the period 1968-2005. These data are updated annually, when CMS provides an additional year of data.

Other input data -

- From the previous year's Trustees Report the July 1, 1995 and 1996 Social Security area population by sex, marital status, and single year of age (5 through 100+). These data are updated when new NCHS death data by marital status are incorporated.

¹⁰ Age groups are: 15-17, 18-19, 20-24, 25-29, ..., 40-44, 45-54, ..., 65-74, and 85+

¹¹ Age groups are: 15-19, 20-24, 25-29, ..., 95+

¹² Age groups are: 0, 1-4, 5-14, 15-24, ..., 75-84, 85+

¹³ Age groups for years prior to 1980 are: 0, 1-4, 5-9, ..., 85+. For years 1980-1982, the age groups are: 0, 1-4, 5-9, ..., 95+. For years 1983 and later, the age groups are 0, 1-4, 5-9, ..., 85+.

1.2.c. Development of Output

Equation 1.2.2 - Average Annual Percentage Reduction (${}_yAA_x$)

The ${}_yAA_x$ by sex and cause are calculated based on the decline in the ${}_yM_x$ for the period 1985 through 2005 and distributed by 21 age groups¹⁴, 2 sexes, and 7 causes of death¹⁵. The values are calculated as the complement of the exponential of the slope of the least-squares line through the logarithms of the ${}_yM_x$.

The ultimate assumed values for the (${}_yAA_x^u$), as set by the Board of Trustees of the OASI and DI Trust Funds, are assumed to be reached in the 25th year of the 75-year projection period. The assumed ultimate values are specified by sex, seven causes of death, and for the following five age groups: under age 15, 15-49, 50-64, and 65-84, and 85 and older.

The values of ${}_yAA_x$ by the 21 age groups, sex, and cause for 2006 through 2008 are assumed to equal the average ${}_yAA_x$ based on the decline in the ${}_yM_x$ for the period 1985-2005. If, however, the average ${}_yAA_x$ for a particular group during the period is negative, then 75 percent of the average rate is used. For years after 2008, a method of graduation is used that causes the absolute difference between the current ${}_yAA_x$ and the ultimate ${}_yAA_x^u$ to decrease rapidly until it reaches the Trustees' ultimate assumed value, ${}_yAA_x^u$. This is accomplished by repeating the following steps for each year of the projection:

1. The absolute value of the distance between the prior year's calculated ${}_yAA_x$ and the ultimate assumed ${}_yAA_x^u$ is calculated.
2. If the ultimate assumed ${}_yAA_x^u$ is greater than the prior year's ${}_yAA_x$, 80 percent of the difference is subtracted from the ultimate assumed ${}_yAA_x^u$. If the ultimate assumed ${}_yAA_x^u$ is less than the prior year's ${}_yAA_x$, then 80 percent of the difference is added to the ultimate assumed ${}_yAA_x^u$.
3. These steps are repeated until the 25th year at which time the ${}_yAA_x$ are set equal to their ultimate assumed values.

¹⁴ Age groups are: 0, 1-4, 5-9, 10-14,, 90-95, and 95+

¹⁵ The seven causes of death are: Heart Disease, Vascular Disease, Violence, Cancer, Respiratory Disease, Diabetes Mellitus, and Other.

Equation 1.2.1 – Central Death Rates (${}_yM_x$)

Values of ${}_yM_x$ are determined for each historical and projected year by the 21 age groups, 2 sexes, and 7 causes of death. The starting year for the projections of the ${}_yM_x$ is 2005, and is the most recent data year in the historical period. However, instead of using the historical data for ${}_yM_x$ in this year as the starting point for mortality projections, starting ${}_yM_x$ are calculated to be consistent with the trend inherent in the last 12 years of available data. Each starting value for the ${}_yM_x$, by sex and cause of death, is computed as the value for the most recent year falling on a weighted least square line, where ${}_yM_x$ is regressed on year, over the last 12 years. The weights are 0.2, 0.4, 0.6, and 0.8 for the earliest four years of the 12 years and are 1.0 for all other years.

For years after 2005, ${}_yM_x$ are projected by sex and cause of death by applying the respective ${}_yAA_x$ to the prior year ${}_yM_x$.

Equations 1.2.3 – Probabilities of death (q_x)

In order to project population by age and sex, probabilities of death are applied to determine the projected number of deaths that will occur in the population. These probabilities, denoted as q_x , reflect the probability a person age x will die within one year, where x refers to age last birthday as of the beginning of each year. For each year in the historical and projection period, separate q_x series are estimated by sex.

Different methods of projecting q_x are used for age 0, for ages 1 through 4, for ages 5 through 94, and for ages 95 and above. The following descriptions provide a brief discussion of these different methods. Additional detail is provided in Actuarial Study number 120 located at the following internet site:

http://www.socialsecurity.gov/OACT/NOTES/as120/LifeTables_Foreword.html.

- *Values for q_x at Age 0:* During the first year of life, mortality starts at an extremely high level, which becomes progressively lower, unlike mortality at other ages which does not change very much within a single year of age. Thus, it is particularly important at age 0 to estimate accurately the pattern of mortality throughout the year of age, as described above, for the calculation of q_0 . For the period 1940 through the last historical year, q_0 is calculated directly from tabulations of births by month and from tabulations of deaths at ages 0, 1-2, 3-6, 7-28 days, 1 month, 2 months, ..., 11 months. After the last historical year, q_0 is calculated from ${}_1M_0$, assuming that the ratio of q_0 to ${}_1M_0$ measured for the last historical year would remain constant thereafter.
- *Values for q_x at Ages 1 – 4:* For the period 1940 through the last year of historical data, probabilities of death at each age 1 through 4 (q_x , $x = 1, 2, 3, 4$) are calculated from tabulations of births by year and from tabulations of deaths at ages 1, 2, 3, and 4 years. After the last historical year, each q_x (where $x = 1, 2, 3, 4$) is calculated from ${}_4M_1$ assuming that the ratio of q_x to ${}_4M_1$ measured for the last historical year would remain constant

thereafter.

- *Values for q_x at Ages 5 – 94:* Probabilities of death for these ages are calculated from the projected central death rates, ${}_5M_x$. As mentioned above, the calculations are discussed in detail in Actuarial Study number 120.
- *Values for q_x at Ages 95+:* It has been observed that the mortality rates of women, though lower than those of men, tend to increase faster with advancing age than those of men. An analysis of Social Security charter Old-Age Insurance beneficiaries has shown that at the very old ages mortality increases about five percent per year of age for men and about six percent per year for women. Probabilities of death at each age 95 and older are calculated as follows for men:

$$q_x = q_{x-1} \cdot \left(\frac{q_{94}}{q_{93}} \cdot \frac{99-x}{5} + 1.05 \cdot \frac{x-94}{5} \right) \quad x = 95, 96, 97, 98, 99$$

$$q_x = 1.05 \cdot q_{x-1} \quad x = 100, 101, 102, \dots$$

For women, the same formulas are used, except that 1.06 is substituted for 1.05. The larger rate of growth in female mortality would eventually, at a very high age, cause female mortality to be higher than male mortality. At the point where this crossover would occur, female mortality is set equal to male mortality.

The values of q_x used in projecting the population are based on age last birthday and are calculated by sex for ${}_{1/2}q_0$ and for q_x where x represents age last birthday and equals 0, 1, 2, ..., 99. Because life table values of probabilities of death are based on exact ages, values for q_x representing age last birthday for ages 0 through 99 are derived by averaging the life table values of probabilities of death at ages x and $x+1$. Deaths occurring during the first six months of life are calculated directly from the life tables' values and are applied to births during the year. Deaths occurring in the population at ages 100 and older are projected as a group.

In addition, probabilities of death are broken down further into marital status. Historical data indicate differential in mortality by marital status is significant. To reflect this, projected relative differences in death rates by marital status are projected to be the same as observed during calendar years 1995 and 1996.

Equation 1.2.4 –Life expectancy

Actuarial Study number 120 presents background information on the calculation of life expectancy, e_x from the probabilities of death (q_x). This study, titled Life Tables for the United

States Social Security area 1900-2100, can be accessed at the following internet site:
http://www.socialsecurity.gov/OACT/NOTES/as120/LifeTables_Foreword.html.

Appendix: 1.2-1

The Board of Trustees of the OASDI Trust Funds sets the ultimate rates of mortality reduction by sex, age group, and cause of death. These ultimate rates are listed in the tables below for the current and prior Trustees Reports. For comparison purposes, rates are also presented for an historical period.

Annual Rates of Reduction in Central Death Rates by Age Group, Sex and Cause

	Historical	Alternative II		Historical	Alternative II	
		2008TR	2009TR		2008TR	2009TR
	1979 to 2005	2032- 2082	2033- 2083	1979 to 2005	2032- 2082	2033- 2083
Under Age 15	Male			Female		
Heart Disease	3.11	1.0	2.2	2.84	1.0	2.2
Cancer	2.64	2.0	1.5	2.23	2.0	1.5
Vascular Disease	1.22	0.6	2.5	1.29	0.6	2.5
Violence	2.96	0.9	1.0	2.36	0.9	1.0
Respiratory Disease	3.16	2.5	2.0	3.17	2.5	2.0
Diabetes Mellitus	1.25	1.8	1.0	2.72	1.8	1.0
Other	2.66	1.8	1.7	2.57	1.8	1.7
Total	2.74	1.55	1.54	2.54	1.58	1.56
Ages 15 - 49	Male			Female		
Heart Disease	2.35	2.3	1.5	1.08	2.2	1.5
Cancer	1.72	1.0	1.5	1.58	1.0	1.5
Vascular Disease	1.64	1.8	1.8	1.83	1.8	1.8
Violence	1.10	0.8	0.7	0.41	0.8	0.6
Respiratory Disease	0.88	0.5	0.5	-0.19	0.5	0.5
Diabetes Mellitus	-1.55	0.3	0.3	-0.89	0.3	0.3
Other	-0.77	0.8	0.8	-0.83	0.8	0.8
Total	1.01	0.93	0.88	0.58	0.93	0.91
Ages 50 - 64	Male			Female		
Heart Disease	3.34	2.2	2.2	2.63	2.2	2.2
Cancer	1.45	1.0	1.5	1.09	1.0	1.5
Vascular Disease	2.35	1.5	2.0	2.10	1.6	2.0
Violence	1.05	0.8	0.5	0.55	0.8	0.5
Respiratory Disease	1.22	0.7	0.7	-0.94	0.7	0.7
Diabetes Mellitus	-2.65	0.3	0.3	-1.35	0.3	0.3
Other	0.08	0.9	0.6	-0.18	0.9	0.6
Total	1.80	1.08	1.08	1.08	1.03	1.08
Ages 65 - 84	Male			Female		
Heart Disease	2.91	1.8	2.2	2.63	1.8	2.2
Cancer	0.42	0.5	0.8	-0.52	0.5	1.0
Vascular Disease	2.76	2.5	2.0	2.10	2.6	2.0

Violence	0.82	1.2	0.5	0.15	1.2	0.5
Respiratory Disease	0.13	0.2	0.3	-3.05	0.2	0.3
Diabetes Mellitus	-2.48	0.6	0.3	-1.44	0.6	0.3
Other	-0.55	0.3	0.3	-1.71	0.3	0.3
Total	1.35	0.71	0.80	0.50	0.67	0.80
Ages 85 and older	Male			Female		
Heart Disease	1.05	1.2	1.2	1.22	1.2	1.2
Cancer	-0.85	0.4	0.5	-0.84	0.3	0.5
Vascular Disease	1.76	1.9	1.8	1.76	1.8	1.8
Violence	-0.74	0.9	0.3	-0.84	0.9	0.3
Respiratory Disease	-1.69	0.2	0.2	-2.95	0.2	0.2
Diabetes Mellitus	-3.18	0.6	0.2	-1.93	0.5	0.2
Other	-2.34	0.2	0.2	-3.52	0.2	0.2
Total	-0.07	0.61	0.59	-0.15	0.59	0.58

1.3. IMMIGRATION

1.3.a. Overview

Immigration consists of legal immigration, legal emigration, other immigration and other emigration. Legal immigration is defined as those persons who have been admitted into the United States and been granted legal permanent resident (LPR) status. Legal emigration consists of legal permanent residents and U.S. Citizens who depart the Social Security area population to reside elsewhere. Other immigrants include persons, other than LPRs, who enter the U.S. and reside for 6 months or longer such as undocumented immigrants, temporary workers, and foreign students. Other emigration includes other immigrants who depart the Social Security area for another country in addition to those who adjust status to become an LPR.

For each year z of the projection period, the IMMIGRATION subprocess produces estimates of legal immigration (L^z), legal emigration (E^z), and other immigration (O^z), by age and sex, based on assumptions set by the Trustees for each category. Estimates of projected other emigration are not developed in this subprocess, but rather are developed in the PROJECTED POPULATION subprocess, documented in section 1.7. In addition, the IMMIGRATION subprocess disaggregates the estimates of L^z into those who have been admitted into the United States during the year (NEW^z) and those who adjusted from the other-immigrant population to LPR status (AOS^z)

Each fiscal year,¹⁶ the Department of Homeland Security (DHS) collects data on the number of persons granted LPR status by age, sex, and class of admission. The U.S Census Bureau provided OCACT with an unpublished estimate of the annual number of legal emigrants, by sex and age, based on the change between the 1980 and 1990 census. The Census Bureau also estimated the aggregate number of net other immigrants who entered the country during 1975-1980, by age and sex. These historical data are used as a basis for developing age-sex distributions that are applied to the Trustees' aggregate immigration assumptions to produce annual immigration and emigration estimates by age and sex.

The primary equations of IMMIGRATION, by age (x) and sex(s), for each year (z) of the 75-year projection period are summarized below:

$$NEW_{x,s}^z = NEW_{x,s}^z(\cdot) \quad (1.3.1)$$

$$AOS_{x,s}^z = AOS_{x,s}^z(\cdot) \quad (1.3.2)$$

$$L_{x,s}^z = NEW_{x,s}^z + AOS_{x,s}^z \quad (1.3.3)$$

¹⁶ The federal fiscal year begins on October 1 of the previous calendar year and ends on September 30 of the specified calendar year.

$$E_{x,s}^z = E_{x,s}^z(\cdot) \quad (1.3.4)$$

$$NL_{x,s}^z = L_{x,s}^z - E_{x,s}^z \quad (1.3.5)$$

$$O_{x,s}^z = O_{x,s}^z(\cdot) \quad (1.3.6)$$

$$OP_{x,s}^z = OP_{x,s}^z(\cdot) \quad (1.3.7)$$

1.3.b. *Input Data*

Trustees Assumptions -

Each year the Board of Trustees of the OASDI Trust Funds specifies the total annual assumed values for legal immigration, legal emigration, and other immigration. For the 2009 Trustees Report, the ultimate values for legal immigration and emigration are 1,000,000 and 250,000, respectively (both reached in 2010). For other immigration, the Trustees set the annual level at 1,500,000 persons per year for each year of the projection period.

Long-Range OASDI Projection Data -

Demography

- Historical probabilities of death by age last birthday (including perinatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex for years 1941-2006. These data are updated each year.
- Social Security area population by single year of age (0-99 and 100+), sex, and marital status for years 1941-2006. These data are updated each year.

Office of Immigration Statistics –

- Historical legal immigration by fiscal year (1941-1995), 5-year age group (0-4, 5-9, ..., 75-79, 80-84), and sex. These data will not be updated.
- Legalizations due to IRCA by 5-year age group and sex for the years 1989-1996. These data will not be updated.

Department of Homeland Security –

- Historical legal immigration by fiscal year (1996-2007), single year of age (0 through 99), sex, and class of admission (New Arrival, Adjustment of Status, Refugee, and Asylee). These data are updated annually, with DHS providing an additional year of data each year.
- Recent arrivals by 5-year age group and sex for 2006. These data are updated

occasionally (based on having new data from an outside source and on OCACT resource time constraints).

- An estimate of the size of the other immigrant population as of January 1, 2006. These data are updated occasionally (based on having new data from an outside source and on OCACT resource time constraints).

U.S. Census Bureau –

- Unpublished estimates of annual legal emigration by five-year age groups (0-4, 5-9, ..., 80-84) and sex for 1990 based on the change between the 1980 and 1990 census. These data are updated occasionally (based on having new data from an outside source and on OCACT resource time constraints).
- Unpublished tabulations of the cumulative number of net other immigrants entering the country during 1975-1980 by five-year age groups (0-4, 5-9, ..., 80-84) and sex. These data are updated occasionally (based on having new data from an outside source and on OCACT resource time constraints).

Other input data -

- Legal emigration conversion factors. These estimates were developed internally by five-year age groups (0-4, 5-9, ..., 80-84) and sex to reflect the fact that the estimated number of people leaving the United States is not equivalent to the number of people leaving the Social Security area. These data are updated when annual legal emigration estimates are updated (see above).
- Internally developed total estimated number of non-immigrants in 1990 and 2000
- Internally developed persistence factors
- Estimate of the residual immigrant population by age group (0-17, 18-49, 50-64 and 65+) and sex as of January 1, 1990 and 2000, from Population Division Working Paper 61¹⁷. These data will not be updated.

1.3.c. *Development of Output*

Equations 1.3.1 and 1.3.2 – Legal Immigration

The Trustees specify the aggregate amount of legal immigration for each year of the 75-year projection period. In order to incorporate the numbers of new immigrants into the Social

¹⁷ See “Evaluating Components of International Migration: The Residual Foreign Born” by Joseph Costanzo, Cynthia J. Davis, Caribert Irazi, Daniel M. Goodkind, and Roberto R. Ramirez. Issued January 2002.

Security area population projections, the total level of new immigrants is disaggregated by age and sex.

There are two ways for an immigrant to be admitted into the U.S. for lawful permanent residence:

- (1) New arrivals such as persons living abroad who are granted an LPR visa and then enter the U.S. through a port of entry. Refugees and asylees that are granted LPR status are also treated as new arrivals in the OCACT model.
- (2) Adjustments of status, who are people already residing in the U.S. as other immigrants and have an application for adjustment to LPR status approved by DHS.

The DHS provides data on legal immigrants by sex, single year of age, classification of admission, and fiscal year of entry. The 10 most recent years of data are used to calculate separate age-sex distributions for both new arrivals and adjustments of status by taking the following steps:

1. The data for the last ten years of single-year of age data are combined into five-year age groups.
2. Refugee and Asylee LPR admissions are subtracted from the adjustment of status data and added into the new arrival category.
3. The data are converted from fiscal year data to calendar year data.
4. For each class of admission, new arrival and adjustment of status, the historical data for the last 10 years (from 1998-2007) are combined into an average age-sex distribution.
5. The distributions by five year age group are disaggregated into a single year of age-sex distribution using the H.S. Beers method of interpolation.

Based on trends in the historical data, it is assumed that fifty percent of all new LPR admissions will be new arrivals and the other fifty percent will consist of adjustments of status. Thus, $NEW_{x,s}^z$, the expected number of new arrival legal immigrants by age (x) and sex (s), is calculated by applying the age-sex distribution for new arrivals to one half of the Trustees assumed level of legal immigration. The remaining half of the Trustees' assumed number of legal immigrants is multiplied by the age-sex distribution of adjustments of status to calculate $AOS_{x,s}^z$.

Equation 1.3.4 – Legal Emigration

The Trustees specify the aggregate amount of legal emigration for each year of the projection period. This is done by setting the ratio of emigration to legal immigration. For the 2009 Trustees Report, the ratio is set at 25 percent.

In order to produce the number of emigrants from the Social Security area population, the total level of emigrants is disaggregated by age and sex. The disaggregation is based on a distribution of emigrants, by sex and five-year age groups, provided to OCACT in unpublished estimates by Census that are based on changes between the 1980 and 1990 census. Since the emigration numbers estimated by Census are for all people leaving the United States, they are adjusted downward by a series of conversion factors so the data correspond to the number of people leaving the Social Security area population.

For each sex, the Beers formula is used to interpolate and distribute each five-year age group into a single year of age-sex distribution, $EDIST_{x,s}$. For each projection year, this distribution is used to distribute the assumed level of total legal emigrants by age and sex using the following equation:

$$E_{x,s}^z = .25 \left(\sum_{s=m}^f \sum_{x=0}^{84} L_{x,s}^z \right) * EDIST_{x,s}$$

Equation 1.3.6 – Other Immigration

The term “other immigration” refers to persons entering the U.S. in a manner other than being lawfully admitted for permanent residence. This includes temporary immigrants (persons legally admitted for a limited period of time) in addition to undocumented immigrants living in the U.S.

The Trustees specify the aggregate amount of other immigrants for each year of the projection period. For each projection year, an age-sex distribution is used to distribute this assumption by age and sex. This age-sex distribution is denoted as $ODIST_{x,s}$ and is developed from a weighted average of the age-sex distribution of adjustments of status from Equation 1.3.2 and an age-sex distribution of non-adjusting other immigrants.

The age distribution of the adjustments of status is modified to incorporate a five year set back based on the assumption that adjustments of status enter the U.S. five years earlier on average. This age-sex distribution is denoted as $ODIST1_{x,s}$. The age distribution of non-adjusting other immigrants, $ODIST2_{x,s}$, is derived from an unpublished census estimate of net other immigration during the period 1975-80 using internally developed levels of persistence. The two age distributions are then combined using the following formula:

$$ODIST_{x,s} = w(ODIST1_{x,s}) + (1 - w)ODIST2_{x,s}$$

Where w is a weighting factor equal to the Trustees ultimate assumed level of adjustments of status increased by a factor¹⁸ of 1.25 divided by the Trustees ultimate assumed level of other

¹⁸ This factor is meant to take into account that a large number of people who have the potential to adjust to LPR status may die or return to their native country prior to doing so.

immigration. For the 2009 Trustees Report, this w equals 0.417. Thus, for each year (z) other immigration is defined by the following equation:

$$O_{x,s}^z = 1,500,000 * ODIST_{x,s}$$

Equation 1.3.7 – Other Than Legal Population

Other than legal population, $OP_{x,s}^z$, is calculated for years 1990 and 2000 as the sum of the unauthorized population and the non-immigrant¹⁹ population for that given year. The unauthorized population for these two years is calculated by taking the following steps:

1. Compute the foreign born non-citizen age distribution by taking the foreign born non-citizen population by 5-year age groups (from the Current Population Survey) and applying the Beers method to get single year of age values,
2. Apply this foreign born non-citizen age distribution to the Census residual unauthorized population (from Technical Working Paper 61), after first grouping ages 18-49 of the residual unauthorized population,
3. Regroup the single age values into 5-year age groups, and smooth the data by applying the Beers method to get single year of ages 0-84,
4. Compute single year of age values for ages 85-100 by applying the same age distribution as the 85-100 SSA population for 2000.

The non-immigrant population is calculated by using the recent arrivals by 5-year age group (from DHS) to break out the total estimate of non-immigrants (from DHS) by 5-year age groups, then applying the Beers method to get single year of age values. Note that in both of the above cases where the Beers method is used, a separate method is used to break out the 0-4 age group, using internally developed factors, since these ages for immigrants would behave differently than the normal population would behave using the Beers method.

Using the calculated total level of the other than legal population in 2000, the DHS estimate of the total other than legal population in 2006, probabilities of death, levels of other immigrants, and adjustments of status, the total number of other emigrants, $OD_{x,s}^z$, for 2000-2006 are calculated. For the 2009 Trustees Report, the assumed fixed level of other emigrants is about 460,000 per year for 2000-2006. The age distribution of other emigrants is internally calculated by applying assumed levels of persistence to the other immigration age distribution.

To calculate the other than legal population by age and sex for a given year 2001-2007, the following equation is used:

¹⁹ The term non-immigrant refers to a foreign national who has entered the United States temporarily for a specific purpose (as defined by DHS).

$$OP_{x,s}^z = OP_{x-1,s}^{z-1} * (1 - q_{x-1,s}^{z-1}) + O_{x-1,s}^{z-1} - OD_{x-1,s}^{z-1} - AOS_{x-1,s}^{z-1}$$

Where $q_{x,s}^z$ is the probability of death (calculated in the MORTALITY subprocess), $O_{x,s}^z$ is the other immigration, $OD_{x,s}^z$ is the number of other departures, and $AOS_{x,s}^z$ is the number of adjustments of status.

Projected other departures are calculated later in the PROJECTED POPULATION subprocess, using two key inputs that are calculated in the IMMIGRATION subprocess. Ideally, projected other departures would be modeled utilizing rates of emigration by age, sex, and year of entry. However due to the scarcity of available data on other immigrants, calculating rates of emigration by this level of detail is not feasible at this time. As an alternative, the total level of departures was modeled based on changes in the estimated size of the other immigration population during the time period 2000-05. The total number of other departures during that time was split into two pieces (1) a fixed component that is expressed as a percentage of recent arrivals and (2) a rate based component that varies by age and sex. The rate based component is calculated later in the PROJECTED POPULATION subprocess.

The fixed component, $FD_{x,s}^z$, is calculated directly from the age distribution of other immigrants by assuming that the number of fixed exits will come from relatively recent arrivals who do not adjust status. The following equation shows the calculation:

$$FD_{x,s}^z = 0.10(O_{x-2,s}^z) * Nadj_{x-2,s}$$

Where $O_{x,s}^z$ is the assumed level of other immigration for a given year z and $Nadj_{x,s}$ is equal to the percentage of new other immigrants that are not expected to have the potential to adjust status. Since $O_{x,s}^z$ is constant for each year of the long-range projection period, the values of $FD_{x,s}^z$ remain fixed for each year z.

The remaining numbers of departures, $RD_{x,s}^z$, are assumed to be directly influenced by the size of the exposed other immigrant population. Thus, rates of emigration by age (x) and sex (s), $r_{x,s}$, were calculated and averaged over the period 2000-05.

$$r_{x,s} = Average \left[\frac{OD_{x,s}^z - FD_{x,s}^z}{OP_{x,s}^z * (1 - q_{x,s}^z) - FD_{x,s}^z - AOS_{x,s}^z} \right] \text{ for } z = 2000, 2005$$

1.4. HISTORICAL POPULATION

1.4.a. Overview

For each historical year, the HISTORICAL subprocess provides estimates of the Social Security area population by for the period 1941 through 2007. The Social Security area population consists of:

- U.S. resident population and armed forces overseas *plus*
- Net census undercount *plus*
- Civilian residents of Puerto Rico, the Virgin Islands, Guam, the Northern Mariana Islands, and American Samoa *plus*
- Federal civilian employees overseas *plus*
- Dependents of armed forces and federal civilian employees overseas *plus*
- Residual beneficiaries living abroad *plus*
- Other citizens overseas

The U.S. Census Bureau collects population data and tabulates it by age, sex, and marital status every ten years for the decennial census. The decennial census includes data from the 50 states, the District of Columbia, U.S. territories and citizens living abroad. Each subsequent year, the Census Bureau publishes an estimate of the post-censal population. This subprocess combines these censal estimates, along with the estimates of the other components of the Social Security area population listed above and components of change described in sections 1.1 to 1.3 to develop smoothed estimates of the Social Security area population by single year of age and sex ($P_{x,s}^z$). Combining this population with an estimated marital status matrix provides the Social Security area population by single year of age, sex, and marital status ($P_{x,s,m}^z$). These estimates are then used as the basis for the PROJECTED POPULATION subprocess described in section 1.7. The primary equations for this subprocess, 1.4.1 and 1.4.2, are given below:

$$P_{x,s}^z = P_{x,s}^z (\cdot) \quad (1.4.1)$$

$$P_{x,s,m}^z = P_{x,s,m}^z (\cdot) \quad (1.4.2)$$

1.4.b. Input Data

Long-Range OASDI Projection Data -

Demography

- Probabilities of death from MORTALITY, by age last birthday and sex, for years 1941-2006. These data are updated every year.
- The number of new legal immigrants by age and sex for years 1941-2006. These data are from the IMMIGRATION subprocess and are updated each year.
- The number of “other immigrants” legalized under the Immigration Reform and Control Act of 1986 (IRCA) from IMMIGRATION. These data are reproduced each year.
- The number of “other immigrants” by single year of age and sex for 1990 and 2000-2007 from the IMMIGRATION subprocess. An additional year of data is added for each Trustees Report, and historical numbers are updated when they become available.

U.S. Census Bureau Data -

- Decennial census total population estimates. A new estimate is available every ten years.
- Estimates of U.S resident population and Armed Forces population overseas as of each July 1 (1980-2007) by sex and single-year of age 0 through 99, and ages 100 and older. Each year, the U.S. Census Bureau restates the data back to the most recent decennial census and includes one additional year of data.
- Estimates of the 1940, 1950, and 1960 counts of the U.S. population and armed forces overseas by sex and single year of age (0-84) and the age group 85+. These data are not updated.
- Estimates of the population by age group²⁰, sex, and marital status for years 1940-2007. An additional year of data is added for each Trustees Report.
- Estimates of the population by marital status, which have more age groups than the CPS, and sex for years 1982-1989 and 1992-2000. These data are not updated.

²⁰ Age groups are: 15-17, 18-19, 20-24, 25-29, ..., 55-64, 65-74, 75-84, 85+

- Undercount factors by single year of age (0-85+) and sex, estimated using post-censal survey data. These data are updated after each decennial census.
- The total annual civilian population estimates for Puerto Rico, Virgin Islands, Guam, Northern Marianas, and American Samoa for years 1940-2007. For each Trustees Report, an additional data year is downloaded from the U.S. Census Bureau's international database.
- Estimates of U.S resident population and Armed Forces population overseas as of each July 1 (1940-1979) by sex and single-year of age through 84, and for the group aged 85 and older. These data are not updated.
- From the U.S. Census Bureau 1980 Census of Population, Subject Report on Marital Status No. PC80-2-4C, number of existing marriages in 1980 by age group of husband crossed with age group of wife. These data are not updated.

Other input data -

- From the Centers for Medicare and Medicaid (CMS), the population insured for Social Security benefits by single year of age (85-100+) and sex for years 1968-2007. The last year of data is provisional. Each year, CMS provides a final year of data to replace the prior year's provisional data, and a new provisional year of data.
- The SSA Annual Statistical Supplement provides estimates of the total number of OASDI Beneficiaries living abroad for years 1987-2007. For each Trustees Report, an additional year of data is available.
- Output from the Urban Institute's microsimulation model, POLISIM, regarding marriage prevalence for the period 1980-2007 by age of husband and age of wife. For each Trustees Report, the data may be revised and an additional year of data may be added.
- "Other immigrant" population by single year of age and sex for 1980. 3.3 million "other immigrants" are assumed for 1980 as estimated by the Census Bureau. The distribution to break the 3.3 million "other immigrants" into single year of age and sex is derived from the 2007 TR otlpop.hist file. Since this file was originally based off a Census Bureau single year of age and sex study of "other immigrants" entering from 1975 to 1979 only, it was assumed that 1.1 million "other immigrants" entered in each of the five year periods from 1965-1969, 1970-1974, and 1975-1979 with the same age distribution as year 1980 in the 2007 TR otlpop.hist file. These data are not updated.
- From Census Bureau, total net number of "other immigrants" each year from 1980-

1999. From 1980-1989, the value is assumed to be 375,000. From 1990-1999, the value is assumed to be 550,000. These data are not updated.

- Derived from Census Bureau data, distribution for distributing total net “other immigrants” into single year of age and sex from the 2007 TR illeg.alt2 file. These data are not updated.

1.4.c. Development of Output

Equation 1.4.1 – Historical Population by age and sex ($P_{x,s}^z$)

The Census Bureau's estimate of the residents of the 50 States, D.C., and U.S. Armed Forces overseas is used as a basis for calculating $P_{x,s}^z$. The base estimate is adjusted for net census undercount and increased for other U.S. citizens living abroad (including residents of US territories) and for non-citizens living abroad who are insured for Social Security benefits.

The estimates of the number of residents of the fifty States and D.C. and Armed Forces overseas, as of July 1 of each year, by sex for single years of age through 84, and for the group aged 85 or older, are obtained from the Census Bureau. Adjustments for net census undercount are estimated using post-censal survey data from the Census Bureau. The numbers of persons in the other components of the Social Security area as of July 1 are estimated by sex for single years of age through 84, and for the group aged 85 or older, from data of varying detail. Numbers of civilian residents of Puerto Rico, the Virgin Islands, Guam, American Samoa, and the Northern Mariana Islands are estimated from data obtained from the Census Bureau. Numbers of Federal civilian employees overseas, dependents of these Federal civilian employees, dependents of Armed Forces overseas, and other citizens overseas covered by Social Security are also based on estimates used by the Census Bureau. The overlap among the components, believed to be small, is ignored.

The first step of the process is to estimate $P_{x,s}^z$ as of January 1st for certain tab years (1941, 1951, 1957, 1961, each decennial Census year from 1970 through 2000, and the last year of historical data [2007 for the 2009 Trustees Report]). For ages 0-84, $P_{x,s}^z$ for each tab year is set equal to the averaged surrounding July 1 U.S. population and armed forces overseas counts from the Census Bureau plus an undercount adjustment plus other component populations. For ages 85 and over, $P_{x,s}^z$ for each tab year is set equal to [Built Up Pops Age x, Sex s] * [Total USAF 85+]/[Total Built Up 85+ for Sex s] where the built up estimates are created by taking into account deaths and immigration from the previous tab year and [Total USAF 85+] is the sum of the averaged surrounding July 1 U.S. population and armed forces overseas counts from the Census Bureau plus an undercount adjustment plus other component populations for ages 85 and over.

For years between the tab years, populations are estimated taking into account the components of changes due to births, deaths, and legal immigration during that time period. These estimates are then multiplied by the appropriate age-sex-specific ratios so that the error of closure at the tab years is eliminated.

This subprocess also estimates historical levels of “other immigrants” in the population by age and sex for input to the Economics process. Census estimated that in 1980 there were 3.3 million other resident immigrants living in the U.S. The age distribution is derived from the 2007 TR otipop.hist 1980 values. An adjustment is made to correct for the fact that the 2007 TR file age distribution was only based on immigrants who entered within the 1975 – 1979 period. This adjustment assumes that 1.1 million “other immigrants” entered with the same age distribution implied by the 2007 TR otipop.hist file during each of the periods 1965-1969, 1970-1974, and 1975-1979. “Other immigrants” for 1990 and 2000-2007 are calculated in the IMMIGRATION subprocess. “Other immigrant” populations for 1981-1989 and 1991-1999 are calculated in a similar manner as the total populations between tab years are calculated (“other immigrant” tab years are 1980, 1990, and 2000). For each non-tab year, the estimates by single age and sex are brought forward from the previous year by adding annual numbers of net other immigrants, subtracting deaths for mortality (using the same death rates as the total population), and subtracting the number legalized by the Immigration Reform and Control Act of 1986. Then, these estimates are multiplied by the appropriate age-sex-specific ratios so that the error of closure at the tab years is eliminated. Total annual numbers of net other immigrants are assumed to be 375,000 from 1980 to 1989 and 550,000 from 1990 to 1999 and are disaggregated by single age and sex by using an age/sex distribution. The historical “other immigrant” population is not disaggregated further by marital status but the total population is as discussed below.

Equation 1.4.2 – Historical Population by age, sex, and marital status ($P_{x,s,m}^z$)

Since eligibility for auxiliary benefits is dependent on marital status, the Social Security area population is disaggregated by marital status. The four marital states are defined as single (having never been married), married, widowed, and divorced.

The distribution of the number of existing marriages is based on the 1980 Census Marital Status Report, which contains the number of married couples in 1980 by age group of husband crossed with age group of wife. Additional tabulations from the POLISM model for 1980 through 2007 are incorporated to adjust these marital prevalence grids for changes since 1980. Multiplying the previous year values by the ratio the current year POLISM value to the previous year POLISM value ensures that the 1980 and later grids are consistent with the pre-1980 grids. The grids are transformed from age grouped numbers to single year of age figures from ages 14 to 100+ for husband and wife using the two dimensional H.S. Beers method of interpolation.

Percentages of single, married, widowed, and divorced persons are calculated by taking the estimate for each marital status category and dividing them by the total number of people for each age group and sex from either the CPS or the more detailed Census numbers, if available.

Then, for each sex, if one age group has a higher or lower percentage than the surrounding age groups, an average of the surrounding groups replaces the original value. These percentages are multiplied by the total populations calculated in Equation 1.4.1 for each age, sex, and year to get a preliminary population for each age, sex, and marital status.

To keep the marriage prevalence grids and the marital status percentages smooth, several smoothing algorithms are used. First, the married population is adjusted so that the number of married males equals the number of married females. Next, the marital prevalence grids are smoothed so that each age of husband crossed with age of wife cell is an average of values in its diagonal in the grid. Finally, the number of married persons for each age and sex is set equal to the marginal total of the associated year's marital prevalence grid.

1.5. MARRIAGE

1.5.a Overview

The National Center for Health Statistics (NCHS) collected detailed data on the annual number of new marriages in the Marriage Registration Area (MRA), by age of husband crossed with age of wife, for the period 1978 through 1988 (excluding 1980). In 1988, the MRA consisted of 42 States and D.C. and accounted for 80 percent of all marriages in the U.S. Estimates of the unmarried population in the MRA were obtained from NCHS by age and sex. Marriage rates for this period are calculated from these data.

NCHS stopped collecting data on the annual number of new marriages in the MRA in 1989. Less detailed data on new marriages from a subset of the MRA were obtained for the years 1989-1995. These data are used to determine marriage rates by adjusting the more detailed age-of-husband crossed with age-of-wife data from the earlier years to match the aggregated levels for these years.

Age-specific marriage rates ($\hat{m}_{x,y}^z$) for a given year z are defined as the ratio of (1) number of marriages for given age-of-husband (x) crossed with age-of-wife (y) to (2) a theoretical midyear unmarried population at that age ($P_{x,y}^z$). The theoretical midyear population is defined as the geometric mean²¹ of the midyear unmarried males and unmarried females.

An age-adjusted central marriage rate (\hat{AMR}^z) summarizes the $\hat{m}_{x,y}^z$ for a given year. The standard population chosen for age adjusting is the unmarried males and unmarried females in the Marriage Registration Area (MRA) as of July 1, 1982. The first step in calculating the total age-adjusted central marriage rate for a particular year is to determine an expected number of marriages by applying the age-of-husband age-of-wife specific central marriage rates for that year to the square root of the product of the corresponding age groups in the standard population. The \hat{AMR}^z is then obtained by dividing:

- The expected number of marriages by
- The square root of the product of (a) the number of unmarried males, ages 15 and older, and (b) the unmarried females, ages 15 and older in the standard population.

The MARRIAGE subprocess projects annual $\hat{m}_{x,y}^z$ by age-of-husband crossed with age-of-wife. The equations for this subprocess, 1.5.1 and 1.5.2, are given below:

$$\hat{m}_{x,y}^z = \hat{m}_{x,y}^z (\cdot) \quad (1.5.1)$$

²¹ The square root of the product of the midyear unmarried male and unmarried female populations.

$$\hat{AMR}^z = \frac{\sum_{x,y} P_{x,y}^S \cdot \hat{m}_{x,y}^z}{\sum_{x,y} P_{x,y}^S} \quad (1.5.2)$$

where x and y refer to the age of males and females, respectively, and $P_{x,y}^S$ is the theoretical unmarried population in the MRA as of July 1, 1982 (the square root of the product of the corresponding age groups in the standard population).

1.5.b. Input Data

Long-Range OASDI Projection Data -

Demography

- Estimates of the Social Security area population as of January 1, by age, sex, and marital status for years 1978-1988, excluding 1980. These data are updated each year based on output of the HISTORICAL POPULATION subprocess.
- Estimates of the Social Security area population as of January 1, by age, sex, and marital status for years 1989-2007. These data are updated each year based on output of the HISTORICAL POPULATION subprocess.

Assumptions -

For each Trustees Report ultimate values for the \hat{AMR}^z are assumed. The \hat{AMR}^z reaches its ultimate value in the 25th year of the 75-year projection period. For 2009, the intermediate ultimate \hat{AMR}^z assumption is 4,000 per 100,000 unmarried couples.

NCHS Data -

- Number of new marriages in the MRA, by age-of-husband crossed with age-of-wife, for calendar years 1978 through 1988, excluding 1980. These data are no longer available for years after 1988. The data vary in detail by year. They are broken out by single year age-of-husband crossed with single year age-of-wife for many ages (particularly younger ages).
- Number of unmarried males and females in the MRA for calendar years 1978 through 1988, excluding 1980. These data are no longer available for years after 1988. The data are generally broken out by single year age for ages under 40 and by age groups 40-44, 45-49, 50-54, 55-59, 60-64, 65-74, and 75+.

- Number of new marriages, in a subset of the MRA, by age-group-of-husband crossed with age-group-of-wife (age groups include 15-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), for calendar years 1989-1995. These data are updated as data becomes available and internal resources are sufficient to examine and interpret such new data.
- The total number of new marriages in the United States for the period 1978-2007. Each year, NCHS publishes the total number of marriages for a more recent year.
- Number of new marriages in the MRA for years 1979 and 1981-1988 by age group (age groups include 14-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), sex, and prior marital status (single, widowed, and divorced). These data are no longer available for years after 1988.
- Number of unmarried people in the MRA for years 1979 and 1981-1988 by age group (age groups include 14-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+), sex, and prior marital status (single, widowed, and divorced). These data are no longer available for years after 1988.

1.5.c. *Development of Output*

Equation 1.5.1 -

Age-specific marriage rates are determined for a given age-of-husband crossed with age-of-wife, where ages range from 14 through 100+. The historical period includes years of complete NCHS data on the number of marriages and the unmarried population in the Marriage Registration Area (MRA) for the period 1978 through 1988, excluding 1980. Provisional data are used for the period 1989 through 1995 and total number of marriages is used for the period 1996 through 2007. The projection period of the MARRIAGE subprocess begins one year after the last historical data year.

The historical age-specific marriage rates are calculated for each year in the historical period based on NCHS data of the number of new marriages by age-of-husband crossed with age-of-wife and the number of unmarried persons by age and sex. The formula used in the calculations is given below:

$$\hat{m}_{x,y}^z = \frac{\hat{M}_{x,y}^z}{P_{x,y}^z}, \text{ where}$$

- x refers to age of males and y refers to the age of females.
- $\hat{M}_{x,y}^z$ is the number of marriages in year z ;
- $(P_{x,y}^z)$ is the geometric mean²² of the midyear unmarried males and unmarried females in year z ; and

²²The square root of the product of the midyear unmarried male and unmarried female populations.

The rates for the period 1978 through 1988²³ are then averaged, graduated, and loaded into an 87 by 87 matrix (age-of-husband crossed with age-of-wife for ages 14 through 100+), denoted as MarGrid. This matrix is used in the calculation of the age-specific marriage rates for all later provisional years and the years in the projection period.

For the first part of the provisional period, 1989-1995, NCHS provided data on the number of marriages in a subset of the MRA by age-group-of-husband crossed with age-group-of-wife (age groups include 15-19, 20-24, 25-29, 30-34, 35-44, 45-54, 55-64, and 65+). These data are used to change the distribution of MarGrid by these age groups. For each age-group-of-husband crossed with age-group-of-wife, the more detailed marriage rates in MarGrid that are contained within this group are adjusted so that the number of marriages obtained by using the rates in MarGrid match the number implied by the MRA subset.

For each year of the provisional period, an expected total number of marriages is calculated by multiplying the rates in the MarGrid (or the adjusted MarGrid for years 1989-1995) by the corresponding geometric mean of the unmarried males and unmarried females in the Social Security area population. All rates in MarGrid (or the adjusted MarGrid for years 1989-1995) are then proportionally adjusted to correspond to the total number of marriages estimated in the year for the Social Security area population. This estimate is obtained by increasing the number of marriages reported in the U.S. to reflect the difference between the Social Security area population and the U.S. population.

The provisional age-specific rates are then graduated using the Whittaker-Henderson method and are used to calculate the age-adjusted rates for each year. The age-adjusted marriage rates are expected to reach their ultimate value in the 25th year of the 75-year projection period. Rather than use the last year of provisional data to calculate the starting rate, the rates for the past five years are averaged to derive the starting value. The annual rate of change in the age-adjusted marriage rate is calculated by taking the 26th root of the ratio of the ultimate value and the starting value. Thus, to calculate the rate for a projected year, the rate of change is applied to the prior year's rate (or the starting value for the first year of the projection period).

To obtain the age-of-husband-age-of-wife-specific rates for a particular year from the age-adjusted rate projected for that year, the age-of-husband-age-of-wife-specific rates in MarGrid are proportionally scaled so as to produce the age-adjusted rate for the particular year.

A complete projection of age-of-husband-age-of-wife-specific marriage rates was not done separately for each previous marital status. However, experience data indicate that the differential in marriage rates by prior marital status is significant. Thus, future relative differences in marriage rates by prior marital status are assumed to be the same as the average of those experienced during 1979 and 1981-1988.

²³ Data for 1980 is not available and is excluded from the calculations.

1.6. DIVORCE

1.6.a. Overview

For the period 1979 through 1988, the National Center for Health Statistics (NCHS) collected data on the annual number of divorces in the Divorce Registration Area (DRA), by age-group-of-husband crossed with age-group-of-wife. In 1988, the DRA consisted of 31 States and accounted for about 48 percent of all divorces in the U.S. These data are then inflated to represent an estimate of the total number of divorces in the Social Security area. This estimate for the Social Security area is based on the total number of divorces during the corresponding calendar year in the 50 States, the District of Columbia, Puerto Rico, and the Virgin Islands. Divorce rates for this period are calculated using this adjusted data on number of divorces and estimates of the married population by age and sex in the Social Security area.

An age-of-husband crossed with age-of-wife specific divorce rate ($\hat{d}_{x,y}^z$) for a given year z is defined as the ratio of (1) the number of divorces in the Social Security area for the given age of husband and wife ($\hat{D}_{x,y}^z$) to (2) the corresponding number of married couples in the Social Security area ($P_{x,y}^z$) with the given age of husband (x) and wife (y). An age-adjusted central divorce rate ($\hat{ADR}_{x,y}^z$) summarizes the $\hat{d}_{x,y}^z$ for a given year.

The \hat{ADR}^z is calculated by determining the expected number of divorces by applying:

- The age-of-husband crossed with age-of-wife specific divorce rates to
- The July 1, 1982, population of married couples in the Social Security area by corresponding age-of-husband and age-of-wife.

The expected number of divorces is then divided by the total number of married couples in that year.

The DIVORCE subprocess projects annual $\hat{d}_{x,y}^z$ by age-of-husband crossed with age-of-wife.

The primary equations, 1.6.1 and 1.6.2, are given below:

$$\hat{d}_{x,y}^z = \hat{d}_{x,y}^z (\cdot) \quad (1.6.1)$$

$$\hat{ADR}^z = \frac{\sum_{x,y} P_{x,y}^S \hat{d}_{x,y}^z}{\sum_{x,y} P_{x,y}^S} \quad (1.6.2)$$

where x and y refer to the age of husband and age of wife, respectively, and $P_{x,y}^S$ is the number of married couples in the Social Security area population as of July 1, 1982.

1.6.b. Input Data

Long-Range OASDI Projection Data -

Demography

- Social Security area population of married couples by age-of-husband crossed with age-of-wife as of January 1 for years 1979-2007. These data are updated each year from the HISTORICAL POPULATION subprocess.
- Social Security area population of married couples by age-of-husband crossed with age-of-wife for some projected years. These data are updated each year from PROJECTED POPULATION subprocess of the prior year's Trustees Report.
- The total population in the Social Security area for years 1979-2007. An additional year of data is added each Trustees Report.

Assumptions -

Each year the ultimate assumed value for the age-adjusted divorce rate is established. The rate reaches its ultimate value in the 25th year of the 75-year projection period. For 2009, the ultimate assumed \hat{ADR}^z is 2,000 per 100,000 married couples.

NCHS Data -

- The number of divorces in the Divorce Registration Area (DRA), by age-of-husband crossed with age-of-wife, for calendar years 1979 through 1988. These data are no longer available for years after 1988. The data are broken out by single year age-of-husband crossed with single year age-of-wife for many ages (particularly younger ages).
- The total number of divorces in the United States for years 1979-1988. No new data are available.
- The total number of divorces in the United States for the period for 1989-2007. Additional years of data are incorporated as they become available, which is generally every year. If no new data is available, data for the most recent year is used as a proxy.
- The total number of divorces in Puerto Rico and the Virgin Islands for years 1989-2007.

The most recent year of data was obtained in 2000; the 2000 figures are used as a proxy for 2001-2007²⁴. New data are incorporated as they become available and resources are sufficient to validate their use.

Other Input Data-

- From the U.S. Census Bureau, the total population in the U.S for years 1979-1988. No new data are needed.
- From the U.S. Census Bureau, the total population in the U.S., in Puerto Rico, and the Virgin Islands for years 1989-2007. The most recent year of data was obtained in 2000; the 2000 figures are used as a proxy for 2001-2007²⁵. New data are incorporated as they become available and resources are sufficient to validate their use.

1.6.c. Development of Output

Equation 1.6.1 -

Age-specific divorce rates are defined for ages 14 through 100+. Detailed NCHS data on the number of divorces by age-group-of-husband crossed with age-group-of-wife are available for the period 1979 through 1988. Provisional data on the total number of divorces in the United States are used for the period 1989 through 2007.

First, the detailed NCHS data on divorces by age group is disaggregated into single year of age x and y (ages 14-100+) using the H.S. Beers method of interpolation. Then, the age-specific divorce rates ($\hat{d}_{x,y}^z$) are calculated for the period 1979-1988 by taking the number of divorces (inflated to represent the Social Security area, $D_{x,y}^z$) and dividing by the married population in the Social Security area at that age-of-husband and age-of-wife ($P_{x,y}^z$). The formula for this calculation is given below:

$$\hat{d}_{x,y}^z = \frac{\hat{D}_{x,y}^z}{P_{x,y}^z} \quad (1.6.3)$$

These rates are then averaged, graduated²⁶, and loaded into an 87 by 87 matrix (age-of-husband crossed with age-of-wife for ages 14 through 100+), denoted as DivGrid. DivGrid will be used in the calculation of the age-specific divorce rates for all later years including the projection period.

²⁴ Data for 1988 was used to estimate the number of Puerto Rico and Virgin Island divorces for 1989-1997.

²⁵ Data for 1988 was used to estimate the number of Puerto Rico and Virgin Island divorces for 1989-1997.

²⁶ Using the Whittaker-Henderson method of graduation.

For each year in the provisional period (1989-2007), an expected number of total divorces in the Social Security area is obtained by applying the age-of-husband crossed with age-of-wife rates in DivGrid to the corresponding married population in the Social Security area. The rates in DivGrid are then proportionally adjusted so that they would yield an estimate of the total number of divorces in the Social Security area. The estimate of total divorces is obtained by adjusting the reported number of divorces in the U.S. for (1) the differences between the total divorces in the U.S. and in the combined U.S., Puerto Rico, and Virgin Islands area and (2) the difference between the population in the combined U.S., Puerto Rico, and Virgin Islands area and in the Social Security area.

The rates over the past five years are averaged and used as the starting value for 2007 and the age-adjusted divorce rate is calculated. The rate is expected to reach its ultimate value in the 25th year of the 75-year projection period. The annual rate of change in the age-adjusted divorce rate is calculated by taking the 26th root of the ratio of the ultimate value and the starting value. Thus, to calculate the age-adjusted rate for a projected year, the rate of change is applied to the prior year's rate (or to the starting value for the first year of the projection period).

To obtain age-specific rates for use in the projections, the age-of-husband-age-of-wife-specific rates in DivGrid are adjusted proportionally so as to produce the age-adjusted rate assumed for that particular year.

1.7. PROJECTED POPULATION

1.7.a. Overview

For the 2009 Trustees Report, the starting population for the population projections is the January 1, 2007, Social Security area population, by age, sex, and marital status, produced by the HISTORICAL POPULATION subprocess. The Social Security area population is then projected using the component method. The components of change include births, deaths, net legal immigration, and net other immigration. The components of change are applied to the starting population by age and sex to prepare an estimated population as of January 1, 2008, and to project the population through the 75-year projection period (years 2009-83). There is a separate equation for each of the components of change as follows:

$$B_s^z = B_s^z(\cdot) \quad (1.7.1)$$

Where B_s^z is the number of births of each sex (s) born in year z.

$$D_{x,s}^z = D_{x,s}^z(\cdot) \quad (1.7.2)$$

Where $D_{x,s}^z$ is the number of deaths at age (x), and sex (s) that are expected to occur in year z.

$$NL_{x,s}^z = L_{x,s}^z - E_{x,s}^z \quad (1.7.3)$$

Where $NL_{x,s}^z$ is equal to net legal immigration, by age (x) and sex (s), that is assumed to occur in year z. $L_{x,s}^z$ and $E_{x,s}^z$ are the respective number of legal immigrants²⁷ and emigrants in year z, by age (x) and sex (s). In addition, $L_{x,s}^z$ is the sum of the number of new arrival legal immigrants ($NEW_{x,s}^z$) and the number of legal immigrants adjusting status ($AOS_{x,s}^z$) that are expected to occur in year z, by age (x) and sex (s). All of these values are calculated in and provided by the IMMIGRATION subprocess (1.3).

$$NO_{x,s}^z = NO_{x,s}^z(\cdot) \quad (1.7.4)$$

Where $NO_{x,s}^z$ is the number of net other immigrants²⁸, by age (x) and sex (s), for year z.

Once the components of change are calculated, the following equation is used to calculate the Social Security area population by age and sex:

²⁷ Legal immigrants are defined as persons being granted legal permanent resident status.

²⁸ Other immigrants include all immigrants, other than legal permanent residents, who stay for 6 months or more. They include unauthorized immigrants, temporary workers, and students.

$$P_{x,s}^z = \begin{cases} B_s^{z-1} - D_{x-1,s}^{z-1} & \text{for ages} = 0 \\ P_{x-1,s}^{z-1} - D_{x-1,s}^{z-1} + NI_{x-1,s}^{z-1} + NO_{x-1,s}^{z-1} & \text{for ages} > 0 \end{cases} \quad (1.7.5)$$

Where $P_{x,s}^z$ is the population, by age (x) and sex (s), as of January 1st of year z. Note that for age equal to zero, $D_{-1,s}^{z-1}$ represents perinatal deaths.

The population is further disaggregated into the following four marital statuses: single (never married), married, widowed, divorced. The following equation shows the population by age (x), sex (s), and marital status (m) for each year z:

$$P_{x,s,m}^z = P_{x,s,m}^z (\cdot) \quad (1.7.6)$$

In addition to projecting the total Social Security area population, this subprocess also projects the other immigrant population by age (x) and sex (s) using the following equation:

$$OP_{x,s}^z = OP_{x-1,s}^{z-1} + NO_{x-1,s}^{z-1} - D_{x-1,s}^{z-1} \quad (1.7.8)$$

Where, $OP_{x,s}^z$ is equal to the other immigrant population, by age(x) and sex (s) as of January 1st of year z. $D_{x,s}^z$ are the number of deaths in the other immigrant population by age (x) and sex (s) (see equation 1.7.2).

1.7.b. Input Data

Long-Range OASDI Projection Data -

Demography

IMMIGRATION

- Projected numbers of legal immigrants, who are new arrivals, by single year of age (0-84) and sex for years beginning with the starting year and ending with 2101. Each year, these data are updated.
- Projected numbers of legal immigrants, who are adjustments of status, by single year of age (0-84) and sex for years beginning with the starting year and ending with 2101. Each year, these data are updated.
- Projected numbers of legal emigrants by single year of age (0-84) and sex for years beginning with the starting year and ending with 2101. Each year, these

data are updated.

- Projected numbers of other immigrants by age (0-84) and sex for years beginning with the starting year and ending with 2101. Each year, these data are updated.
- Other than legal population by age and sex for 2007. Each year, these data are updated.
- Fixed departures by age and sex. Each year, these data are updated.
- Rates of emigration by age and sex. Each year, these data are updated.

FERTILITY

- Historical birth rates by single year of age of mother (14-49) for the years beginning with 1941 and ending with the year prior to the starting year. Each year, these data are updated.

- Projected birth rates by single year of age of mother (14-49) for the years beginning with the starting year and ending with 2101. Each year, these data are updated.

MORTALITY

- Historical probabilities of death by age last birthday (including perinatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex for years beginning with 1941 and ending with the year prior to the starting year. Each year, these data are updated.
- Projected probabilities of death by age last birthday (including perinatal mortality factor, single year of age for ages 0-99, and age group 100+) and sex for the years beginning with the starting year and ending with 2101. Each year, these data are updated.
- Marital factors to distribute probabilities of death by marital status. Factors are dimensioned by sex, single year of age (ages 14-100+) and marital status. Each year, these data are updated.

HISTORICAL POPULATION

- Social Security area population by single year of age (0-99 and 100+), sex, and marital status as of the starting date and one year prior to the starting date. These data are updated each year.
- Married couples by single year of age of husband crossed with single year of age of wife as of the starting date and one year prior to the starting date. These data are updated each year.
- Women of childbearing years (14-49) by single year of age as of January 1 beginning with 1941 and ending with the starting year. Each year, these data are updated.
- Children 18 and under by single year of age as of January 1 beginning with 1941 and ending with the starting year. Each year, these data are updated.
- Married couples with children by single year of age of husband (ages 14-83) crossed with single year of age of wife (ages 14-49) as of January 1 beginning with 1941 and ending with the starting year. Each year, these data are updated.
- Married men by age group (15-24, 25-29, 30-34, ..., 60-64) as of January 1 beginning with 1960 and ending with the starting year. Each year, these data are updated.

MARRIAGE

- Projected central marriage rates by single year of age of husband (ages 14-100+) crossed with single year of age of wife (ages 14-100+) for each year of the projection period. These data are updated annually.

- Averaged and graduated marriage rates for the period 1979 and 1981-1988 by single year of age (ages 14-100+), sex, and prior marital status (single, divorced, and widowed). These data are updated annually.

DIVORCE

- Projected central divorce rates by single year of age of husband (14-100+) crossed with single year of age of wife (14-100+) for each year of the projection period. These data are updated annually.

U.S. Census Bureau Data -

- CPS data on the average number of children per married couple with children by age group of householder (age groups 20-24, 25-29, 30-34, ..., 55-64) for 1960-2006. Each year, an additional year of data is added.

1.7.c. Details of Output

Equation 1.7.1 - Births

The number of births for the Social Security area, B_x^z , is computed for each year z of the projection period by applying the age-specific birth rate to the mid year female population aged 14 to 49.

$$B_x^z = b_x^z \left(\frac{FP_x^z + FP_x^{z+1}}{2} \right)$$

where,

b_x^z = birth rate of mothers age x in year z ;

B_x^z = number of births to mothers age x in year z ; and

FP_x^z = female population age x at the beginning of year z .

The number of births in a given year is disaggregated by sex by assuming a gender ratio of 105 male births for every 100 female births.

Equation 1.7.2 – Deaths

The number of deaths for the Social Security area by age (x) and sex (s), $D_{x,s}^z$, is computed for each projection year by applying the death probabilities for each age (x) and sex (s), $q_{x,s}^z$, to the exposed population at the beginning of the year.

$$D_{x,s}^z = q_{x,s}^z P_{x,s}^z$$

Deaths for the other immigrant population are calculated in the same manner.

$$OD_{x,s}^z = q_{x,s}^z OP_{x,s}^z$$

Equation 1.7.4 – Net Other Immigration

Net other immigration by age (x) and sex (s) for year z, $NO_{x,s}^z$, consists of other immigration less other emigration. Thus we have the following equation:

$$NO_{x,s}^z = O_{x,s}^z - OE_{x,s}^z$$

Other immigration by age (x) and sex (s), $O_{x,s}^z$ is calculated in the IMMIGRATION subprocess (1.3) for each year z of the long-range projection period. Other emigration by age (x) and sex (s), $OE_{x,s}^z$, consists of departures from the Social Security area population and those who adjust status to legal permanent residents. Therefore, other emigration can be expressed via the following equation:

$$OE_{x,s}^z = AOS_{x,s}^z + OD_{x,s}^z$$

Where $AOS_{x,g}^z$ is the annual number of people who adjust status to legal permanent resident status. Adjustments of status are assumed to equal one half of all legal immigrants and are calculated in the IMMIGRATION subprocess.

$OD_{x,s}^z$ is equal to the annual number of other immigrants who depart the Social Security area by age (x) and sex (s).

$$OD_{x,s}^z = FD_{x,s}^z + RD_{x,s}^z$$

Where $FD_{x,s}^z$ is the number of fixed departures (calculated in the IMMIGRATION subprocess) and $RD_{x,s}^z$ is the varying number of departures based on the rates of emigration (these rates of emigration are calculated in the IMMIGRATION subprocess). Modeling the $OD_{x,s}^z$ in this manner recognizes two key points of other emigration: (1) as the size of the exposed population increases the overall number of departures is likely to increase and (2) as the average duration of stay increases the overall gross departure rate is likely to decrease.

$RD_{x,s}^z$ is calculated by applying the rates of emigration to the exposed population. The exposed population is calculated by subtracting deaths, fixed departures, and adjustments of status from the beginning of year other immigrant population.

$$RD_{x,s}^z = r_{x,s} * (OP_{x,s}^z * (1 - q_{x,s}^z) - FD_{x,s} - AOS_{x,s}^z)$$

Equation 1.7.6 – Disaggregating the population by marital status

Once the population is projected by single year of age and sex, it is then disaggregated into the following four marital states; single, married, widowed, and divorced. Estimates of the Social Security area population by single year of age (0-99 and 100+), sex, and marital status as of the starting date of the population projection are obtained from the HISTORICAL POPULATION subprocess. In addition, the HISTORICAL POPULATION subprocess provides the number of married couples by single year of age of husband crossed with single year of age of wife as of the starting date.

All births are assigned to the single category. For a given age and sex, deaths are assigned by marital status according to the relative differences in death rates by marital status observed for that age and sex during the calendar years 1995 and 1996, as determined in the MORTALITY subprocess. For a given age and sex, immigrants are assigned by marital status according to the beginning of year marital distribution of the Social Security area population for that age and sex.

Once the number of marriages, divorces, and widowings during a year are determined, the population by sex, age, and marital status is updated to represent end of year. The unmarried population at the end of the year is estimated from the population at the beginning of the year by subtracting deaths and marriages and adding new immigrants, widows (or widowers), and divorces during the year. The married population at the end of the year is estimated from the population at the beginning of the year by reducing the population for divorces, widows (or widowers), dissolutions of marriages when both husband and wife dies, and by increasing the population for new immigrants and marriages during the year.

Numbers of new marriages are determined for each projection year. The annual number of marriages occurring at each age of husband crossed with each age of wife is obtained by multiplying the age-of-husband and-age-of-wife-specific marriage rates with the geometric mean of the midyear unmarried male population and the midyear unmarried female population.

The midyear unmarried male and female populations²⁹ are estimated from the beginning of the year unmarried populations. The age-specific midyear male unmarried population is calculated by adjusting the number of unmarried males at the beginning of the year to represent midyear using the relationship between the two points in time in the prior year. The midyear female unmarried population is approximated similarly.

The numbers of marriages are then distributed by previous marital status in the same proportions as would have been produced by applying the previous marital-status-specific marriage rates from the MARRIAGE subprocess to the population by marital status at the beginning of the year.

Numbers of new divorces are determined for each projection year. The number of divorces during a year, occurring at each age of husband crossed with each age of wife, is obtained by multiplying the age-of-husband by age-of-wife divorce rates for that year with the midyear number of married couples in that age crossing.

The midyear married couples are estimated from the beginning of the year married couples. The number of age-of-husband by age-of-wife midyear married couples is calculated by adjusting the number of married couples at the beginning of the year to represent midyear using the relationship between the two points in time in the prior year.

Widowings are computed by applying general population probabilities of death to the marriage prevalence at the beginning of the year. Widowings and deaths by marital status are then reconciled for internal consistency.

²⁹ The midyear populations exposed to marriage are the unmarried populations (sum of those single, widowed, and divorced).

Process 2:

Economics

2. Economic

The Office of the Chief Actuary uses the Economic process to project OASDI employment and earnings-related variables, such as the average wage for indexing and the effective taxable payroll. The Economic process receives input data from the Demography process and provides output data to the Beneficiaries and the Trust Fund Operations & Actuarial Status processes.

The Economic Process is composed of four subprocesses, U.S. EMPLOYMENT, U.S. EARNINGS, COVERED EMPLOYMENT AND EARNINGS, and TAXABLE PAYROLL. As a rough overview, U.S. EMPLOYMENT and U.S. EARNINGS project U.S. employment and earnings data, respectively, while COVERED EMPLOYMENT AND EARNINGS converts these employment and earnings variables to OASDI covered concepts. TAXABLE PAYROLL, in turn, converts OASDI covered earnings to taxable concepts, which are eventually used to estimate future payroll tax income and future benefit payments.

U.S. EMPLOYMENT and U.S. EARNINGS produce quarterly output, while the output from COVERED EMPLOYMENT AND EARNINGS is annual. TAXABLE PAYROLL produces both.

Two appendices are at the end of this documentation. The first appendix, 2-1, provides details of the equations mentioned below. The second appendix, 2-2, provides a listing with explanations of acronyms used in this documentation.

2.1. U.S. EMPLOYMENT (USEMP)

2.1.a. Overview

The Bureau of Labor Statistics (BLS) publishes historical monthly estimates for civilian U.S. employment-related concepts from the Current Population Survey (CPS). The principal measures include the civilian labor force (LC) and its two components – employment (E) and unemployment (U), along with the civilian noninstitutional population (N). Historical estimates for N by age, sex, and marital status are tabulated within the Economic Process from the CPS. For each disaggregated age/sex/marital status group, N is also projected, using the military population (M) and the growth rate projected for the Social Security area population (P) (see Demography Process input). The BLS also publishes values for the civilian labor force participation rate (LFPR) and the civilian unemployment rate (RU). The LFPR is defined as the ratio of LC to N, while the RU is the ratio of U to LC, expressed to a base of 100.

For each age/sex group, USEMP projects quarterly and annual values for these principal measures of U.S. employment. Equations 2.1.1 through 2.1.5 outline the subprocess' overall structure and solution sequence for the total economy.

$$N_{\text{year}} = [(N_{\text{year-1}} + M_{\text{year-1}}) * (P_{\text{year}} / P_{\text{year-1}})] - M_{\text{year}} \quad (2.1.1)$$

$$\text{LFPR} = \text{LFPR}(\cdot) \quad (2.1.2)$$

$$\text{LC} = \text{LFPR} * \text{N} \quad (2.1.3)$$

$$\text{RU} = \text{RU}(\cdot) \quad (2.1.4)$$

$$\text{E} = \text{LC} * (1 - \text{RU} / 100) \quad (2.1.5)$$

where the subscript year in equation (2.1.1) represents the projection year.

The Demography Process estimates historical values for the total SSA area population (P) and the other immigrant population (OP). USEMP projects annual values for total employed OP (EO) and various sub-components, including the portion of EO whose earnings are reported and posted to the Master Earnings File (EO_MEF), those EO whose earnings are reported posted to the Earnings Suspense File (EO_ESF), those EO in the underground economy (EO_UND), and EO_MEF who are OASDI covered (EO_MEFC). Equations 2.1.6 through 2.1.10 outline the overall structure of the subprocess used to estimate EO and its sub-components.

$$\text{EO} = \text{E} * \text{OP} / \text{P} \quad (2.1.6)$$

$$\text{EO_MEF} = \text{EO_MEF}(\cdot) \quad (2.1.7)$$

$$EO_MEFC = EO_MEFC(\cdot) \quad (2.1.8)$$

$$EO_ESF = EO_ESF(\cdot) \quad (2.1.9)$$

$$EO_UND = EO_UND(\cdot) \quad (2.1.10)$$

Finally, for each age/sex group, USEMP projects “at-any-time” employed other immigrant population (TEO). EO represents the average annual number of individuals in the other immigrant population who were employed each month during a calendar year. TEO represents the total number of individuals in the other immigrant population who had any employment during the calendar year. (EO can be roughly viewed as the average number of jobs worked by OP during a calendar year, while TEO represents the total number of individuals who worked those jobs.) Effectively, Equations 2.1.11 through 2.1.15 convert every EO age-sex sub-component to an at-any-time TEO age-sex sub-component counterpart.

$$TEO = TEO(\cdot) \quad (2.1.11)$$

$$TEO_MEF = TEO_MEF(\cdot) \quad (2.1.12)$$

$$TEO_MEFC = TEO_MEFC(\cdot) \quad (2.1.13)$$

$$TEO_ESF = TEO_ESF(\cdot) \quad (2.1.14)$$

$$TEO_UND = TEO_UND(\cdot) \quad (2.1.15)$$

2.1.b. *Input Data*

Long-Range OASDI Projection Data

These data are updated each year.

Demography

- Social Security area population as of year end (1941 – 2099) by age, marital status (single, married, widowed, divorced) and sex (M, F).
- “Other immigrant” population as of year end (1980 – 2099) by age, marital status (single, married, widowed, divorced) and sex (M, F).
- Number of children by age of child and age of mother.
- Life expectancy by age and sex.
- Exit rates (probability of leaving the “other immigrant” population by other than death) by age and sex.

Trust Fund Operations and Actuarial Status – The Trust Fund Operations and Actuarial Status Process provides no input to the Economic Process sections. However, the LFPRs use input from the Outgo Process from the prior year’s Trustees Report. That is, the projected LFPRs for the 2009 Trustees Report use input from the 2008 Trustees Report that includes projections for the disability prevalence rates by age and sex, and the primary insurance amount (PIA) replacement rates by age and sex. The disability prevalence rate is defined as the ratio of the number of disabled worker beneficiaries to the Social Security area population. The PIA replacement rate is defined as the ratio of a hypothetical medium scaled worker’s PIA to his/her career-average indexed earnings level.

Trustees Assumptions

Each year the Board of Trustees of the OASDI Trust Funds sets the ultimate assumed values for key economic variables:

- Real wage differential
- Annual percentage change in total economy productivity
- Annual percentage change in average hours worked
- Ratio of wages to compensation (RWSD)
- Ratio of compensation to GDP (RWSSY)
- Annual percentage change in the price differential
- Annual percentage change in CPI
- Annual trust fund real interest rate
- Unemployment rate

These ultimate values are generally implemented during the last half of the short-range (first 10 years) of the projection horizon. Earlier projected values are set to provide a smooth transition from the latest actual historical values to the assumed long-range ultimate ones. As a by-product of this process, values for the GDP deflator (PGDP), real GDP, and potential GDP are set. The ratio (RTP) of real to potential GDP is an important summary measure of the business cycle.

It is important to note that the Trustees also agree on the assumed short-range values for the above variables.

Addfactors

Addfactors are adjustments that move an estimate closer to an expected value. Addfactors may be used for a variety of reasons associated with data availability, structural changes in the data and/or model, and perceived temporary aberrations in recent historical data. Addfactors were included directly in equation 21.2 for male and female LFPRs ages 16 to 19,

and on the aggregate LC and RU. Addfactors were also included on all male and female LFPRs age about 40 and over to reflect changes in life expectancy.

The need for addfactors is reviewed each year and addfactors are implemented if necessary.

Other input data

- U.S. armed forces (EDMIL) by age and sex, estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and sex. These data are no longer produced by Census.
- EDMIL by age and sex, estimated by the Economic Process as the difference in monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to October 2008) by single year of age (16 to 69) and sex. These data are updated several times a year.
- Data for the mobilized military reservist population, by branch of service (September 2001-October 2008). These data are available from the US Department of Defense news release website. These data are updated by the Department of Defense weekly. This subprocess updates the data several times a year.
- Data from the March Supplement of the Current Population Survey (CPS) by year (1968-2007), for the civilian noninstitutional population, labor force, military, and unemployment. These data are available from the BLS by single year of age (16 to 85+), sex, marital status (never married, married with a spouse present, and married with no spouse present), and presence of children. These data are updated by the BLS annually. This subprocess updates the data every other year (or more often, based on its time constraints).
- Data from the CPS (1948-2008) for civilian employment, civilian labor force, civilian unemployment, and civilian noninstitutional population. These data are available from the BLS by age group and sex. These data are updated by the BLS monthly. This subprocess updates the data several times a year.
- Data from the Current Employment Statistics survey (CES) (1964 (varies) to 2008) for establishment employment and average hourly and weekly earnings. These data are available from the BLS by sector. These data are updated by the BLS monthly. This subprocess updates the data several times a year.
- Unpublished data from the CPS (1965-2008) for male and female civilian labor force participation rates for older workers. These data are available from the BLS by single year of age (ages 55-74). These data are updated by the BLS monthly. This subprocess updates the data several times a year.

2.1.c. *Development of Output*

Equation 2.1.1 - Labor Force Participation Rate (LFPR)

The LFPR is disaggregated by age and sex. Age groups include 16 to 17 (i.e., 16-17), 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55, 56, ... , 74, 75-79, 80-84, and 85 and over. For age groups between 20 and 54, male and female LFPRs are further disaggregated by marital status, categories of which include never married, ever married with spouse present, and ever married with spouse absent (includes separated, widowed, and divorced). Female LFPRs disaggregated by age (between 20 and 44) and by marital status are further disaggregated by presence of own child. The groups for presence of own child include females with at least one own child under the age of six and females with no own child(ren) under the age of six. Thus, USEMP contains 107 LFPR equations, 46 for males and 61 for females.

Given the level of demographic disaggregation, the aggregate LFPR is dependent on the projected distribution of the population by age, sex, marital status, and presence of own child. Each disaggregated LFPR, however, is dependent on the input variables that are most relevant to the demographic group. For example, only the LFPRs for younger males are dependent on the military, while only the LFPRs for relevant older workers are dependent on changes to the normal retirement age (NRA). Specific examples of the impact of input data on the disaggregated LFPRs are presented below.

- Military Ratio (RM) is defined as the ratio of U.S. Armed Forces to the total noninstitutional population. An increase in RM lowers the LFPR. The RM affects the LFPRs for younger aged males.
- Disability prevalence ratio (RD) is defined as the ratio of disabled worker beneficiaries to the Social Security area population. An increase in RD lowers the LFPR. The RD affects the LFPRs for males and females below the NRA.
- Ratio of real to potential GDP (RTP) is a measure of the business cycle. An increase in RTP leads to an increase in the LFPR. The RTP affects most LFPRs.
- The PIA replacement rate (PIARR) is defined as the average ratio of a hypothetical worker's PIA to career-average wage level. This value is projected for hypothetical workers with medium-scaled earnings patterns³⁰ who retire at ages 62 through NRA. The PIARR is adjusted to include the reduction for early retirement and the delayed retirement credit, two adjustments that are affected by the NRA. An increase in the adjusted PIARR leads to a decrease in the LFPR. The adjusted PIARR affects the LFPRs for those between the ages of 62 and 70. This variable needs to be considered due to the increase in NRA and other changes that are scheduled in present law.
- The Earnings Test Ratio (RET) is defined as the ratio of the maximum amount of earnings before an OASDI benefit is reduced to the average wage index. An increase in the RET

³⁰ More details on the hypothetical scaled workers are provided in Actuarial Note #2005.3, located at the following internet address: www.socialsecurity.gov/OACT/NOTES/ran3/index.html.

leads to an increase in the LFPR. The RET is used in LFPRs for those between the NRA and age 69. This variable needs to be considered due to the increase in this maximum amount that is scheduled in present law.

- The Family Size Ratio (RFS) is defined as the ratio of the number of children under 6 to mothers of a certain age. An increase in the RFS lowers the LFPR. The RFS affects the LFPRs for females aged 20 to 44 with at least one own child.
- An LFPR increases with its lagged cohort. For example, the change in the LFPR for females aged 25-29 between 2005 and 2010 is dependent on the change in the LFPR for females aged 20-24 between 2000 and 2005. Lagged cohort variables affect female LFPRs age 20 and over, and male LFPRs age 55 and over.
- The LFPR for males ever married spouse present falls with increases in their spouse's LFPR.
- An increase in life expectancy leads to an increase in LFPRs. The LFPRs for those approximately aged 40 and over are affected.

Equation 2.1.3 - Unemployment Rate (RU)

The RU is disaggregated by age and sex. The age groups include 16-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, and 75 and over. Thus, USEMP contains 28 RU equations, 14 for males and 14 for females.

The aggregate RU is dependent on the projected distribution of the labor force by age and sex. Each disaggregated RU is specified using a first-difference model and is only dependent on one input, the distributed lag in the change in the RTP. Coefficients are estimated by regression and constrained to an expected aggregate behavior whereby a 2.0 percentage point change in the RTP elicits a 1.0 percentage point change in the RU. Furthermore, projections are constrained to the ultimate age-sex-adjusted RU set by the Trustees.

Equation 2.1.6 to 2.1.14 – Employed Other Immigrant Population (EO) and At-Any-Time Employed other Immigrant Population (TEO)

EO is estimated by sex and single-year of age from 16 to 100. Thus, USEMP contains 170 equations, 85 for males and 85 for females. Based on various assumptions, we separate each age-sex EO into visa-status components including those authorized to work, those not authorized to work but previously authorized, and those never authorized to work. We separate EO never authorized to work into those who worked in 2001 and earlier and those who began working in 2002 and later, since we believe that those who worked in 2001 and earlier are more likely to have OASDI covered wages. Each disaggregated employment group is then further separated into EO_MEF, EO_MEFC, EO_ESF, and EO_UND.

Every age-sex EO sub-component is converted to its age-sex TEO sub-component counterpart using an age-sex “conversion weight.” For example, if the sub-component of EO is for males age 20 to 24, the conversion weight is defined as the ratio of total economy-wide at-any-time employed males age 20 to 24 (TEM2024) to the sum of CPS and military male

employment age 20 to 24. However, TEM2024 is by definition the sum of at-any-time employment posted and not posted to the MEF, of which, the latter group includes the portion of TEO who are male age 20 to 24 in the underground economy. Thus, we must use proxies for the conversion weights. For males age 20 to 24, our proxy conversion rate is defined as the ratio of the sum of OASDI covered employment posted to the MEF, EO_ESF, and EO_UND to the sum of CPS and military employment. Effectively, this method assumes that the conversion weight for each age-sex sub-component of EO is approximately equal to the conversion rate for each age-sex sub-component of E.

The estimation methodology for the other immigrant employment and earnings will be described in detail in appendix 2.3. (forthcoming)

2.2. U.S. EARNINGS (USEAR)

2.2.a. Overview

In the CPS data, E is separated by class of worker. The broad categories include wage and salary workers (EW), the self-employed (ES), and unpaid family workers (EU). For the nonagricultural sector, a self-employed participation rate (SEPR) is defined as the ratio of ES to E, the proportion of employed persons who are self-employed. For the agricultural sector, the SEPR is defined as the ratio of ES to the civilian noninstitutional population.

USEAR projects quarterly values for these principle classes of employment. Equations 2.2.1 through 2.2.4 outline the subprocess' overall structure and solution sequence.

$$SEPR = SEPR(\cdot) \quad (2.2.1)$$

$$ES = SEPR * E \quad (2.2.2)$$

$$EU = EU(\cdot) \quad (2.2.3)$$

$$EW = E - ES - EU \quad (2.2.4)$$

In the National Income and Product Accounts (NIPA), the Bureau of Economic Analysis (BEA) publishes historical quarterly estimates for gross domestic product (GDP), real GDP, and the GDP price deflator (PGDP). PGDP is equal to the ratio of nominal to real GDP. Potential (or full-employment) GDP is a related concept defined as the level of real GDP that is consistent with a full-employment aggregate RU.

USEAR projects quarterly values for these output measures. Potential GDP is based on the change in (1) full-employment values for E (including U.S. armed forces), (2) average hours worked per week, and (3) productivity. Full-employment values for E are derived by solving USEMP under full-employment conditions, while the full-employment values for the other variables (average hours worked and productivity) are set by assumption. Projected real GDP is set equal to the product of potential GDP and RTP. RTP reaches 1.0 in the short-range period and remains at 1.0 thereafter. Nominal GDP is the product of real GDP and PGDP. The growth rate in PGDP is set by assumptions.

The BEA also publishes quarterly values for the principal components of U.S. earnings, including total wage worker compensation (WSS), total wage and salary disbursements (WSD), and total proprietor income (Y). These concepts can be aggregated and rearranged. Total compensation (WSSY) is defined as the sum of WSS and Y. The total compensation ratio

(RWSSY) is defined as the ratio of WSSY to the GDP. The income ratio (RY) is defined as the ratio of Y to WSSY. The earnings ratio (RWSD) is defined as the ratio of WSD to WSS.

USEAR projects quarterly values for these principle components of U.S. earnings using Equations 2.2.5 through 2.2.11.

$$RWSSY = RWSSY(\cdot) \quad (2.2.5)$$

$$WSSY = RWSSY * GDP \quad (2.2.6)$$

$$RY = RY(\cdot) \quad (2.2.7)$$

$$Y = RY * WSSY \quad (2.2.8)$$

$$WSS = WSSY - Y \quad (2.2.9)$$

$$RWSD = RWSD(\cdot) \quad (2.2.10)$$

$$WSD = RWSD * WSS \quad (2.2.11)$$

2.2.b. Input Data

Long-Range OASDI Projection Data

Demography- (See Section 2.1.b.)

Economics - Data from Section 2.1 include the total employed (E), E by age and sex, LFPRs by age and sex, the aggregate unemployment rate (RU), and the full-employment concepts for LC, RU, and E.

Trustees Assumptions - (See Section 2.1.b.)

Addfactors

Addfactors were included on some employment and output variables to smooth the transition between the latest historical data and the projected values. The need for addfactors is reviewed each year and addfactors are implemented if necessary.

Other input data

- Data from the NIPA (1929 (varies) to 2007) for GDP, income, wages, compensation, personal consumption expenditures, investment, employer contributions for employee pension and insurance funds, and employer contributions for government social insurance. They are published by the BEA quarterly and/or annually. This subprocess updates the data several times a year.
- Ratio of OASDI covered to NIPA wages, and ratio of OASDI taxable to covered wages. NIPA wages by sector are available quarterly and annually from 1947 to 2005. They are published by the BEA and updated several times during the year. OASDI covered and taxable wages (1971 to 2004) are updated annually by the Economic process. Covered and taxable data for more recent historical years are estimated from preliminary tabulations of Form 941 and W-2 data. Projected values for covered ratios are set to the latest historical year.
- OASDI employee, employer, and self-employed tax rates from 1937 to 2080. These contribution rates are set according to the Social Security Act of 1935 as amended through 2005. The rates are updated when legislation mandates a change.
- The October ratio of the number of teenagers enrolled in school to the civilian noninstitutional population by sex and age group (16-17 and 18-19) for the period 1947 to 2006. An additional new year of data from the Census Bureau is usually available for including in preparation of the next annual Trustees Report. Projected values are set to levels from the latest historical year.
- The historical CPI for medical services is published monthly by the BLS. Quarterly values are projected based on the projected growth in the aggregate CPI and an additional amount defined as the growth rate differential in the two price measures that was assumed in the latest President's Fiscal Year Budget. The series is updated annually.
- U.S. armed forces (EDMIL) by age and sex were estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and sex. These data are no longer produced by Census.
- EDMIL by age and sex are estimated by the Economic process as the difference in the monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to October 2008) by single year of age (16 to 69) and sex. These data are updated several times a year.
- Wages for railroad workers are wages covered by the Railroad Retirement Act. The

annual data are for the period 1971 to 2006. An additional year of data from the Railroad Retirement Board is usually available for including in preparation of the next annual Trustees Report.

- Unpublished data from the CPS (1988-2007) on employment by class of worker (i.e., agricultural, non-agricultural, unpaid family, private industry, government, wage and salary, self-employed). These data are available from the BLS by age group and sex. These data are updated by the BLS annually. This subprocess updates the data annually.
- Data from the NIPA (1947-2008) on wages and compensation of households and institutions are published by the BEA quarterly. This subprocess updates the data several times a year.
- Other program-related parameters, including the average indexing wage, the benefit increase, the taxable maximum, and the annual retirement earnings test exempt amounts, are obtained annually from the Short-Range section of OCACT. This subprocess updates the data annually.
- Ratios of OASDI covered wages to NIPA wages are used from the prior year's Trustees Report as input to the current year's calculations, for the following sectors: the ratio of Federal civilian, Federal military, state and local, and private industry.
- Recent historical values (2000-2008) for quarterly real potential GDP are obtained from the Congressional Budget Office. This subprocess updates the data annually and adjusts the series to create a series usable for the current Trustees Report.
- Unpublished data from the CES & CPS for total hours worked in the economy. These data are available from the BLS. These data are updated by the BLS quarterly (1948-2008) and annually (1948-2007). This subprocess updates the data several times a year.
- The Federal minimum hourly wage is based on the Fair Labor Standards Act from the Department of Labor for 1938 to 2007. The wage is updated when there is legislation mandating a change.
- Time trends (set by Economic process) are used in the agriculture sector for employment, real output, and compensation in the short-range period. These short-range trends are extended for each year's Trustees Report, reflecting a new short-range period.

2.2.c. *Development of Output*

Equation 2.2.1 - Self-Employed Participation Rate (SEPR)

The SEPR is disaggregated by age, sex, and industry. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The industry groups include agriculture and non-agriculture.

For the non-agriculture sector, the SEPRs by age and sex are defined as the ratio of the non-agriculture self-employment to total employment. Thus, the aggregate non-agriculture SEPR is dependent on the projected distribution of employment by age and sex. All non-agriculture SEPRs by age and sex are dependent on the RTP. Increases in the RTP lead to decreases in the SEPRs. For female age groups between 20 and 64, the non-agriculture SEPRs are also dependent on the groups LFPRs. Increases in the LFPRs lead to increases in the SEPRs.

For the agriculture sector, the male SEPRs by age are defined as the ratio of agriculture self-employment to the civilian noninstitutional population. Thus, the aggregate agriculture SEPR for males is dependent on the projected distribution of the population by age. The agriculture SEPRs for males by age are dependent on the ratio of total agriculture employment (EA) to the total civilian population aged 16 and over. (EA is projected in a farm sub-program. Real farm output is projected to increase with the population, while farm productivity, defined as output per worker, is projected to continue to follow its historical trend. EA is projected as the ratio of farm output to farm productivity.) An increase in the ratio of EA to the total civilian population aged 16 and over leads to an increase in the agriculture SEPRs for males.

The female SEPRs by age for the agriculture sector are defined as the ratio of the female to male agriculture self-employment. Thus, the aggregate agriculture SEPR for females is dependent on the projected distribution of male agriculture employment by age. For female age groups between 18 and 64, the SEPRs are dependent on the RTP and the corresponding ratio of total female to male employment. Generally, an increase in the RTP leads to increases in the SEPRs. An increase in the total employment ratio also leads to an increase in the SEPR.

Equation 2.2.3 - Unpaid Family Workers (EU)

EU is disaggregated by age, sex, and industry. The age groups include 16-17, 18-19, 20-24, 25-34, 35-44, 45-54, 55-64, and 65 and over. The industry groups include agriculture and non-agriculture.

From 1970 to 2005, the level of EU fell from about 0.5 to 0.03 million in the agriculture sector and from about 0.5 to 0.09 million in the nonagriculture sector. For projections, the levels of EU by age and sex in the agriculture sector are assumed to be constant and about five thousand or less. The EUs by age and sex in the nonagriculture sector are projected as a constant ratio to ES.

Equation 2.2.5 - Total Compensation Ratio (RWSSY)

The Trustees set the ultimate annual growth rate for RWSSY. For the short-range period, total WSS, WSD, and Y are aggregated from sector components. Total GDP, WSS, and WSD are divided into the farm and nonfarm sectors. The nonfarm sector is further separated into the government and government enterprises, households, non-profit institutions, and residual (private nonfarm business excluding government enterprises (PBNFXGE)) sectors. Total Y is divided into the farm and residual (i.e., PBNFXGE) sectors.

The methodology used to estimate GDP, WSS, WSD, and Y differs by sector.

Farm - Nominal GDP is the product of real GDP and the farm price deflator. Real farm GDP is projected from estimates of real farm per capita output. EA is projected from estimates of farm productivity. EAW is projected to continue its historical increase relative to EA. Farm compensation (WSSPF) is the product of estimates for average farm compensation (AWSSPF) and EAW, while farm proprietor income (YF) is the product of estimates of average farm proprietor income (AYF) and EAS. AYF is projected based, in part, on the growth in average compensation in the private sector.

Government and Government Enterprises - This sector is further disaggregated to Federal Civilian, Federal Military, and State and Local. In each sector, WSD is the product of estimates for average wages and employment. WSS is the sum of WSD and estimates for non-wage components of compensation. GDP is the sum of WSS and estimates of consumption of fixed capital.

Household - WSS is the product of estimates for average compensation and employment. WSD is WSS less employer contributions for the OASDHI tax. GDP is the sum of WSS and the gross value added of owner-occupied housing.

Nonprofit Institutions - The Nonprofit Institutions sector is further disaggregated to Health, Education, and Social Services sectors. In each sector, WSS is the product of estimates for average compensation and employment. WSD is WSS less the estimates for non-wage components of compensation. GDP is WSS plus a residual component of output.

Private Nonfarm Business Excluding Government Enterprises (PBNFXGE) - GDP in the PBNFXGE sector is total economy-wide GDP less the sum of the other sector GDPs. WSS is projected as a ratio to GDP less Y. The ratio is projected to be mostly stable, varying only temporarily with changes in RTP. Y is projected as a ratio to GDP.

Thus, total labor compensation (WSSY) is summed from sector components, while the total compensation ratio (RWSSY) is the ratio of total WSSY to total GDP. It is important to note that the pure program-generated estimate for the total RWSSY is adjusted to ensure a smooth transition between the latest historical data and the Trustees' ultimate assumptions.

Equation 2.2.7 - Income Ratio (RY)

Y is disaggregated to the farm and PBNFXGE sectors. In the PBNFXGE sector, the Y is projected as a ratio to GDP in the sector. In the farm sector, Y is projected based, in part, on the growth in average compensation in the private sector.

Equation 2.2.10 - Earnings Ratio (RWSD)

In the NIPA, the difference between the WSS and WSD is defined as employer contributions for employee pension and insurance funds (OLI) and employer contributions for government social insurance (SOC). OLI is mostly health and life insurance, and pension and profit sharing. SOC is composed of employer contributions to Federal and State & Local government social insurance funds. Federal government funds include OASDI, HI, UI, and other small groups. State and Local government funds mostly include workers' compensation.

RWSD is defined as the ratio of WSD to WSS and is projected for the federal military, federal civilian government, state and local government, and private sectors of the economy. Over the long-range period, the projected values for RWSD in each sector are set to the trustees' assumed ultimate annual growth rate. Over the short-range period, each projected value for RWSD for each sector is the sum of the model's "raw" estimate and an adjustment. The adjustment guarantees that the projected value for RWSD matches the BEA's latest published historical quarterly value for the variable. For subsequent quarters, this adjustment or "difference" may be increased, decreased, or held constant, depending on the assumed nature of the difference. For example, the projected value for the RWSD for the federal military sector has the largest adjustment, averaging about -0.01 for CY 2007. We believe this difference is associated with a temporary federal government contribution to the military's thrift savings plan to meet manpower requirements for the wars in Iraq and Afghanistan. Consequently, this adjustment to RWSD for the federal military sector is eliminated in 2008 and later. The adjustments for RWSD in state and local government and private sectors were about -0.01 and 0.01, respectively, and are held constant in subsequent quarters of the short-range projection horizon.

2.3. OASDI COVERED EMPLOYMENT AND EARNINGS (COV)

2.3.a. Overview

Total at-any-time employment (TE) is defined as the sum of total OASDI covered employment (TCE) and total noncovered employment (NCE). TCE can be decomposed to workers who only report OASDI covered self-employed earnings (SEO) and to wage and salary workers who report some OASDI covered wages (WSW). Combination workers (CMB_TOT) are those who have both OASDI covered wages and self-employed income. Workers with some self-employment income (CSW) are the sum of SEO and CMB_TOT.

Some of these concepts can be rearranged. The total employed ratio (RTE) is defined as the ratio of TE to the sum of EW, ES, and EDMIL, while the combination employment ratio (RCMB) is defined as the ratio of CMB_TOT to WSW.

COV projects annual values for TE and the principle measures of OASDI covered employment. Equations 2.3.1 through 2.3.9 outline the overall structure and solution sequence used to project these concepts.

$$RTE = RTE(\cdot) \quad (2.3.1)$$

$$TE = RTE * (EW + ES + EDMIL) \quad (2.3.2)$$

$$NCE = NCE(\cdot) \quad (2.3.3)$$

$$TCE = TE - NCE \quad (2.3.4)$$

$$SEO = SEO(\cdot) \quad (2.3.5)$$

$$WSW = TCE - SEO \quad (2.3.6)$$

$$RCMB = RCMB(\cdot) \quad (2.3.7)$$

$$CMB_TOT = RCMB * WSW \quad (2.3.8)$$

$$CSW = SEO + CMB_TOT \quad (2.3.9)$$

Total OASDI covered earnings is defined as the sum of OASDI covered wages (WSC) and total covered self-employed income (CSE_TOT). Both components can be expressed as ratios to their U.S. earnings counterparts. The covered wage ratio (RWSC) is defined as the ratio of WSC to WSD, while the covered self-employed ratio (RCSE) is the ratio of CSE_TOT to Y.

COV projects annual values for the principal measures of OASDI covered earnings using Equations 2.3.10 through 2.3.13.

$$RWSC = RWSC(\cdot) \quad (2.3.10)$$

$$WSC = RWSC * WSD \quad (2.3.11)$$

$$RCSE = RCSE(\cdot) \quad (2.3.12)$$

$$CSE_TOT = RCSE * Y \quad (2.3.13)$$

COV can now project various annual measures of average OASDI covered earnings, including the average covered wage (ACW), average covered self-employed income (ACSE), and average covered earnings (ACE).

$$ACW = WSC / WSW \quad (2.3.14)$$

$$ACSE = CSE_TOT / CSW \quad (2.3.15)$$

$$ACE = (WSC + CSE_TOT) / TCE \quad (2.3.16)$$

The average wage index (AWI) is based on the average wage of all workers with wages from Forms W-2 posted to the Master Earnings File (MEF). By law, it is used to set the OASDI contribution and benefit base (TAXMAX).

COV projects annual values for the AWI and TAXMAX.

$$AWI = AWI(\cdot) \quad (2.3.17)$$

$$TAXMAX = TAXMAX(\cdot) \quad (2.3.18)$$

2.3.b. Input Data

Long-Range OASDI Projection Data

Demography - (See Section 2.1.b.)

Economics- Employment and earnings-related data from Sections 1.1 and 1.2.

Trustees Assumptions - (See Section 2.1.b.)

Addfactors

Addfactors were included on some employment variables to smooth the transition from the latest historical data to program estimates. The need for addfactors is reviewed each year and addfactors are implemented if necessary.

Other input data

- The ratio of maximum quarterly population within a year to the annual value for that year is calculated from 1959 through the year of latest available data by 5-year age group and sex. The population, defined as the sum of the civilian noninstitutional population and the military is updated quarterly from CPS and military data.
- Ratios of OASDI covered to NIPA wages by sector. NIPA wages by sector are available quarterly and annually from 1947 to 2007. They are published by the BEA and updated several times during the year. OASDI covered wages (1971 to 2004) are updated annually by the Economic process. Covered data for the latest historical year are estimated from tabulations of Form 941 and W-2 data.
- Ratio of disabled worker beneficiaries to the Social Security area population is produced by 5-year age group and sex for the years 1970 to 2099. The Social Security area population is produced by the Demography process and updated annually for the Trustees Report. The number of disabled worker beneficiaries are annual data from the prior year's Trustees Report (Disability subprocess), and are updated annually.
- U.S. armed forces (EDMIL) by age and sex were estimated by the Department of Defense and published by the Census Bureau on a monthly basis (1948-2000) by single year of age (17 to 64) and sex. These data are no longer produced by Census.
- EDMIL by age and sex are estimated by the Economic process as the difference in the monthly resident plus Armed Forces overseas population and the monthly civilian population. These two populations are available from the Census Bureau on a monthly basis (April 2000 to October 2008) by single year of age (16 to 69) and sex. These data are updated several times a year.
- Railroad employment is covered by the Railroad Retirement Act. The annual

historical data are for the period 1971 to 2007. An additional new year of historical data from the Railroad Retirement Board is usually available for inclusion in preparation of the next annual Trustees Report.

- Data obtained from Office of Research, Evaluation, and Statistics (ORES) are tabulations of quarterly Form 941 data. Data currently used are the OASDI, HI, and income taxable wages by sector for the most recent five years. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI covered wages by sector for the latest historical years.
- Data obtained from the most recently available 1.0% CWHS active file, maintained on Social Security's mainframe and made available by ORES. The years of data are 1951 to the third year prior to the current Trustees Report year. The data are used for comparison of OASDI covered earnings from other sources.
- Data obtained from extracting information from the 1.0% Employee-Employer Files, maintained on Social Security's mainframe and made available by ORES. Each year two files are created: a Version 1 file for the third year prior to the current Trustees Report and a Version 3 file for the fifth year prior to the current Trustees Report. Data currently being used are government and farm sector OASDI, HI, and total wages and employment. Data from the latest files are used to estimate OASDI covered wages for the years available on each file.
- Data obtained from quarterly IRS Form 941 file, provided by Office of Systems (OS). Data currently used are the OASDI and HI taxable wages for 1978 to the most recent year available. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of HI taxable wages, which are then used to develop OASDI covered wages for the most recent historical years.
- Data for the most recent ten years from the Quarterly EPOXY Report, received in hard-copy and, more recently, electronic formats obtained from OS. The data currently used are the total number of workers with OASDI taxable earnings, total number of workers with OASDI self-employed taxable earnings, distribution of number of HI workers by wage intervals, distribution of number of OASDI workers by wage intervals, number of persons with OASDI taxable wages, number of persons with HI taxable wages, number of persons with OASDI taxable self-employment, and number of persons with HI taxable self-employment.
- Data obtained from the Quarterly Trust Fund Letter, received from Office of Financial Policy and Operations (OFPO). Data currently used are OASDI and HI taxable wages accumulated from all Forms 941 and W-2 to date, and changes in self-

employment earnings and self-reported tips since the prior Letter. These data are for years 1978 to the most recent year available.

- Data obtained from OS on amounts of OASDI taxable wages on the Earnings Suspense File for 1937 through the second year prior to the current Trustees Report year. The data are used in estimating total OASDI covered employment.

2.3.c. Development of Output

Equation 2.3.1 - Ratio of Total Employment (RTE)

Since CPS data is only available for those aged 16 and over, RTEs could not be constructed or projected. For those aged 9 and under, we used covered population ratios (CPR), defined as the ratio of OASDI covered employment to the Social Security area population. CPRs for those 9 and younger are disaggregated by single-year of age and sex. Projected values for CPRs by age and sex are set to their latest historical levels. Thus, the aggregate CPR for those aged 9 and under is dependent on the projected age-sex distribution of the population. For those aged 10-15, we used total population ratios (TPR), defined as the ratio of TE to the Social Security area population. The TPRs are disaggregated by sex and age, including those aged 10-13 and 14-15. The TPR for males aged 10-13 is projected to remain constant at its latest historical level. The TPR for females aged 10-13, and the TPRs for males and females aged 14-15, are dependent on the RTP. An increase in the RTP leads to an increase in the TPRs.

The RTE for those aged 16 and over is defined as the ratio of TE to the sum of EW and ES (or alternatively, E less EU) and EDMIL. The RTE is disaggregated by sex and age, including those aged 16-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, and 70 and over. Thus, COV contains 26 RTE equations, 13 for males, and 13 for females.

The aggregate RTE for those aged 16 and over is dependent on the projected age-sex distribution of employment. Each disaggregated RTE is dependent on its corresponding age-sex employment ratio. The employment ratio is roughly defined as the ratio of E to N. Increases in the employment ratio lead to decreases in the RTEs.

Equation 2.3.3 - Non-Covered Employment (NCE)

NCE is disaggregated by age and sex. Age groups include 14-15, 16-17, 18-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, and 70 and over. Employment may not be OASDI covered for a variety of reasons mostly related to the type of work. Consequently, NCE is further disaggregated to the type-of-work components listed below.

Federal Civilian Government - All Federal civilian employees are HI (i.e., Medicare) covered. All Federal Civilian employees hired in January 1984 and later are covered under the Federal Employees Retirement System (FERS) and are OASDI covered. Employees hired before January 1984 are covered under the Civil Service Retirement System (CSRS) and are not OASDI covered. This “closed group” of relatively older CSRS employees is projected to fall to near zero by 2030.

State and Local Government - In 1983, about 70 percent of State and Local Government (S&L) employment and wages were covered under OASDI and HI. Beginning April 1986, all newly hired S&L employees were covered under HI. Beginning January 1990, all S&L employees not under an S&L retirement system were covered under OASDHI.

By 2002, about 30 and 10 percent of S&L employment (and also wages) are still not covered under OASDI, and HI respectively. The closed group of relatively older S&L employees not covered under HI is projected to fall to near zero by 2020. S&L employment not covered under OASDI is projected to grow at about the same rate as the labor force.

Students at Public Schools - Prior to 2000, students working at S&L public schools were covered under OASDI and HI if the other school employees were covered. In 2000, legislation offered an “open season” allowing schools to remove their students from coverage. Virtually all major schools opted for removal. Hence, almost no students working at their public schools are covered under OASDI or HI. Students at public schools are projected to grow at about the same rate as the population aged 18 to 24.

Election Workers - Most S&L election workers are subject to an earnings test and are not covered under OASDHI. The earnings test was raised from \$100 to \$1,000 beginning January 1995 and indexed beginning in 2000. Election workers are projected to grow at about the same rate as LC.

Private Household - The threshold for coverage of domestic employees’ earnings was raised from \$50 per calendar quarter to \$1,000 per calendar year (CY) per employee. Domestic workers are no longer covered if under age 18. Private household employment is projected to grow at about the same rate as E and vary with RTP.

Students at Private Schools - All students working in private schools are not covered under OASDHI. Students at private schools are projected to grow at about the same rate as the population aged 18 to 24.

Railroad - Employers do not submit payments for payroll taxes to the IRS for railroad employees. Railroad employees are projected by the Railroad Retirement Board.

Other Noncovered Workers - Set at a constant amount of 0.4 million to reflect relatively small and shrinking groups such as paperboys.

Underground Economy Workers - Set to the at-any-time employed in the other immigrant population who have no reported earnings and therefore are part of the underground economy (i.e., TEO_UND).

Equation 2.3.5 - Self-Employed Only (SEO)

SEO is projected to grow at the same rate as ES.

Equation 2.3.7 - Ratio of Combination Workers (RCMB)

Total CMB_TOT can be separated into two groups depending on whether they have OASDI covered wages under or over the TAXMAX. CMB_TOT with covered wages under the TAXMAX have taxable wages and self-employed income. CMB_TOT with covered wages over the TAXMAX have taxable wages only. CMB_TOT with covered wages over the TAXMAX would have paid taxes on their self-employed income if the TAXMAX had been eliminated.

Total CMB_TOT is projected as a ratio to WSW. This ratio is dependent on the RTP. If RTP rises, then the CMB_TOT falls.

Equation 2.3.10 - Ratio of Covered Wages (RWSC)

RWSC is disaggregated by the following sectors: Federal Civilian government, Federal Military, S&L government, and Private.

Federal Civilian government - Total Federal civilian employment and wages are split by retirement system. Those under FERS are OASDI covered, while those under CSRS are not. Employment and wages are projected for workers under each retirement system. Employment under CSRS is a closed group that is expected to fall to zero by about 2030. Employment under FERS is defined as total Federal employment less employment under CSRS. Total Federal civilian employment is projected to grow at an average annual rate of about 0.2 percent from 2004 to the end of the short-range period, and about equal to the growth in the LC thereafter. The growth rates in the average wage for those under CSRS and FERS are projected based on, for the first five years, pay raises assumed under the most recent OMB FY Budget and on the growth rate in the CPI. Hence, the RWSC for the Federal civilian employment is defined as the ratio of wages for employment under FERS to total Federal civilian wages.

Federal Military - The RWSC for the Federal military sector is projected to remain constant at its latest actual historical level.

S&L government - The RWSC for the S&L government sector is projected to remain constant at its latest actual historical level.

Private - The private sector is separated into sub-sectors including private households, farm, railroad, tips, and a residual private “base”. The RWSCs for the private household and farm sub-sectors are projected to remain constant at their latest actual historical levels. By definition, the RWSCs for the railroad and tips sub-sectors are projected to remain constant at 0.0 and 1.0, respectively. The projected RWSC for the private base sub-sector is dependent on the ratio of EO wage workers in the private base sub-sector who are covered under the OASDI program to all EO wage workers in the private base sub-sector. We assume that all of EO will be wage workers employed in the private residual base sub-sector of the economy and that the proportion of EO that is covered under the OASDI program will decrease. Therefore, we assume that the RWSC for the private residual base sector will also decrease.

The estimation methodology for the other immigrant employment and earnings will be described in detail in Appendix 2.3. (forthcoming)

Equation 2.3.12 - Ratio of Covered Self-Employed Earnings (RCSE)

The RCSE is projected to remain constant at its latest actual historical level.

Equation 2.3.17 - Average Wage Index (AWI)

The growth in the AWI is projected to be equal to the growth in an economy-wide average wage (ACWC) defined as the ratio of WSD to total wage workers (covered and non-covered). Total wage workers are projected as the sum of WSW and the various groups of non-covered wage workers discussed above.

Equation 2.3.18 - OASDI Taxable Maximum (TAXMAX)

By law, the growth in the AWI is used to increase the TAXMAX.

2.4. Effective TAXABLE PAYROLL (TAXPAY)

2.4.a. Overview

TAXPAY estimates historical annual taxable earnings data including total employee OASDI taxable wages (WTEE), total employer taxable wages (WTER), and total self-employed taxable income (SET). By law, each employee is required to pay the OASDI tax on wages from all covered jobs up to the TAXMAX, while each employer is required to pay the OASDI tax on the wages of each worker up to the TAXMAX. If an employee works more than one covered wage job and the sum of all covered wages exceeds the TAXMAX, the employee but not the employer is due a refund. Hence, WTER is greater than WTEE. The difference (i.e., WTER less WTEE) is defined as multi-employer refund wages (MER).

TAXPAY also estimates the historical annual effective OASDI taxable payroll (ETP). ETP is the amount of earnings in a year which, when multiplied by the combined employee-employer tax rate, yields the total amount of taxes due from wages and self-employed income in the year. ETP is used in estimating OASDI income and in determining income and cost rates and the actuarial balance. ETP is defined as WTER plus SET less one-half of MER.

TAXPAY projects annual values for ETP after first estimating its components. The components in turn are estimated by a collection of ratios. The employee taxable ratio (RWTEE) is defined as the ratio of WTEE to WSC. The multi-employer refund wage ratio (RMER) is defined as the ratio of MER to WSC. The self-employed net income taxable ratio (RSET) is defined as the ratio of SET to CSE_TOT. Equations 2.4.1 through 2.4.8 outline the projection methodology.

$$RWTEE = RWTEE(\cdot) \quad (2.4.1)$$

$$WTEE = RWTEE * WSC \quad (2.4.2)$$

$$RMER = RMER(\cdot) \quad (2.4.3)$$

$$MER = RMER * WSC \quad (2.4.4)$$

$$WTER = WTEE + MER \quad (2.4.5)$$

$$RSET = RSET(\cdot) \quad (2.4.6)$$

$$SET = RSET * CSE_TOT \quad (2.4.7)$$

$$ETP = WTER + SET - 0.5 * MER \quad (2.4.8)$$

Over the short-range projection horizon (i.e., first 10 years), TAXPAY also projects annual OASDI wage tax liabilities (WTL) and self-employment tax liabilities (SEL). In Equation 2.4.9, WTL is the product of the effective taxable wages, defined as WTER less one-half of MER, and the combined OASDI employee-employer tax rate (TRW). In Equation 2.4.10, SEL is the product of SET and the OASDI self-employed tax rate (TRSE).

$$WTL = WTER * TRW \quad (2.4.9)$$

$$SEL = SET * TRSE \quad (2.4.10)$$

Also over the short-range horizon, TAXPAY decomposes WTL into quarterly wage tax liabilities (WTLQ) then to quarterly wage tax collections (WTLQC). TAXPAY also decomposes SEL into quarterly self-employed net income tax collections (SELQC).

$$WTLQ = WTLQ(\cdot) \quad (2.4.11)$$

$$WTLQC = WTLQC(\cdot) \quad (2.4.12)$$

$$SELQC = SELQC(\cdot) \quad (2.4.13)$$

Finally, over the first two projected quarters, TAXPAY estimates of WTLQC and SELQC are replaced with ones from the most recent OMB FY Budget. And, over the first four projected quarters, TAXPAY includes estimates for appropriation adjustments (AA).

$$AA = AA(\cdot) \quad (2.4.14)$$

2.4.b. *Input Data*

Data used to obtain values input directly to model

- Data obtained from ORES by email for the amounts of single and multi-employer refunds for the latest 5 years. Each year, data are updated.
- Data obtained from ORES are tabulations of quarterly Form 941 data. Data currently used are the OASDI, HI, and income taxable wages by sector for the most recent five years. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI taxable wages by sector for the latest historical years.
- Data obtained from the most recently available 1.0% CWHS active file, maintained on Social Security's mainframe and made available by ORES. The years of data are

1951 to the third year prior to the current Trustees Report year. The data are used for comparison of OASDI taxable earnings from other sources.

- Data obtained from quarterly IRS Form 941 file, provided by OS. Data currently used are the OASDI and HI taxable wages for 1978 to the most recent year available. The data represent changes in reported wages since the prior quarterly report. The most recent data are appended to previously reported data. Annual totals are computed and used to derive estimates of OASDI taxable wages for the most recent historical years.
- Data for the most recent ten years from the Quarterly EPOXY Report, received in hard-copy and, more recently, electronic formats obtained from OS. The data currently used are the total number of workers with OASDI taxable earnings, total number of workers with OASDI self-employed taxable earnings, distribution of number of HI workers by wage intervals, distribution of number of OASDI workers by wage intervals, number of persons with OASDI taxable wages, number of persons with HI taxable wages, number of persons with OASDI taxable self-employment, number of persons with HI taxable self-employment, number of workers with single-employer excess wages, and number of workers with multi-employer excess wages.
- Data obtained from the Quarterly Trust Fund Letter, received from OFPO. Data currently used are OASDI and HI taxable wages accumulated from all Forms 941 and W-2 to date, and changes in self-employment earnings and self-reported tips since the prior Letter. These data are for years 1978 to the most recent year available.

Long-Range OASDI Projection Data

Historical and projected data from Sections 2.1, 2.2, and 2.3. The data for the following variables have final year of 2099. Each variable is shown with starting year.

AIW	Average wage for indexing (\$), 1971
AWSCFM	Average covered wage for farm workers (\$), 1971
AWSCML	Average covered wage for military (\$), 1971
DMWCHI	Deemed military wage credits for HI (\$ millions), 1978
DMWCOD	Deemed military wage credits for OASDI (\$ millions), 1978
ECFCHO	Number of HI-only covered Federal Civilian workers (millions), 1983
ECFCOD	Number of OASDI covered Federal Civilian workers (millions), 1983
ECHITOT	Number of HI covered workers (millions), 1971
ECSEHI	Number of HI covered self-employed workers (millions), 1994
ECSENO MAX	Number of covered self-employed workers if no taxable maximum (millions), 1983
ECSEO	Number of OASDI covered self-employed only workers (millions), 1971

ECSEOD	Number of OASDI covered self-employed workers (millions), 1971
ECSLNOIS	Number of non-OASDI covered State and Local workers including students (millions), 1983
ECSLNRP	Number of OASDI covered State and Local workers with no retirement plan (millions), 1983
ECSLOD	Number of OASDI covered State and Local workers (millions), 1983
ECSLP91	Number of State and Local workers covered under OASDI under pre-1991 law (millions), 1983
ECWSHI	Number of HI covered wage workers (millions), 1983
ECWSOD_MEF	Number of OASDI covered wage workers on the Master Earnings File (MEF) in millions, 1990
ECWSODFU16_MEF	Number of OASDI covered wage workers on the MEF, female under 16 years old (millions), 1990
ECWSODF1619_MEF	Number of OASDI covered wage workers on the MEF, female 16 to 19 years old (millions), 1990
ECWSODF2024_MEF	Number of OASDI covered wage workers on the MEF, female 20 to 24 years old (millions), 1990
ECWSODF2529_MEF	Number of OASDI covered wage workers on the MEF, female 25 to 29 years old (millions), 1990
ECWSODF3034_MEF	Number of OASDI covered wage workers on the MEF, female 30 to 34 years old (millions), 1990
ECWSODF3539_MEF	Number of OASDI covered wage workers on the MEF, female 35 to 39 years old (millions), 1990
ECWSODF4044_MEF	Number of OASDI covered wage workers on the MEF, female 40 to 44 years old (millions), 1990
ECWSODF4549_MEF	Number of OASDI covered wage workers on the MEF, female 45 to 49 years old (millions), 1990
ECWSODF5054_MEF	Number of OASDI covered wage workers on the MEF, female 50 to 54 years old (millions), 1990
ECWSODF5559_MEF	Number of OASDI covered wage workers on the MEF, female 55 to 59 years old (millions), 1990
ECWSODF6064_MEF	Number of OASDI covered wage workers on the MEF, female 60 to 64 years old (millions), 1990
ECWSODF6569_MEF	Number of OASDI covered wage workers on the MEF, female 65 to 69 years old (millions), 1990
ECWSODF700_MEF	Number of OASDI covered wage workers on the MEF, female 70 years old and older (millions), 1990
ECWSODMU16_MEF	Number of OASDI covered wage workers on the MEF, male under 16 years old (millions), 1990
ECWSODM1619_MEF	Number of OASDI covered wage workers on the MEF, male 16 to 19 years old (millions), 1990

ECWSODM2024_MEF	Number of OASDI covered wage workers on the MEF, male 20 to 24 years old (millions), 1990
ECWSODM2529_MEF	Number of OASDI covered wage workers on the MEF, male 25 to 29 years old (millions), 1990
ECWSODM3034_MEF	Number of OASDI covered wage workers on the MEF, male 30 to 34 years old (millions), 1990
ECWSODM3539_MEF	Number of OASDI covered wage workers on the MEF, male 35 to 39 years old (millions), 1990
ECWSODM4044_MEF	Number of OASDI covered wage workers on the MEF, male 40 to 44 years old (millions), 1990
ECWSODM4549_MEF	Number of OASDI covered wage workers on the MEF, male 45 to 49 years old (millions), 1990
ECWSODM5054_MEF	Number of OASDI covered wage workers on the MEF, male 50 to 54 years old (millions), 1990
ECWSODM5559_MEF	Number of OASDI covered wage workers on the MEF, male 55 to 59 years old (millions), 1990
ECWSODM6064_MEF	Number of OASDI covered wage workers on the MEF, male 60 to 64 years old (millions), 1990
ECWSODM6569_MEF	Number of OASDI covered wage workers on the MEF, male 65 to 69 years old (millions), 1990
ECWSODM700_MEF	Number of OASDI covered wage workers on the MEF, male 70 years old and older (millions), 1990
ESLCG	Number of State and Local workers not covered under HI (millions), 1983
ESLSTUD	Number of noncovered students at public schools employed by their school (millions), 1983
GAPLAG	Ratio of real to potential GDP lagged (units), 1971
RTP	Ratio of real to potential GDP (units), 1971
RU	Civilian unemployment rate (percent), 1971
SEECOMB	Self-employed earnings of all SE workers who also earned wages in same year (\$ millions), 1991
SEECHI	HI covered self-employed earnings (\$ millions), 1991
SEECNOMAX	Covered self-employed earnings if no taxable maximum (\$ millions), 1991
SEECOD	OASDI covered self-employed earnings (\$ millions), 1991
SEECOD_OLD	OASDI covered self-employed earnings excluding self-employed earnings of workers with covered wages greater than or equal to the OASDI taxable maximum (\$ millions), 1971
TAXMAXHI	HI taxable maximum (\$) – 0 indicates no maximum, 1971
TAXMAXOD	OASDI taxable maximum (\$), 1971
TCFCD	Proportion of annual Federal Civilian wages earned in each quarter (units), 1971

TCMD	Proportion of annual military wages earned in each quarter (units), 1971
TCPD	Proportion of annual private sector wages earned in each quarter (units), 1971
TCSLD	Proportion of annual State and Local wages earned in each quarter (units), 1971
WSCCMB	Wages earned in same year by all SE workers with both types of earnings (\$ millions), 1991
WSCFCHO	HI Covered wages of Federal Civilian HI-only workers (\$ millions), 1983
WSCFCOD	OASDI Covered wages of Federal Civilian workers (\$ millions), 1971
WSCFM	Covered wages of farm workers (\$ millions), 1971
WSCHI	HI covered wages (\$ millions), 1983
WSCML	Covered wages of members of the Armed Forces (\$ millions), 1971
WSCOD	OASDI covered wages (\$ millions), 1971
WSCOD_SF	OASDI covered wages on the Suspense File (\$ millions), 1971
WSCP HH	Covered wages of private household workers (\$ millions), 1971
WSCP NF	Covered wages of private nonfarm workers (\$ millions), 1971
WCSLHI	HI covered State and Local wages (\$ millions), 1983
WCSLNRP	Covered wages of State and Local workers with no retirement plan (\$ millions), 1971
WCSLOD	OASDI covered State and Local wages (\$ millions), 1971
WCSL P91	Wages of State and Local workers covered under OASDI under pre-1991 law (\$ millions), 1971
WSD	Total NIPA wages (\$ millions), 1971
WSP	Total NIPA private sector wages (\$ millions), 1971
WSS	Total NIPA compensation (\$ millions), 1971
WSSLCG	Wages of State and Local workers not covered under HI (\$ millions), 1983
WSSLNOIS	Wages of non-OASDI covered State and Local workers including students (\$ millions), 1983
WSSLSTUD	Wages of noncovered students at public schools employed by their school (\$ millions), 1983
WSTTIPSSR	Taxable tips reported by tip earner instead of employer (\$ millions), 1978
WTWPO	Proportion of annual Postal Service wages earned in each quarter (units), 1971

Other direct input data

- FICA, SECA, and Federal Employer tax transfers by month from the Department of the Treasury for years 1984 to 2008.
- FICA, SECA, and Federal Employer tax transfers by month from the Department of the Treasury for January 2009 and estimated transfers for February 2009 to June 2009.
- FICA and SECA tax transfers by month split by liability period from the Department of the Treasury for 1984 to 2007.
- FICA and SECA tax transfers by month split by liability period from the Department of the Treasury for January 2008 to June 2008.
- Monthly OASI, DI, and HI deposits by States, received periodically throughout the year from the Department of the Treasury for 1984 to 2008. (This is an obsolete type of revenue which has had no valid non-zero amount since 2002.)
- Historical annual HI taxable self-employment earnings for 1983 to 2006.
- Historical annual OASDI taxable self-employment earnings for 1971 to 2006.
- Historical annual OASDI multi-employer refund wages for 1971 to 2007.
- Historical annual HI taxable wages for 1983 to 2007.
- Historical annual OASDI taxable wages for 1971 to 2007.
- Historical annual HI-only taxable Federal Civilian wages for 1987 to 2003.
- Historical annual OASDI taxable Federal Civilian wages for 1971 to 2003.
- Historical annual HI taxable Federal Civilian wages for 1987 to 2003.
- Historical annual OASDI taxable farm sector wages for 1971 to 2003.
- Historical annual HI taxable farm sector wages for 1991 to 2003.
- Historical annual OASDI taxable military sector wages for 1971 to 2003.
- Historical annual HI taxable military sector wages for 1991 to 2003.
- Historical annual OASDI taxable State and Local government sector wages for 1971

- to 2003.
- Historical annual HI taxable State and Local government sector wages for 1987 to 2003.
 - Historical and projected annual OASDI taxable tips for employees as reported by employers for 1971 to 2018.
 - Historical and projected annual OASDI taxable tips for employers as reported by employers for 1971 to 2018.
 - Annual and quarterly OASDI and HI taxable wages for calendar year 2008.
 - Estimated annual HI taxable payroll for railroad workers for 2007 to 2099.
 - Historical FICA and SECA appropriation adjustments for OASI, DI, and HI by month for 1968 to 2008.
 - Historical FICA revenues for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical SECA revenues for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical Federal Employer revenues for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical Deposits by States for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical single-employer refunds of excess taxes for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical FICA credits for OASI and DI by quarter for 1984 to 2008.
 - Historical SECA credits for OASI, DI, and HI by quarter for 1984 to 2008.
 - Historical multi-employer refunds of excess taxes for OASI, DI, and HI by month for 1968 to 2008.
 - Data used to estimate future FICA appropriation adjustments for 2008 to 2010 – These adjustments are used to make sure that transfers to the Trust Funds reflect taxable wage amounts reported to SSA. The data input includes estimated quarterly tax liabilities ultimately owed the Trust Funds from President’s Budget model solutions, quarterly tax liabilities for 2009Q1 from IRS-941 tabulations, estimates of tax liabilities for all prior years for 2009Q1 to 2010Q2, and estimated ratios of

quarterly liability amounts reported to SSA over ultimate values for 2008Q2 to 2009Q2. Values are estimated for the four quarters of the Trustees Report year and for the following two quarters.

- Miscellaneous historical covered employment and earnings data:
 - HI Covered self-employed workers for 1986 to 1993.
 - Number of OASDI covered wage workers by age group and sex for 1996.
 - HI covered self-employment earnings for 1971 to 1993.
 - Covered self-employment earnings if there were no taxable maximum for 1971 to 1993.
 - OASDI covered self-employment earnings for 1971 to 1993.

- Miscellaneous historical and fixed projected data:
 - Quarterly distribution of annual OASDI taxable farm wages for 1971 to 2020.
 - Quarterly OASDI covered private nonfarm sector wages for 1971 to 1977.
 - Quarterly OASDI covered State and Local government sector wages for 1971 to 1977.
 - Quarterly OASDI covered military sector wages for 1971 to 1977.
 - Quarterly OASDI covered Federal Civilian sector wages for 1971 to 1977.
 - Quarterly OASDI taxable private nonfarm sector wages for 1971 to 1977.
 - Quarterly OASDI taxable State and Local government sector wages for 1971 to 1977.
 - Quarterly OASDI taxable military sector wages for 1971 to 1977.
 - Quarterly OASDI taxable Federal Civilian sector wages for 1971 to 1977.
 - Quarterly OASDI taxable farm sector wages for 1971 to 1977.
 - OASDI employee, employer, and self-employment tax rates from 1937 to 2100. These contribution rates are set according to the Social Security Act of 1935 and amendments to the Act through 2004. The rates are updated when

legislation mandates a change (which hasn't occurred since 2000).

- Annual OASDI employee credit tax rate for 1984.
- Annual OASDI self-employment credit tax rates for 1984 to 1989.
- Annual trend variable for taxable to covered wage ratio calculation for 1971 to 2100 (no longer used)
- Annual trend variable for taxable to covered self-employment earnings ratio calculation for 1971 to 2100 (no longer used)

Proportions of OASDI tax liabilities for self-employment earnings for the current and prior calendar year estimated to be collected in each quarter through the end of the short-range period for 1971 to 2100. (In any particular quarter, some self-employed individuals are paying taxes on earnings from the prior year and some are paying from the current year's earnings.) Values are derived from historical data from the Office of Tax Analysis (OTA) in the Department of the Treasury for the amount of self-employment taxes transferred to the OASDI Trust Funds in each month split by the calendar year (either the current or the prior) in which the self-employment income was earned. The data are updated every year after historical information for a complete new year is received (usually in March).

- Average OASDI covered wages by age groups and sex for 1996.
- Average OASDI covered wages by age groups and sex for 1996.
- Ratio of OASDI taxable to covered wages by age groups and sex for 1996.
- Corrections to prior FICA appropriation adjustments made in March 2000.
- Projected single-employer refunds of excess tax by month for September 2009 to September 2020.
- An adjustment factor to trend variable for ratio of OASDI taxable to covered wages for 1971 to 2100. A factor of 0.4 is input for years 2002 to 2100 and is applied to model-computed time trend adjustments to RWTEE. Historical analysis suggests a "trend" rate of decline in RWTEE of about 0.0016 per year through 2001. This trend reflects a movement toward greater amounts of wages being paid to workers earning above the taxable maximum. We expect this trend to continue to the end of the short-range projection horizon, but at a slower rate. Consequently, we assume an adjustment factor of 0.4 such that the trend annual rate of decline from 2002 to the end of the short-range projection horizon is about 0.0006 per year (i.e., $0.0016 * 0.4$). The adjustment factor has not been updated since first decided upon. It will be

changed if and when it is deemed appropriate.

- Add factor for ratio of OASDI taxable to covered Federal Civilian government sector wages for 2002 to 2020.
- Add factor for ratio of OASDI taxable to covered State and Local government sector wages for 2002 to 2020.
- Add factor to adjust model-computed add factor for the ratio of OASDI taxable to covered self-employment earnings for 2007.

2.4.c. *Development of Output*

Equation 2.4.1 - Employee Taxable Ratio (RWTEE)

Over the short-range projection horizon, the projected value for RWTEE is the sum of the model's "raw" estimate and an "addfactor." The raw estimate for RWTEE is dependent on the distribution of workers by wage interval, the RELMAX, RTP, the age-sex distribution of wage workers, and a time trend adjustment. The projected distribution of workers by wage interval is an average (or amalgam) distribution over the 1993 through 2004 period. Holding other factors constant, a distribution with relatively more workers with wages over the TAXMAX leads to a lower RWTEE. The RELMAX is defined as the ratio of the TAXMAX to the ACW. A higher RELMAX leads to a higher RWTEE. An increase in the RTP also leads to a higher RWTEE. The change in the projected RWTEE due to the change in the age-sex distribution of wage workers is calculated by allowing employment by age and sex to change while holding taxable ratios (and average covered wages) by age and sex constant to levels in 1996. The time trend adjustment reduces the level of RWTEE by about 0.6 percentage point over the short-range projection horizon.

The addfactor is the product of a "base" adjustment and an assumed weight. The base adjustment is defined as the actual value for RWTEE in the latest historical (or base) year less the raw estimated value for the same period. The weight has an assumed value of 0.9 for the first projected year, then values of 0.8 in the second projected year, 0.7 in the third projected year, ..., and 0.0 in the tenth projected year (i.e., the end of the short-range projection horizon). The addfactor is necessary because we assume that the distribution of wage workers by earnings interval in the base year will gradually change to the amalgam distribution by the end of the short-range period and remain constant thereafter.

Over the long-range projection horizon, RWTEE is assumed to remain constant.

RWTEEs are also projected for various sub-aggregates including Federal Civilian employees under FERS and CSRS, Federal Civilian employees under CSRS only, S&L employees

covered under OASDI, S&L employees covered under HI only, U.S. armed forces, and agriculture. The RWTEE for each sub-aggregate is dependent only on its sub-aggregate RELMAX, that is, the ratio of the TAXMAX to the sub-aggregate's average covered wage.

Equation 2.4.3 - Multi-Employer Refund Wage Ratio (RMER)

The RMER is functionally related to the RWTEE. As RWTEE approaches one, then RMER approaches zero. In between the limit values, RMER is positive. Given the present position of RWTEE and RMER on the function, a projected decline in RWTEE leads to an increase in RMER.

The projected RMER is also dependent on RU. An increase in RU leads to a decrease in RMER.

Equation 2.4.6 - Self-Employed Net Income Taxable Ratio (RSET)

The RSET is disaggregated by type of self-employed worker, SEO and CMB_TOT.

SEO - The RSET is dependent on the distribution of self-employed workers by income interval and a RELMAX. The projected distribution of self-employed workers by income interval is set to the 1996 distribution. The RELMAX is defined here as the ratio of the TAXMAX to the average income for SEO. A higher RELMAX leads to a higher RSET.

CMB_TOT - Taxable self-employed net income for CMB_TOT is projected in two steps. First, a taxable earnings (wages and self-employed income) ratio for CMB_TOT is projected based on the 1996 distribution and a RELMAX defined as the ratio of the TAXMAX to the average covered earnings. The projected level of taxable earnings for CMB_TOT is the product of the estimated taxable earnings ratio for CMB_TOT and their covered earnings. Second, a taxable wage ratio for CMB_TOT is projected based on a RELMAX defined as the ratio of the TAXMAX to the average covered wage for CMB_TOT. The projected level of taxable wages for CMB_TOT is the product of the estimated taxable wage ratio for CMB_TOT and their covered wages.

Taxable self-employed net income for CMB_TOT is obtained by subtracting taxable wages from taxable earnings for CMB_TOT.

A "combined" RSET is calculated as the ratio of taxable self-employed net income for SEO and CMB_TOT to CSE_TOT. As with the RWTEE, the combined RSET is adjusted over the short-range period due to other factors (i.e., RTP, the age-sex distribution of workers, and a trend). The effect of the other factors are taken from RWTEE and "scaled." That is, RSET is adjusted by a percent effect (as opposed to percentage point) that is equal to the percent change in RWTEE due to changes in these other factors.

It is important to note that while the RWTEE is held constant after the short-range period, the RSETs for self-employed workers are not. After the short-range period, the projected RSETs for SEO and CMB_TOT continue to be dependent on their respective RELMAXs. Since by law the TAXMAX grows at the rate of the AWI and since ACSE is assumed to grow faster than the ACW (since only ACW declines with the growth in fringe benefits), the RELMAXs for self-employed workers decline over the long-range period while the RELMAX for wage workers is approximately constant. Hence, the RSETs for SEO and CMB_TOT are projected to decline over the long-range period while the RWTEE is held constant.

Equation 2.4.11 - Quarterly Wage Tax Liabilities (WTLQ)

The total WTLQ is summed from sector components that include the Federal Civilian, Federal Military, S&L, Private Household, Farm, Self-reported Tips, and residual Private Nonfarm. Sector WTLQ is determined by computing ratios of quarterly to annual liabilities for each quarter. For the Private Nonfarm, S&L, Federal Civilian and Military sectors, these “WTLQ ratios” are calculated for each quarter. Each is dependent on the quarterly distribution of WSD and the RWTEE for the relevant sector, and on a payday adjustment that takes into account the actual number of paydays that fall into a particular calendar quarter. WTLQ ratios are also calculated for each quarter for the other sectors. However, these are expected to be constants over the projection horizon.

Equation 2.4.12 - Quarterly Wage Tax Collections (WTLQC)

Employers incur tax liabilities when they pay wages to their employees. These liabilities are required to be deposited with the U.S. Treasury by employers based on the amount of total payroll tax liability (income taxes plus Social Security and Medicare taxes withheld) accumulated. Some very large employers must deposit their tax liabilities the next banking day after paying their employees. Other levels of accumulated tax liabilities require depositing within three days, by the middle of the following month, or by the end of the month following the quarter. If employers follow these deposit requirements, the result is that all tax liability for a particular quarter is deposited by the last day of the month following the end of the quarter. Thus, the WTLQC for any particular quarter are the sum of the tax liabilities deposited for wages paid in the same quarter and the liabilities deposited for wages paid in the prior quarter.

WTLQC is summed from sector components that include the Federal Civilian, Federal Military, Farm, S&L, and residual Private Nonfarm (including Private Household and Self-reported Tips). For the Federal Civilian and Military sectors, the WTLQC is set equal to its respective WTLQ since tax liabilities for the two sectors are collected immediately. The WTLQC for Farm is also set equal to its WTLQ, due in part to the fact that farms report tax liabilities annually. For the S&L and Private Nonfarm sectors, WTLQC is the product of WTLQ and the proportion of WTLQ that should be deposited in the month following the end of each quarter. This proportion is based on the deposit requirements and estimates of accumulated tax liabilities, which in turn are based on firm size (or total wages paid).

Equation 2.4.13 - Quarterly Self-Employed Net Income Tax Collections (SELQC)

For wage workers, annual liabilities (WTL) are distributed to quarterly liabilities (WTLQ), which in turn are distributed to quarterly collections (WTLQC). However, for self-employed workers, annual liabilities (SEL) are distributed directly to SELQC, since the SSA only receives self-employed liability amounts on an annual basis (from tabulations of Form 1040 Schedule SE provided by IRS).

SEL for a particular calendar year is distributed as collections to the four quarters of that year and to the first three quarters of the next year. This distribution uses quarterly proportions that are based on an historical pattern of the amount of SEL collected in each month, as estimated by the OTA. The OTA estimates reflect IRS regulations that require self-employed workers to deposit estimated tax liabilities four times a year (January, April, June, and September).

Equation 2.4.14 - Appropriation Adjustments (AA)

We estimate WTLQC and SELQC for the next two quarters based on projected levels of WSD and Y in the various OMB FY Budgets. The Treasury uses these estimates to make *initial appropriations* to the OASDI Trust Funds over the period. Roughly one year after an initial appropriation has been made for a quarter, the initial appropriation is adjusted based on certified taxable wages reported on Forms 941 and W-2 and taxable self-employment income reported on Forms 1040 Schedule SE. This *appropriation adjustment* occurs quarterly and includes amounts due to newly-reported wages and self-employment income for all earlier periods.

FICA appropriation adjustments for wages are projected for all quarters for which we know what the Treasury will transfer. This generally means that we make estimates for adjustments to occur through the second quarter of the year following the Trustees Report year. The projected adjustment for each quarter is composed of two parts. The first estimates the error in the initial appropriation for the fourth earlier quarter. This error is defined as the difference between the Treasury's initial appropriation and that portion of TAXPAY's latest estimate for that quarter which is expected to be certified at the point when the adjustment is made. The second part is an estimate of the tax liability from additional certified wages for all previous periods and is based on an average of additional amounts over a recent historical period. These estimates are currently made in separate Excel files (generally named "EstimatedAppAdjAlt2.xls" and placed in a folder unique to the particular Trustees Report, e.g., \\wmpiet\usr\wmpiet\Excel\TR06), where they are further documented in a "Notes" sheet in the Excel file.

Appropriation adjustments for self-employment income are projected over the entire projection horizon based on the historical pattern of reporting. These estimates are made in the TAXPAY.

Appendix 2-1

Equations

2.1 U.S. Employment (USEMP)

MALE LFPR Equations - Annual

Regressions using March CPS Data

Aged 20 to 44

$$PM2024NM = 0.55951 + 0.00198 - 1.0 * RM2024D - 0.91435 * RMM2024NM - 0.33947 * RNM2024S.1 + 0.16484 * NM2024NM / (NM2024NM + NM2024MS + NM2024MA) + 0.21031$$

$$* RTP4Q.1 + PM2024_A94C + PM2024_A94M$$

$$PM2529NM = 0.71186 - 1.0 * RM2529D - 0.92991 * RMM2529NM + 0.06580 * NM2529NM / (NM2529NM + NM2529MS + NM2529MA) + 0.15942 * RTPL2T5 + PM2554_A94C$$

$$+ PM2554_A94M$$

$$PM3034NM = 0.83231 - 1.0 * RM3034D - 0.71864 * RMM3034NM + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM3539NM = 0.82140 - 0.02915 - 1.0 * RM3539D - 0.84057 * RMM3539NM + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM4044NM = 0.73248 + 0.02540 - 1.0 * RM4044D - 0.00000 * RMM4044NM + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM2024MS = 1.00368 + 0.01443 - 1.0 * RM2024D - 1.05842 * RMM2024MS - 0.14902 * RNM2024S.1 + PM2024_A94C + PM2024_A94M$$

$$PM2529MS = 0.99457 - 1.0 * RM2529D - 1.22879 * RMM2529MS - 0.01937 * (PF2529MS - PF2554_A94C - PF2554_A94M) +$$

$$PM2554_A94C + PM2554_A94M$$

$$PM3034MS = 0.96492 - 1.0 * RM3034D - 1.26554 * RMM3034MS - 0.03628 * (PF3034MS - PF2554_A94C - PF2554_A94M) + 0.05 *$$

$$RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM3539MS = 0.94746 - 1.0 * RM3539D - 1.02117 * RMM3539MS - 0.01377 * (PF3539MS - PF2554_A94C - PF2554_A94M) + 0.05 *$$

$$RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM4044MS = 0.95152 - 0.00165 - 1.0 * RM4044D - 1.14303 * RMM4044MS - 0.02197 * (PF4044MS - PF2554_A94C - PF2554_A94M) +$$

$$0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM2024MA = 0.55549 + 0.04400 - 1.0 * RM2024D - 0.30242 * RMM2024MA + 0.38966 * RTP4Q.1 - 0.20116 * RNM2024S.1 +$$

$$PM2024_A94C + PM2024_A94M$$

$$PM2529MA = 0.73674 - 1.0 * RM2529D - 0.89617 * RMM2529MA + 0.19750 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM3034MA = 0.87897 - 1.0 * RM3034D - 0.59344 * RMM3034MA + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM3539MA = 0.87523 - 1.0 * RM3539D - 0.20124 * RMM3539MA + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

$$PM4044MA = 0.87250 - 0.02214 - 1.0 * RM4044D - 1.41032 * RMM4044MA + 0.05 * RTPL2T5 + PM2554_A94C + PM2554_A94M$$

Aged 75 and Over

$$PM7579 = 0.58899 * PM65O_A94MM * (PM7074.5 / PM65O_A94MM.5)$$

$$PM8084 = 0.52749 * PM65O_A94MM * (PM7579.5 / PM65O_A94MM.5)$$

$$PM850 = 0.48912 * PM65O_A94MM * (PM8084.5 / PM65O_A94MM.5)$$

Regressions using Annual CPS Data

Aged 16 to 19

$$PM1617 = 0.35974 * DIFF (RTP) + 0.31790 * DIFF (RTP.1) + 0.46318 * DIFF (DROPOUT1012M) + PM1617.1$$

$$TPM1819 = 0.25962 * DIFF (RTP) + 0.26610 * DIFF (DROPOUT1012M) - 1.44724 * DIFF (NM1819M / (NM1819M + NM1819)) +$$

$$TPM1819.1$$

Aged 45 to 54

$$PM4549NM = -1.0 * RM4549D + 0.02887 * RTPL1T4 + 0.75041$$

$$PM5054NM = -1.0 * RM5054D + 0.77072$$

$$PM4549MS = -1.0 * RM4549D - 0.01488 * PF4549MS + 0.15238 * RTPL1T4 + 0.83034$$

$$PM5054MS = -1.0 * RM5054D - 0.00463 * PF5054MS + 0.19639 * RTPL1T4 + 0.76277$$

$$PM4549MA = -1.0 * RM4549D + 0.00850 * 84 + 0.18962$$

$$PM5054MA = -1.0 * RM5054D + 0.43104 * RTPL1T4 + 0.42746$$

Aged 55 to 74

$$PM55 = 0.11202 * RTP.3 - 1.0 * RM55D + 0.01523 - 0.27650 + (0.24398 - 0.11202) * -0.01 + (MOVAVG (3, PM5054_FE.2 +$$

$$PM2554_94M_TE.2 + PM2554_90C_TE.2 + RM5054D.2) - 0.05338$$

* RTP.3 - 1))

PM56 = 0.10041 * RTP.3 - 1.0 * RM56D - 0.00180 + (PM55.1 + RM55D.1 - 0.11202 * RTP.4)

PM57 = 0.13081 * RTP.3 - 1.0 * RM57D - 0.04182 + (PM56.1 + RM56D.1 - 0.10041 * RTP.4)

PM58 = 0.24841 * RTP.3 - 1.0 * RM58D - 0.12905 + (PM57.1 + RM57D.1 - 0.13081 * RTP.4)

PM59 = 0.25992 * RTP.3 - 1.0 * RM59D - 0.02818 + (PM58.1 + RM58D.1 - 0.24841 * RTP.4)

PM60 = 0.31289 * RTP.3 - 1.0 * RM60D - 0.08886 + (PM59.1 + RM59D.1 - 0.25992 * RTP.4)

PM61 = 0.32660 * RTP.3 - 1.0 * RM61D - 0.04412 + (PM60.1 + RM60D.1 - 0.31289 * RTP.4)

PM62 = (1.0 - 1.0 * DIFF (RRADJ_M62) + 0.100 * DIFF (RETEST64U) + 0.15 * DIFF (RTP.2)) * (PM62.1 + RM62D.1) - RM62D

PM63 = (1.0 - 1.0 * DIFF (RRADJ_M63) + 0.100 * DIFF (RETEST64U) + 0.15 * DIFF (RTP.2)) * (PM63.1 + RM63D.1) - RM63D

PM64 = (1.0 - 1.0 * DIFF (RRADJ_M64) + 0.100 * DIFF (RETEST64U) + 0.15 * DIFF (RTP.2)) * (PM64.1 + RM64D.1) - RM64D

PM65 = (1.0 - 1.0 * DIFF (RRADJ_M65) + 0.100 * DIFF (RETEST65) + 0.15 * DIFF (RTP.2)) * PM65.1

PM66 = (1.0 - 1.0 * DIFF (RRADJ_M66) + 0.100 * DIFF (RETEST66) + 0.15 * DIFF (RTP.2)) * PM66.1

PM67 = (1.0 - 1.0 * DIFF (RRADJ_M67) + 0.100 * DIFF (RETEST67) + 0.15 * DIFF (RTP.2)) * PM67.1

PM68 = (1.0 - 1.0 * DIFF (RRADJ_M68) + 0.100 * DIFF (RETEST68) + 0.15 * DIFF (RTP.2)) * PM68.1

PM69 = (1.0 - 1.0 * DIFF (RRADJ_M69) + 0.100 * DIFF (RETEST69) + 0.15 * DIFF (RTP.2)) * PM69.1

PM70 = (1.0 - 1.0 * DIFF (RRADJ_M70) + 0.15 * DIFF (RTP.2)) * PM70.1

PM71 = (0.04127 + 0.91152) * PM70.1

PM72 = (0.02632 + 0.90027) * PM71.1

PM73 = (0.05478 + 0.91456) * PM72.1

PM74 = (-0.02105 + 0.87355) * PM73.1

MALE LFPR Equations - Quarterly

Regressions using March CPS Data

Aged 20 to 44

PM2024NM = 0.55951 + 0.00198 - 1.0 * RM2024D - 0.91435 * RMM2024NM - 0.33947 * RNM2024S + 0.16484 * NM2024NM / (NM2024NM + NM2024MS + NM2024MA) + 0.21031 * RTP.1 + PM2024_A94C + PM2024_A94M

PM2529NM = 0.71186 - 1.0 * RM2529D - 0.92991 * RMM2529NM + 0.06580 * NM2529NM / (NM2529NM + NM2529MS + NM2529MA) + 0.15942 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM3034NM = 0.83231 - 1.0 * RM3034D - 0.71864 * RMM3034NM + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM3539NM = 0.82140 - 0.02915 - 1.0 * RM3539D - 0.84057 * RMM3539NM + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM4044NM = 0.73248 + 0.02540 - 1.0 * RM4044D - 0.00000 * RMM4044NM + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM2024MS = 1.00368 + 0.01443 - 1.0 * RM2024D - 1.05842 * RMM2024MS - 0.14902 * RNM2024S + PM2024_A94C + PM2024_A94M

PM2529MS = 0.99457 - 1.0 * RM2529D - 1.22879 * RMM2529MS - 0.01937 * MOVAVG (4, (PF2529MS.1 - PF2554_A94C.1 - PF2554_A94M.1)) + PM2554_A94C + PM2554_A94M

PM3034MS = 0.96492 - 1.0 * RM3034D - 1.26554 * RMM3034MS - 0.03628 * MOVAVG (4, (PF3034MS.1 - PF2554_A94C.1 - PF2554_A94M.1)) + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM3539MS = 0.94746 - 1.0 * RM3539D - 1.02117 * RMM3539MS - 0.01377 * MOVAVG (4, (PF3539MS.1 - PF2554_A94C.1 - PF2554_A94M.1)) + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM4044MS = 0.95152 - 0.00139 - 1.0 * RM4044D - 1.14303 * RMM4044MS - 0.02197 * MOVAVG (4, (PF4044MS.1 - PF2554_A94C.1 - PF2554_A94M.1)) + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM2024MA = 0.55549 + 0.04400 - 1.0 * RM2024D - 0.30242 * RMM2024MA + 0.38966 * RTP.1 - 0.20116 * RNM2024S + PM2024_A94C + PM2024_A94M

PM2529MA = 0.73674 - 1.0 * RM2529D - 0.89617 * RMM2529MA + 0.19750 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM3034MA = 0.87897 - 1.0 * RM3034D - 0.59344 * RMM3034MA + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM3539MA = 0.87523 - 1.0 * RM3539D - 0.20124 * RMM3539MA + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

PM4044MA = 0.87250 - 0.02214 - 1.0 * RM4044D - 1.41032 * RMM4044MA + 0.05 * MOVAVG (4, RTP.2) + PM2554_A94C + PM2554_A94M

Aged 75 and Over

PM7579 = 0.58899 * PM65O_A94MM * MOVAVG (4, PM7074.20)

PM8084 = 0.52749 * PM65O_A94MM * MOVAVG (4, PM7579.20)

PM850 = 0.48912 * PM65O_A94MM * MOVAVG (4, PM8084.20)

Regressions using Annual CPS Data

Aged 16 to 19

$$\begin{aligned} \text{PM1617} &= 0.35974 * \text{DIFF (RTP.1)} + 0.31790 * \text{DIFF (RTP.5)} + 0.46318 * \text{DIFF (DROPOUT1012M)} + \text{PM1617.1} \\ \text{TPM1819} &= 0.25962 * \text{DIFF (RTP.1)} + 0.26610 * \text{DIFF (DROPOUT1012M)} - 1.44724 * \text{DIFF (RM1819M)} + \text{TPM1819.1} \end{aligned}$$

Aged 45 to 54

$$\begin{aligned} \text{PM4549NM} &= -1.0 * \text{RM4549D} + 0.02887 * \text{MOVAVG (4, RTP.1)} + 0.75041 \\ \text{PM5054NM} &= -1.0 * \text{RM5054D} + 0.77072 \end{aligned}$$

$$\begin{aligned} \text{PM4549MS} &= -1.0 * \text{RM4549D} - 0.01488 * \text{PF4549MS} + 0.15238 * \text{MOVAVG (4, RTP.1)} + 0.83034 \\ \text{PM5054MS} &= -1.0 * \text{RM5054D} - 0.00463 * \text{PF5054MS} + 0.19639 * \text{MOVAVG (4, RTP.1)} + 0.76277 \end{aligned}$$

$$\begin{aligned} \text{PM4549MA} &= -1.0 * \text{RM4549D} + 0.00850 * 84 + 0.18962 \\ \text{PM5054MA} &= -1.0 * \text{RM5054D} + 0.43104 * \text{MOVAVG (4, RTP.1)} + 0.42746 \end{aligned}$$

Aged 55 to 74

$$\begin{aligned} \text{PM55} &= 0.11202 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM55D} + 0.01523 - 0.27650 + (0.24398 - 0.11202) - 0.01 + (\text{PM5054L_FE}) \\ \text{PM56} &= 0.10041 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM56D} - 0.00180 + (\text{PM55.4} + \text{RM55D.4} - 0.11202 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM57} &= 0.13081 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM57D} - 0.04182 + (\text{PM56.4} + \text{RM56D.4} - 0.10041 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM58} &= 0.24841 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM58D} - 0.12905 + (\text{PM57.4} + \text{RM57D.4} - 0.13081 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM59} &= 0.25992 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM59D} - 0.02818 + (\text{PM58.4} + \text{RM58D.4} - 0.24841 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM60} &= 0.31289 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM60D} - 0.08886 + (\text{PM59.4} + \text{RM59D.4} - 0.25992 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM61} &= 0.32660 * \text{MOVAVG (5, RTP.10)} - 1.0 * \text{RM61D} - 0.04412 + (\text{PM60.4} + \text{RM60D.4} - 0.31289 * \text{MOVAVG (5, RTP.14)}) \\ \text{PM62} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M62)} + 0.100 * \text{DIFF (RETEST64U)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * (\text{PM62.1} + \text{RM62D.1}) - \text{RM62D} \\ \text{PM63} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M63)} + 0.100 * \text{DIFF (RETEST64U)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * (\text{PM63.1} + \text{RM63D.1}) - \text{RM63D} \\ \text{PM64} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M64)} + 0.100 * \text{DIFF (RETEST64U)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * (\text{PM64.1} + \text{RM64D.1}) - \text{RM64D} \\ \text{PM65} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M65)} + 0.100 * \text{DIFF (RETEST65)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM65.1} \\ \text{PM66} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M66)} + 0.100 * \text{DIFF (RETEST66)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM66.1} \\ \text{PM67} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M67)} + 0.100 * \text{DIFF (RETEST67)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM67.1} \\ \text{PM68} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M68)} + 0.100 * \text{DIFF (RETEST68)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM68.1} \\ \text{PM69} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M69)} + 0.100 * \text{DIFF (RETEST69)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM69.1} \\ \text{PM70} &= (1.0 - 1.0 * \text{DIFF (RRADJ_M70)} + 0.15 * \text{DIFF (MOVAVG (5, RTP.6))}) * \text{PM70.1} \\ \text{PM71} &= (0.04127 + 0.91152) * \text{PM70.4} \\ \text{PM72} &= (0.02632 + 0.90027) * \text{PM71.4} \\ \text{PM73} &= (0.05478 + 0.91456) * \text{PM72.4} \\ \text{PM74} &= (-0.02105 + 0.87355) * \text{PM73.4} \end{aligned}$$

Aggregation of Equations from Regressions using March CPS Data
Using Quarterly Seasonal Adjustment Weights

Aged 20 to 44

$$\begin{aligned} \text{TPM2024} &= ((\text{PM2024NM} * \text{RM2024NM} + \text{PM2024MS} * \text{RM2024MS} + \text{PM2024MA} * \text{RM2024MA}) + 0.006) * 1.021 \\ \text{TPM2529} &= ((\text{PM2529NM} * \text{RM2529NM} + \text{PM2529MS} * \text{RM2529MS} + \text{PM2529MA} * \text{RM2529MA}) + 0.005) * 0.997 \\ \text{TPM3034} &= ((\text{PM3034NM} * \text{RM3034NM} + \text{PM3034MS} * \text{RM3034MS} + \text{PM3034MA} * \text{RM3034MA}) + 0.005) * 0.997 \\ \text{TPM3539} &= ((\text{PM3539NM} * \text{RM3539NM} + \text{PM3539MS} * \text{RM3539MS} + \text{PM3539MA} * \text{RM3539MA}) + 0.010) * 0.999 \\ \text{TPM4044} &= ((\text{PM4044NM} * \text{RM4044NM} + \text{PM4044MS} * \text{RM4044MS} + \text{PM4044MA} * \text{RM4044MA}) + 0.010) * 0.999 \\ \text{PM4549} &= ((\text{PM4549NM} * \text{RM4549NM} + \text{PM4549MS} * \text{RM4549MS} + \text{PM4549MA} * \text{RM4549MA}) + 0.003) * 0.999 \\ \text{PM5054} &= ((\text{PM5054NM} * \text{RM5054NM} + \text{PM5054MS} * \text{RM5054MS} + \text{PM5054MA} * \text{RM5054MA}) + 0.003) * 0.999 \end{aligned}$$

Aged 75 and Over

$$\text{PM750} = ((\text{PM7579} * \text{NM7579/NM750} + \text{PM8084} * \text{NM8084/NM750} + \text{PM850} * \text{NM850/NM750}) + 0.000) * 1.000$$

Aggregation of Equations from Regressions using Annual CPS Data

Aged 55 to 74

$$\begin{aligned} \text{PM5559} &= (\text{PM55} * \text{NM55} + \text{PM56} * \text{NM56} + \text{PM57} * \text{NM57} + \text{PM58} * \text{NM58} + \text{PM59} * \text{NM59}) / (\text{NM55} + \text{NM56} + \text{NM57} + \text{NM58} + \text{NM59}) \\ \text{PM6064} &= (\text{PM60} * \text{NM60} + \text{PM61} * \text{NM61} + \text{PM62} * \text{NM62} + \text{PM63} * \text{NM63} + \text{PM64} * \text{NM64}) / (\text{NM60} + \text{NM61} + \text{NM62} + \text{NM63} + \text{NM64}) \end{aligned}$$

$$\begin{aligned}
PM6569 &= (PM65 * NM65 + PM66 * NM66 + PM67 * NM67 + PM68 * NM68 + PM69 * NM69) / (NM65 + NM66 + NM67 + NM68 + NM69) \\
PM7074 &= (PM70 * NM70 + PM71 * NM71 + PM72 * NM72 + PM73 * NM73 + PM74 * NM74) / (NM70 + NM71 + NM72 + NM73 + NM74)
\end{aligned}$$

Male LFPRs - full employment

$$\begin{aligned}
PM1617_FE &= PM1617 + 0.35974 * (1 - RTP.1) + 0.31790 * (1 - RTP.5) \\
TPM1819_FE &= TPM1819 + 0.25962 * (1 - RTP.1) \\
TPM2024_FE &= TPM2024 + (0.21031 * (NM2024NM + NM2024M - MM2024MS - MM2024MA) + 0.00000 * (NM2024MS + MM2024MS) + 0.38966 * (NM2024MA + MM2024MA)) / ((NM2024NM + NM2024MS + NM2024MS) + NM2024M) * (1 - RTP.1) \\
TPM2529_FE &= TPM2529 + (0.15942 * (NM2529NM + NM2529M - MM2529MS - MM2529MA) + 0.00000 * (NM2529MS + MM2529MS) + 0.19750 * (NM2529MA + MM2529MA)) / ((NM2529NM + NM2529MS + NM2024MA) + NM2529M) * (1 - MOVAVG (4, RTP.2)) \\
TPM3034_FE &= TPM3034 + (0.05000 * (NM3034NM + NM3034M - MM3034MS - MM3034MA) + 0.05000 * (NM3034MS + MM3034MS) + 0.05000 * (NM3034MA + MM3034MA)) / ((NM3034NM + NM3034MS + NM3034MA) + NM3034M) * (1 - MOVAVG (4, RTP.2)) \\
TPM3539_FE &= TPM3539 + (0.05000 * (NM3539NM + NM3539M - MM3539MS - MM3539MA) + 0.05000 * (NM3539MS + MM3539MS) + 0.05000 * (NM3539MA + MM3539MA)) / ((NM3539NM + NM3539MS + NM3539MA) + NM3539M) * (1 - MOVAVG (4, RTP.2)) \\
TPM4044_FE &= TPM4044 + (0.05000 * (NM4044NM + NM4044M - MM4044MS - MM4044MA) + 0.05000 * (NM4044MS + MM4044MS) + 0.05000 * (NM4044MA + MM4044MA)) / ((NM4044NM + NM4044MS + NM4044MA) + NM4044M) * (1 - MOVAVG (4, RTP.2)) \\
PM4549_FE &= PM4549 + (0.02887 * NM4549NM + 0.15238 * NM4549MS + 0.00000 * NM4549MA) / (NM4549NM + NM4549MS + NM4549MA) * (1 - MOVAVG (4, RTP.1)) \\
PM5054_FE &= PM5054 + (0.00000 * NM5054NM + 0.19639 * NM5054MS + 0.43104 * NM5054MA) / (NM5054NM + NM5054MS + NM5054MA) * (1 - MOVAVG (4, RTP.1)) \\
PM5559_FE &= PM5559 + (0.11202 * NM55 + 0.10041 * NM56 + 0.13081 * NM57 + 0.24841 * NM58 + 0.25992 * NM59) / (NM55 + NM56 + NM57 + NM58 + NM59) * (1 - MOVAVG (5, RTP.10)) \\
PM6064_FE &= PM6064 + (0.31289 * NM60 + 0.32660 * NM61 + 0.15 * 0.55 * NM62 + 0.15 * 0.48 * NM63 + 0.15 * 0.44 * NM64) / (NM60 + NM61 + NM62 + NM63 + NM64) * (1 - MOVAVG (5, RTP.10)) \\
PM6569_FE &= PM6569 + (0.15 * 0.38 * NM65 + 0.15 * 0.34 * NM66 + 0.15 * 0.33 * NM67 + 0.15 * 0.30 * NM68 + 0.15 * 0.24 * NM69) / (NM65 + NM66 + NM67 + NM68 + NM69) * (1 - MOVAVG (5, RTP.10)) \\
PM7074_FE &= PM7074 + (0.15 * 0.18 * NM70 + 0.00000 * NM71 + 0.00000 * NM72 + 0.00000 * NM73 + 0.00000 * NM74) / (NM70 + NM71 + NM72 + NM73 + NM74) * (1 - MOVAVG (5, RTP.10)) \\
PM5054L_FE &= MOVAVG (8, PM5054_FE.8 + PM2554_A90C.8 + PM2554_A94M.8 + RM5054D.8) \\
PM7074L &= MOVAVG (8, PM7074.16 / PM65O_A94MM.16) \\
PM7579L &= MOVAVG (8, PM7579.16 / PM65O_A94MM.16) \\
PM8084L &= MOVAVG (8, PM8084.16 / PM65O_A94MM.16)
\end{aligned}$$

FEMALE LFPR Equations - Annual

Regressions using March CPS Data

Aged 20 to 44

$$\begin{aligned}
PF2024NMC6U &= -1.00 * RF2024D + 0.96222 * RTPL1T4 - 1.0801 * PFW2024NM - 0.1 * IF2024NMC6U + 0.11117 - 0.25881 + 0.005 \\
PF2529NMC6U &= -1.00 * RF2529D + 0.91191 * RTPL1T4 - 0.06000 * IF2529NMC6U - 0.61008 * PFW2529NM + 0.13672 - 0.21084 \\
PF3034NMC6U &= -1.00 * RF3034D - 0.06000 * IF3034NMC6U - 0.46290 * PFW3034NM + 1.30921 * MOVAVG (3, PF2529NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) + 0.14328 - 0.37975 \\
PF3539NMC6U &= -1.00 * RF3539D - 0.65032 * PFW3539NM + 0.75635 \\
PF4044NMC6U &= -1.00 * RF4044D - 0.06000 * IF4044NMC6U + 2.80447 * MOVAVG (3, PF3539NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) + 0.21194 * RTPL1T4 - 1.92826 \\
PF2024NMNC6 &= -1.00 * RF2024D + 0.13031 * RTPL1T4 + 0.55986 * MOVAVG (2, PF1819_FE.3 + PF1619_94M_TE.3 + PF1619_90C_TE.3) + 0.28086 + 0.005 \\
PF2529NMNC6 &= -1.00 * RF2529D + 0.87626
\end{aligned}$$

PF3034NMNC6 = -1.00 * RF3034D + 1.75050 * MOVAVG (3, PF2529NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 +
 RF2529D.4) + 0.19793 * RTPL1T4 - 0.81095
 PF3539NMNC6 = -1.00 * RF3539D + 0.65953 * MOVAVG (3, PF3034NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 +
 RF3034D.4) + 0.32043 * RTPL1T4 - 0.02382
 PF4044NMNC6 = -1.00 * RF4044D + 0.57971 * MOVAVG (3, PF3539NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 +
 RF3539D.4) + 0.34356

 PF2024MSC6U = -1.00 * RF2024D - 0.1 * IF2024MSC6U + 0.01086 * 87 + 0.04411 * RTPL1T4 + 0.03335 - 0.32002 + 0.005
 PF2529MSC6U = -1.00 * RF2529D + 1.0 * MOVAVG (3, PF2024_FE.4 + PF2024_94M_TE.4 + PF2024_94C_TE.4 + RF2024D.4) - 0.1 *
 IF2529MSC6U + 0.11923 * RTPL1T4 + 0.00572 * 88
 + 0.05218 - 0.59301
 PF3034MSC6U = -1.00 * RF3034D + 1.03276 * MOVAVG (3, PF2529_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) -
 0.06606 * IF3034MSC6U + 0.06988 * RTPL1T4
 - 0.10472
 PF3539MSC6U = -1.00 * RF3539D - 0.1 * IF3539MSC6U + 1.05734 * MOVAVG (3, PF3034_FE.4 + PF2554_94M_TE.4 +
 PF2554_94C_TE.4 + RF3034D.4) + 0.15040 * RTPL1T4 - 0.15161
 PF4044MSC6U = -1.00 * RF4044D + 0.93189 * MOVAVG (3, PF3539_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3539D.4) +
 0.05375 - 0.09385

 PF2024MSNC6 = -1.00 * RF2024D + 0.66513 * MOVAVG (2, PF1819_FE.3 + PF1619_94M_TE.3 + PF1619_90C_TE.3) - 0.02309 +
 0.40392 + 0.005
 PF2529MSNC6 = -1.00 * RF2529D + 0.75346 * MOVAVG (3, PF2024_FE.4 + PF2024_94M_TE.4 + RF2024D.4) + 0.15260 * RTPL1T4 +
 0.15786
 PF3034MSNC6 = -1.00 * RF3034D + 0.72921 * MOVAVG (3, PF2529_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) +
 0.27929
 PF3539MSNC6 = -1.00 * RF3539D + 0.70122 * MOVAVG (3, PF3034_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3034D.4) +
 0.33338 * RTPL1T4 - 0.01765 - 0.03387
 PF4044MSNC6 = -1.00 * RF4044D + 0.83686 * MOVAVG (3, PF3539_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3539D.4) +
 0.16837

 PF2024MAC6U = -1.00 * RF2024D + 0.41498 * RTPL1T4 - 0.31052 * IF2024MAC6U - 0.08075 * PFW2024MA + 0.68113 * MOVAVG
 (2, PF1819_FE.3 + PF1619_94M_TE.3
 + PF1619_90C_TE.3) + 0.09208 + 0.11197 + 0.263 + 0.005
 PF2529MAC6U = -1.00 * RF2529D + 0.32708 * RTPL1T4 + 0.39869 * MOVAVG (3, PF2024_FE.4 + PF2024_94M_TE.4 +
 PF2024_94C_TE.4 + RF2024D.4) - 0.19425 * IF2529MAC6U
 - 0.27997 * PFW2529MA + 0.39831
 PF3034MAC6U = -1.00 * RF3034D + 0.15332 * MOVAVG (3, PF2529_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) -
 0.21611 * IF3034MAC6U - 0.48469 * PFW3034MA
 + 0.09521 + 0.90844
 PF3539MAC6U = -1.00 * RF3539D + 0.45383 * MOVAVG (3, PF3034_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3034D.4) +
 0.10467 + 0.30563
 PF4044MAC6U = -1.00 * RF4044D + 0.83908 * MOVAVG (3, PF3539_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3539D.4) +
 0.20826 * RTPL1T4 - 0.18854

 PF2024MANC6 = -1.00 * RF2024D + 0.87285 * MOVAVG (2, PF1819_FE.3 + PF1619_94M_TE.3 + PF1619_90C_TE.3) - 0.07951 +
 0.2418 + 0.005
 PF2529MANC6 = -1.00 * RF2529D + 0.82169
 PF3034MANC6 = -1.00 * RF3034D + 0.05157 * MOVAVG (3, PF2529_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF2529D.4) +
 0.80559
 PF3539MANC6 = -1.00 * RF3539D + 0.20191 * MOVAVG (3, PF3034_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3034D.4) +
 0.70206
 PF4044MANC6 = -1.00 * RF4044D + 0.41386 * MOVAVG (3, PF3539_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF3539D.4) +
 0.07702 * RTPL1T4 - 0.030022 + 0.48116

 Aged 45 to 54
 PF4549NM = -1.00 * RF4549D + 0.26468 * MOVAVG (3, PF4044NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4044D.4) +
 0.57522
 PF5054NM = -1.00 * RF5054D + 0.37747 * MOVAVG (3, PF4549NM_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4549D.4) +
 0.68899 * RTPL1T4 - 0.22031

 PF4549MS = -1.00 * RF4549D + 0.92026 * MOVAVG (3, PF4044_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4044D.4) +
 0.16000 * RTPL1T4 - 0.09969
 PF5054MS = -1.00 * RF5054D + 0.97734 * MOVAVG (3, PF4549_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4549D.4) -
 0.01843

$PF4549MA = -1.00 * RF4549D + 0.39804 * MOVAVG(3, PF4044_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4044D.4) + 0.22670 * RTPL1T4 + 0.30580$
 $PF5054MA = -1.00 * RF5054D + 0.46765 * MOVAVG(3, PF4549_FE.4 + PF2554_94M_TE.4 + PF2554_94C_TE.4 + RF4549D.4) + 0.24122 * RTPL1T4 + 0.20279$

Aged 75 and Over

$PF7579 = 0.51019 * PF7074.5$
 $PF8084 = 0.50704 * PF7579.5$
 $PF850 = 0.55432 * PF8084.5$

Regressions using Annual CPS Data

Aged 16 to 19

$PF1617 = 0.12385 * DIFF(RTP) + 0.17046 * DIFF(RTP.1) + 0.56079 * DIFF(DIFFDONM1617) + PF1617.1$
 $PF1819 = 0.22735 * DIFF(RTP) + 0.14711 * DIFF(RTP.1) + 0.15129 * DIFF(RAM1819) - 0.14961 * DIFF(RNF1819S) + PF1819.1$

Aged 55 to 74

$PF55 = 0.80973 * MOVAVG(3, PF5054.2 + PF2554_90C_TE.2 + PF2554_94M_TE.2 + RF5054D.2) - 1.0 * RF55D + 0.16573 * RTP.1 - 0.05150$
 $PF56 = 0.94452 * (PF55.1 + RF55D.1 + 0.16573 * (1 - RTP.2)) - 1.0 * RF56D + 0.09741 * RTP.2 - 0.07766$
 $PF57 = 1.01984 * (PF56.1 + RF56D.1 + 0.09741 * (1 - RTP.3)) - 1.0 * RF57D + 0.14837 * RTP.2 - 0.17199$
 $PF58 = 0.83467 * (PF57.1 + RF57D.1 + 0.14837 * (1 - RTP.3)) - 1.0 * RF58D + 0.09913 * RTP.2 - 0.01364$
 $PF59 = 0.82721 * (PF58.1 + RF58D.1 + 0.09913 * (1 - RTP.3)) - 1.0 * RF59D + 0.14296 * RTP.2 - 0.06068$
 $PF60 = 0.74197 * (PF59.1 + RF59D.1 + 0.14296 * (1 - RTP.3)) - 1.0 * RF60D + 0.12539 * RTP.2 - 0.01822$
 $PF61 = 1.03326 * (PF60.1 + RF60D.1 + 0.12539 * (1 - RTP.3)) - 1.0 * RF61D + 0.25650 * RTP.2 - 0.29862$
 $PF62 = ((PF61 + RF61D + 0.25650 * (1 - RTP.2)) / (PM61 + RM61D + 0.32660 * (1 - RTP.3))) * (PM62 + RM62D + 0.15 * (1 - RTP.2)) - RF62D$
 $PF63 = ((PF62.1 + RF62D.1) / (PF62.2 + RF62D.2) - 1.0 * DIFF(RRADJ_F63 - RRADJ_F62.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST64U - RETEST64U.1)) * (PF63.1 + RF63D.1) - RF63D$
 $PF64 = ((PF63.1 + RF63D.1) / (PF63.2 + RF63D.2) - 1.0 * DIFF(RRADJ_F64 - RRADJ_F63.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST64U - RETEST64U.1)) * (PF64.1 + RF64D.1) - RF64D$
 $PF65 = (0.75538 * (PF64.1 + RF64D.1) / (PM64.1 + RM64D.1) + 0.15565) * PM65$
 $PF66 = (PF65.1 / PF65.2 - 1.0 * DIFF(RRADJ_F66 - RRADJ_F65.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST66 - RETEST65.1)) * PF66.1$
 $PF67 = (PF66.1 / PF66.2 - 1.0 * DIFF(RRADJ_F67 - RRADJ_F66.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST67 - RETEST66.1)) * PF67.1$
 $PF68 = (PF67.1 / PF67.2 - 1.0 * DIFF(RRADJ_F68 - RRADJ_F67.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST68 - RETEST67.1)) * PF68.1$
 $PF69 = (PF68.1 / PF68.2 - 1.0 * DIFF(RRADJ_F69 - RRADJ_F68.1) + 0.15 * DIFF(RTP.2 - RTP.3) + 0.1 * DIFF(RETEST69 - RETEST68.1)) * PF69.1$
 $PF70 = (PF69.1 / PF69.2 - 1.0 * DIFF(RRADJ_F70 - RRADJ_F69.1) + 0.15 * DIFF(RTP.2 - RTP.3)) * PF70.1$
 $PF71 = 0.89763 * PF70.1$
 $PF72 = 0.91226 * PF71.1$
 $PF73 = 0.86723 * PF72.1$
 $PF74 = 0.88162 * PF73.1$

FEMALE LFPR Equations - Quarterly

Regressions using March CPS Data

Aged 20 to 44

$PF2024NMC6U = -1.00 * RF2024D + 0.96222 * MOVAVG(4, RTP.1) - 1.0801 * PFW2024NM - 0.1 * IF2024NMC6U + 0.11117 - 0.25881 + 0.005$
 $PF2529NMC6U = -1.00 * RF2529D + 0.91191 * MOVAVG(4, RTP.1) - 0.06000 * IF2529NMC6U - 0.61008 * PFW2529NM + 0.13672 - 0.21084$
 $PF3034NMC6U = -1.00 * RF3034D - 0.06000 * IF3034NMC6U - 0.46290 * PFW3034NM + 1.30921 * PF2529NML_FE + 0.14328 - 0.37975$
 $PF3539NMC6U = -1.00 * RF3539D - 0.65032 * PFW3539NM + 0.75635$
 $PF4044NMC6U = -1.00 * RF4044D - 0.06000 * IF4044NMC6U + 2.80447 * PF3539NML_FE + 0.21194 * MOVAVG(4, RTP.1) - 1.92826$

 $PF2024NMNC6 = -1.00 * RF2024D + 0.13031 * MOVAVG(4, RTP.1) + 0.55986 * PF1819L_FE + 0.28086 + 0.005$

PF2529NMNC6 = -1.00 * RF2529D + 0.87626
 PF3034NMNC6 = -1.00 * RF3034D + 1.75050 * PF2529NML_FE + 0.19793 * MOVAVG (4, RTP.1) - 0.81095
 PF3539NMNC6 = -1.00 * RF3539D + 0.65953 * PF3034NML_FE + 0.32043 * MOVAVG (4, RTP.1) - 0.02382
 PF4044NMNC6 = -1.00 * RF4044D + 0.57971 * PF3539NML_FE + 0.34356

 PF2024MSC6U = -1.00 * RF2024D - 0.1 * IF2024MSC6U + 0.01086 * 87 + 0.04411 * MOVAVG (4, RTP.1) + 0.03335 - 0.32002 + 0.005
 PF2529MSC6U = -1.00 * RF2529D + 1.0 * PF2024L_FE - 0.1 * IF2529MSC6U + 0.11923 * MOVAVG (4, RTP.1) + 0.00572 * 88 +
 0.05218 - 0.59301
 PF3034MSC6U = -1.00 * RF3034D + 1.03276 * PF2529L_FE - 0.06606 * IF3034MSC6U + 0.06988 * MOVAVG (4, RTP.1) - 0.10472
 PF3539MSC6U = -1.00 * RF3539D - 0.1 * IF3539MSC6U + 1.05734 * PF3034L_FE + 0.15040 * MOVAVG (4, RTP.1) - 0.15161
 PF4044MSC6U = -1.00 * RF4044D + 0.93189 * PF3539L_FE + 0.05375 - 0.09385

 PF2024MSNC6 = -1.00 * RF2024D + 0.66513 * PF1819L_FE - 0.02309 + 0.40392 + 0.005
 PF2529MSNC6 = -1.00 * RF2529D + 0.75346 * PF2024L_FE + 0.15260 * MOVAVG (4, RTP.1) + 0.15786
 PF3034MSNC6 = -1.00 * RF3034D + 0.72921 * PF2529L_FE + 0.27929
 PF3539MSNC6 = -1.00 * RF3539D + 0.70122 * PF3034L_FE + 0.33338 * MOVAVG (4, RTP.1) - 0.01765 - 0.03387
 PF4044MSNC6 = -1.00 * RF4044D + 0.83686 * PF3539L_FE + 0.16837

 PF2024MAC6U = -1.00 * RF2024D + 0.41498 * MOVAVG (4, RTP.1) - 0.31052 * IF2024MAC6U - 0.08075 * PFW2024MA + 0.68113 *
 PF1819L_FE + 0.09208 + 0.11197 + 0.263 + 0.005
 PF2529MAC6U = -1.00 * RF2529D + 0.32708 * MOVAVG (4, RTP.1) + 0.39869 * PF2024L_FE - 0.19425 * IF2529MAC6U - 0.27997 *
 PFW2529MA + 0.39831
 PF3034MAC6U = -1.00 * RF3034D + 0.15332 * PF2529L_FE - 0.21611 * IF3034MAC6U - 0.48469 * PFW3034MA + 0.09521 + 0.90844
 PF3539MAC6U = -1.00 * RF3539D + 0.45383 * PF3034L_FE + 0.10467 + 0.30563
 PF4044MAC6U = -1.00 * RF4044D + 0.83908 * PF3539L_FE + 0.20826 * MOVAVG (4, RTP.1) - 0.18854

 PF2024MANC6 = -1.00 * RF2024D + 0.87285 * PF1819L_FE - 0.07951 + 0.2418 + 0.005
 PF2529MANC6 = -1.00 * RF2529D + 0.82169
 PF3034MANC6 = -1.00 * RF3034D + 0.05157 * PF2529L_FE + 0.80559
 PF3539MANC6 = -1.00 * RF3539D + 0.20191 * PF3034L_FE + 0.70206
 PF4044MANC6 = -1.00 * RF4044D + 0.41386 * PF3539L_FE + 0.07702 * MOVAVG (4, RTP.1) - 0.03002 + 0.48116

Aged 45 to 54

PF4549NM = -1.00 * RF4549D + 0.26468 * PF4044NML_FE + 0.57522
 PF5054NM = -1.00 * RF5054D + 0.37747 * PF4549NML_FE + 0.68899 * MOVAVG (4, RTP.1) - 0.22031
 PF4549MS = -1.00 * RF4549D + 0.92026 * PF4044L_FE + 0.16000 * MOVAVG (4, RTP.1) - 0.09969
 PF5054MS = -1.00 * RF5054D + 0.97734 * PF4549L_FE - 0.01843
 PF4549MA = -1.00 * RF4549D + 0.39804 * PF4044L_FE + 0.22670 * MOVAVG (4, RTP.1) + 0.30580
 PF5054MA = -1.00 * RF5054D + 0.46765 * PF4549L_FE + 0.24122 * MOVAVG (4, RTP.1) + 0.20279

Aged 75 and Over

PF7579 = 0.51019 * MOVAVG (4, PF7074.20)
 PF8084 = 0.50704 * MOVAVG (4, PF7579.20)
 PF850 = 0.55432 * MOVAVG (4, PF8084.20)

Regressions using Annual CPS Data

Aged 16 to 19

PF1617 = 0.12385 * DIFF (RTP.1) + 0.17046 * DIFF (RTP.5) + 0.56079 * DIFF (DIFFDONM1617) + PF1617.1
 PF1819 = 0.22735 * DIFF (RTP.1) + 0.14711 * DIFF (RTP.5) + 0.15129 * DIFF (RAM1819) - 0.14961 * DIFF (RNF1819S) + PF1819.1

Aged 55 to 74

PF55 = 0.80973 * PF5054L_FE - 1.0 * RF55D + 0.16573 * MOVAVG (5, RTP.2) - 0.05150
 PF56 = 0.94452 * (PF55.4 + RF55D.4 + 0.16573 * (1 - MOVAVG (5, RTP.6))) - 1.0 * RF56D + 0.09741 * MOVAVG (5, RTP.6) - 0.07766
 PF57 = 1.01984 * (PF56.4 + RF56D.4 + 0.09741 * (1 - MOVAVG (5, RTP.10))) - 1.0 * RF57D + 0.14837 * MOVAVG (5, RTP.6) - 0.17199
 PF58 = 0.83467 * (PF57.4 + RF57D.4 + 0.14837 * (1 - MOVAVG (5, RTP.10))) - 1.0 * RF58D + 0.09913 * MOVAVG (5, RTP.6) - 0.01364
 PF59 = 0.82721 * (PF58.4 + RF58D.4 + 0.09913 * (1 - MOVAVG (5, RTP.10))) - 1.0 * RF59D + 0.14296 * MOVAVG (5, RTP.6) - 0.06068
 PF60 = 0.74197 * (PF59.4 + RF59D.4 + 0.14296 * (1 - MOVAVG (5, RTP.10))) - 1.0 * RF60D + 0.12539 * MOVAVG (5, RTP.6) - 0.01822
 PF61 = 1.03326 * (PF60.4 + RF60D.4 + 0.12539 * (1 - MOVAVG (5, RTP.10))) - 1.0 * RF61D + 0.25650 * MOVAVG (5, RTP.6) - 0.29862
 PF62 = ((PF61 + RF61D + 0.25650 * (1 - MOVAVG (5, RTP.10))) / (PM61 + RM61D + 0.32660 * (1 - MOVAVG (9, RTP.10)))) * (PM62 +
 RM62D + 0.15 * (1 - MOVAVG (5, RTP.10))) - RF62D

$PF63 = ((PF62.4 + RF62D.4) / (PF62.5 + RF62D.5) - 1.0 * DIFF (RRADJ_F63 - RRADJ_F62.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST64U - RETEST64U.4) * (PF63.1 + RF63D.1) - RF63D$
 $PF64 = ((PF63.4 + RF63D.4) / (PF63.5 + RF63D.5) - 1.0 * DIFF (RRADJ_F64 - RRADJ_F63.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST64U - RETEST64U.4) * (PF64.1 + RF64D.1) - RF64D$
 $PF65 = (0.75538 * (PF64.4 + RF64D.4) / (PM64.4 + RM64D.4) + 0.15565) * PM65$
 $PF66 = (PF65.4 / PF65.5 - 1.0 * DIFF (RRADJ_F66 - RRADJ_F65.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST66 - RETEST65.4) * PF66.1$
 $PF67 = (PF66.4 / PF66.5 - 1.0 * DIFF (RRADJ_F67 - RRADJ_F66.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST67 - RETEST66.4) * PF67.1$
 $PF68 = (PF67.4 / PF67.5 - 1.0 * DIFF (RRADJ_F68 - RRADJ_F67.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST68 - RETEST67.4) * PF68.1$
 $PF69 = (PF68.4 / PF68.5 - 1.0 * DIFF (RRADJ_F69 - RRADJ_F68.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) + 0.1 * DIFF (RETEST69 - RETEST68.4) * PF69.1$
 $PF70 = (PF69.4 / PF69.5 - 1.0 * DIFF (RRADJ_F70 - RRADJ_F69.4) + 0.15 * DIFF (MOVAVG (5, RTP.6) - MOVAVG (5, RTP.10))) * PF70.1$
 $PF71 = 0.89763 * PF70.4$
 $PF72 = 0.91226 * PF71.4$
 $PF73 = 0.86723 * PF72.4$
 $PF74 = 0.88162 * PF73.4$

Maximums for Female LFPRs

$PF2024NMC6U_M = MIN (PF2024NMC6U, 0.75)$
 $PF2529NMC6U_M = MIN (PF2529NMC6U, 0.75)$
 $PF3034NMC6U_M = MIN (PF3034NMC6U, 0.75)$
 $PF3539NMC6U_M = MIN (PF3539NMC6U, 0.75)$
 $PF4044NMC6U_M = MIN (PF4044NMC6U, 0.75)$
 $PF2024NMNC6_M = MIN (PF2024NMNC6, 0.80)$
 $PF2529NMNC6_M = MIN (PF2529NMNC6, 0.89)$
 $PF3034NMNC6_M = MIN (PF3034NMNC6, 0.89)$
 $PF3539NMNC6_M = MIN (PF3539NMNC6, 0.85)$
 $PF4044NMNC6_M = MIN (PF4044NMNC6, 0.85)$
 $PF4549NM_M = MIN (PF4549NM, 0.85)$
 $PF5054NM_M = MIN (PF5054NM, 0.85)$

 $PF2024MSC6U_M = MIN (PF2024MSC6U, 0.65)$
 $PF2529MSC6U_M = MIN (PF2529MSC6U, 0.70)$
 $PF3034MSC6U_M = MIN (PF3034MSC6U, 0.70)$
 $PF3539MSC6U_M = MIN (PF3539MSC6U, 0.70)$
 $PF4044MSC6U_M = MIN (PF4044MSC6U, 0.70)$
 $PF2024MSNC6_M = MIN (PF2024MSNC6, 0.85)$
 $PF2529MSNC6_M = MIN (PF2529MSNC6, 0.85)$
 $PF3034MSNC6_M = MIN (PF3034MSNC6, 0.85)$
 $PF3539MSNC6_M = MIN (PF3539MSNC6, 0.85)$
 $PF4044MSNC6_M = MIN (PF4044MSNC6, 0.85)$
 $PF4549MS_M = MIN (PF4549MS, 0.85)$
 $PF5054MS_M = MIN (PF5054MS, 0.85)$

 $PF2024MAC6U_M = MIN (PF2024MAC6U, 0.85)$
 $PF2529MAC6U_M = MIN (PF2529MAC6U, 0.75)$
 $PF3034MAC6U_M = MIN (PF3034MAC6U, 0.75)$
 $PF3539MAC6U_M = MIN (PF3539MAC6U, 0.75)$
 $PF4044MAC6U_M = MIN (PF4044MAC6U, 0.75)$
 $PF2024MANC6_M = MIN (PF2024MANC6, 0.87)$
 $PF2529MANC6_M = MIN (PF2529MANC6, 0.88)$
 $PF3034MANC6_M = MIN (PF3034MANC6, 0.88)$
 $PF3539MANC6_M = MIN (PF3539MANC6, 0.88)$
 $PF4044MANC6_M = MIN (PF4044MANC6, 0.88)$
 $PF4549MA_M = MIN (PF4549MA, 0.88)$
 $PF5054MA_M = MIN (PF5054MA, 0.85)$

Quarterly Weighted Equations from Regressions using March CPS Data

Aged 20 to 44 by Marital Status

PF2024NM = (PF2024NMC6U_M * RF2024NMC6U + (1 - RF2024NMC6U) * PF2024NMNC6_M)
PF2024MS = (PF2024MSC6U_M * RF2024MSC6U + (1 - RF2024MSC6U) * PF2024MSNC6_M)
PF2024MA = (PF2024MAC6U_M * RF2024MAC6U + (1 - RF2024MAC6U) * PF2024MANC6_M)

PF2529NM = (PF2529NMC6U_M * RF2529NMC6U + (1 - RF2529NMC6U) * PF2529NMNC6_M)
PF2529MS = (PF2529MSC6U_M * RF2529MSC6U + (1 - RF2529MSC6U) * PF2529MSNC6_M)
PF2529MA = (PF2529MAC6U_M * RF2529MAC6U + (1 - RF2529MAC6U) * PF2529MANC6_M)

PF3034NM = (PF3034NMC6U_M * RF3034NMC6U + (1 - RF3034NMC6U) * PF3034NMNC6_M)
PF3034MS = (PF3034MSC6U_M * RF3034MSC6U + (1 - RF3034MSC6U) * PF3034MSNC6_M)
PF3034MA = (PF3034MAC6U_M * RF3034MAC6U + (1 - RF3034MAC6U) * PF3034MANC6_M)

PF3539NM = (PF3539NMC6U_M * RF3539NMC6U + (1 - RF3539NMC6U) * PF3539NMNC6_M)
PF3539MS = (PF3539MSC6U_M * RF3539MSC6U + (1 - RF3539MSC6U) * PF3539MSNC6_M)
PF3539MA = (PF3539MAC6U_M * RF3539MAC6U + (1 - RF3539MAC6U) * PF3539MANC6_M)

PF4044NM = (PF4044NMC6U_M * RF4044NMC6U + (1 - RF4044NMC6U) * PF4044NMNC6_M)
PF4044MS = (PF4044MSC6U_M * RF4044MSC6U + (1 - RF4044MSC6U) * PF4044MSNC6_M)
PF4044MA = (PF4044MAC6U_M * RF4044MAC6U + (1 - RF4044MAC6U) * PF4044MANC6_M)

Aged 20 to 54

PF2024 = (PF2024NM * RF2024NM + PF2024MS * RF2024MS + PF2024MA * RF2024MA) * 1.015
PF2529 = (PF2529NM * RF2529NM + PF2529MS * RF2529MS + PF2529MA * RF2529MA) * 0.999
PF3034 = (PF3034NM * RF3034NM + PF3034MS * RF3034MS + PF3034MA * RF3034MA) * 0.999
PF3539 = (PF3539NM * RF3539NM + PF3539MS * RF3539MS + PF3539MA * RF3539MA) * 0.996
PF4044 = (PF4044NM * RF4044NM + PF4044MS * RF4044MS + PF4044MA * RF4044MA) * 0.996
PF4549 = (PF4549NM_M * RF4549NM + PF4549MS_M * RF4549MS + PF4549MA_M * RF4549MA) * 0.998
PF5054 = (PF5054NM_M * RF5054NM + PF5054MS_M * RF5054MS + PF5054MA_M * RF5054MA) * 0.998

Aged 55 to 74

PF5559 = (PF55 * NF55 + PF56 * NF56 + PF57 * NF57 + PF58 * NF58 + PF59 * NF59) / (NF55 + NF56 + NF57 + NF58 + NF59)
PF6064 = (PF60 * NF60 + PF61 * NF61 + PF62 * NF62 + PF63 * NF63 + PF64 * NF64) / (NF60 + NF61 + NF62 + NF63 + NF64)
PF6569 = (PF65 * NF65 + PF66 * NF66 + PF67 * NF67 + PF68 * NF68 + PF69 * NF69) / (NF65 + NF66 + NF67 + NF68 + NF69)
PF7074 = (PF70 * NF70 + PF71 * NF71 + PF72 * NF72 + PF73 * NF73 + PF74 * NF74) / (NF70 + NF71 + NF72 + NF73 + NF74)

Aged 75 and Over

PF750 = (PF7579 * NF7579/NF750 + PF8084 * NF8084/NF750 + PF850 * NF850/NF750) * 1.006

Female LFPRs - full employment

PF1617_FE = PF1617 + 0.12385 * (1 - RTP.1) + 0.17046 * (1 - RTP.5)
PF1819_FE = PF1819 + 0.22735 * (1 - RTP.1) + 0.14711 * (1 - RTP.5)
PF2024_FE = PF2024 + (0.04411 * NF2024MSC6U + 0.00000 * NF2024MSNC6 + 0.41498 * NF2024MAC6U + 0.00000 * NF2024MANC6 + 0.96222 * NF2024NMC6U + 0.13031 * NF2024NMNC6) / (NF2024NM + NF2024MS + NF2024MA) * (1 - MOVAVG (4, RTP.1))
PF2529_FE = PF2529 + (0.11923 * NF2529MSC6U + 0.15260 * NF2529MSNC6 + 0.32708 * NF2529MAC6U + 0.00000 * NF2529MANC6 + 0.91191 * NF2529NMC6U + 0.00000 * NF2529NMNC6) / (NF2529NM + NF2529MS + NF2529MA) * (1 - MOVAVG (4, RTP.1))
PF3034_FE = PF3034 + (0.06988 * NF3034MSC6U + 0.00000 * NF3034MSNC6 + 0.00000 * NF3034MAC6U + 0.00000 * NF3034MANC6 + 0.00000 * NF3034NMC6U + 0.19793 * NF3034NMNC6) / (NF3034NM + NF3034MS + NF3034MA) * (1 - MOVAVG (4, RTP.1))
PF3539_FE = PF3539 + (0.15040 * NF3539MSC6U + 0.33338 * NF3539MSNC6 + 0.00000 * NF3539MAC6U + 0.00000 * NF3539MANC6 + 0.00000 * NF3539NMC6U + 0.32043 * NF3539NMNC6) / (NF3539NM + NF3539MS + NF3539MA) * (1 - MOVAVG (4, RTP.1))
PF4044_FE = PF4044 + (0.00000 * NF4044MSC6U + 0.00000 * NF4044MSNC6 + 0.20826 * NF4044MAC6U + 0.07702 * NF4044MANC6 + 0.21194 * NF4044NMC6U + 0.00000 * NF4044NMNC6) / (NF4044NM + NF4044MS + NF4044MA) * (1 - MOVAVG (4, RTP.1))
PF4549_FE = PF4549 + (0.00000 * NF4549NM + 0.16000 * NF4549MS + 0.22670 * NF4549MA) / (NF4549NM + NF4549MS + NF4549MA) * (1 - MOVAVG (4, RTP.1))
PF5054_FE = PF5054 + (0.68899 * NF5054NM + 0.00000 * NF5054MS + 0.24122 * NF5054MA) / (NF5054NM + NF5054MS + NF5054MA) * (1 - MOVAVG (4, RTP.1))
PF5559_FE = PF5559 + (0.16573 * NF55 + 0.09741 * NF56 + 0.14837 * NF57 + 0.09913 * NF58 + 0.14296 * NF59) / (NF55 + NF56 + NF57 + NF58 + NF59) * (1 - MOVAVG (5, RTP.4))

PF6064_FE = $PF6064 + (0.12539 * NF60 + 0.25650 * NF61 + 0.15 * 0.41 * NF62 + 0.15 * 0.36 * NF63 + 0.15 * 0.35 * NF64) / (NF60 + NF61 + NF62 + NF63 + NF64) * (1 - MOVAVG(5, RTP.4))$
 PF6569_FE = $PF6569 + (0.15 * 0.26 * NF65 + 0.15 * 0.23 * NF66 + 0.15 * 0.21 * NF67 + 0.15 * 0.18 * NF68 + 0.15 * 0.15 * NF69) / (NF65 + NF66 + NF67 + NF68 + NF69) * (1 - MOVAVG(5, RTP.4))$
 PF7074_FE = $PF7074 + (0.15 * 0.13 * NF70 + 0.00000 * NF71 + 0.00000 * NF72 + 0.00000 * NF73 + 0.00000 * NF74) / (NF70 + NF71 + NF72 + NF73 + NF74) * (1 - MOVAVG(5, RTP.4))$
 PF1819L_FE = MOVAVG(8, PF1819_FE.10 + PF1619_94M_TE.10 + PF1619_90C_TE.10)
 PF2024L_FE = MOVAVG(13, PF2024_FE.14 + PF2024_90C_TE.14 + PF2024_94M_TE.14 + RF2024D.14 - PF2024_ADJWF.14)
 PF2024NMC6UL_FE = MOVAVG(13, PF2024NMC6U.14 + PF2024_94C_TE.14 + PF2024C_94M_TE.14 + RF2024D.14 - PF2024_ADJWF.14 + (0.96222 * NF2024NMC6U.14) / NF2024NMC6U.14 * (1 - MOVAVG(4, RTP.1)))
 PF2529L_FE = MOVAVG(13, PF2529_FE.14 + PF2554_90C_TE.14 + PF2554_94M_TE.14 + RF2529D.14 - PF2529_ADJWF.14)
 PF2529NML_FE = MOVAVG(13, PF2529NM.14 + PF2554_94C_TE.14 + PF2554_94M_TE.14 + RF2529D.14 - PF2529_ADJWF.14 + (0.91191 * NF2529NMC6U.14 + 0.00000 * NF2529NMNC6.14) / NF2529NM.14 * (1 - MOVAVG(4, RTP.1)))
 PF3034L_FE = MOVAVG(13, PF3034_FE.14 + PF2554_90C_TE.14 + PF2554_94M_TE.14 + RF3034D.14 - PF3034_ADJWF.14)
 PF3034NML_FE = MOVAVG(13, PF3034NM.14 + PF2554_94C_TE.14 + PF2554_94M_TE.14 - PF3034_ADJWF.14 + RF3034D.14 + (0.00000 * NF3034NMC6U.14 + 0.19793 * NF3034NMNC6.14) / NF3034NM.14 * (1 - MOVAVG(4, RTP.1)))
 PF3539L_FE = MOVAVG(13, PF3539_FE.14 + PF2554_90C_TE.14 + PF2554_94M_TE.14 + RF3539D.14 - PF3539_ADJWF.14)
 PF3539NML_FE = MOVAVG(13, PF3539NM.14 + PF2554_94C_TE.14 + PF2554_94M_TE.14 + RF3539D.14 - PF3539_ADJWF.14 + (0.00000 * NF3539NMC6U.14 + 0.32043 * NF3539NMNC6.14) / NF3539NM.14 * (1 - MOVAVG(4, RTP.1)))
 PF4044L_FE = MOVAVG(13, PF4044_FE.14 + PF2554_90C_TE.14 + PF2554_94M_TE.14 + RF4044D.14)
 PF4044NML_FE = MOVAVG(13, PF4044NM.14 + PF2554_94C_TE.14 + PF2554_94M_TE.14 + RF4044D.14 + (0.21194 * NF4044NMC6U.14 + 0.00000 * NF4044NMNC6.14) / NF4044NM.14 * (1 - MOVAVG(4, RTP.1)))
 PF4549L_FE = MOVAVG(13, PF4549_FE.14 + PF2554_90C_TE.14 + PF2554_94M_TE.14 + RF4549D.14)
 PF4549NML_FE = MOVAVG(13, PF4549NM.14 + PF2554_94C_TE.14 + PF2554_94M_TE.14 + RF4549D.14 + (0.00000 * NF4549NM.14) / NF4549NM.14 * (1 - MOVAVG(4, RTP.1)))
 PF5054L_FE = MOVAVG(8, PF5054_FE.8 + PF2554_90C_TE.8 + PF2554_94M_TE.8 + RF5054D.8)

Civilian Labor Force

LM1617 = PM1617 * NM1617
 LM1819 = TPM1819 * (NM1819 + NM1819M)
 LM2024 = TPM2024 * (NM2024 + NM2024M)
 LM2529 = TPM2529 * (NM2529 + NM2529M)
 LM3034 = TPM3034 * (NM3034 + NM3034M)
 LM3539 = TPM3539 * (NM3539 + NM3539M)
 LM4044 = TPM4044 * (NM4044 + NM4044M)
 LM4549 = PM4549 * NM4549
 LM5054 = PM5054 * NM5054
 LM5559 = PM5559 * NM5559
 LM6064 = PM6064 * NM6064
 LM6569 = PM6569 * NM6569
 LM7074 = PM7074 * NM7074
 LM750 = PM750 * NM750
 LM7579 = PM7579 * NM7579
 LM8084 = PM8084 * NM8084
 LM850 = PM850 * NM850
 LCM = LM1617 + LM1819 + LM2024 + LM2529 + LM3034 + LM3539 + LM4044 + LM4549 + LM5054 + LM5559 + LM6064 + LM6569 + LM7074 + LM750
 LM2534 = LM2529 + LM3034
 LM3544 = LM3539 + LM4044
 LM4554 = LM4549 + LM5054
 LM5564 = LM5559 + LM6064
 LM650 = LM6569 + LM7074 + LM750

Females

LF1617 = PF1617 * NF1617
 LF1819 = PF1819 * NF1819
 LF2024 = PF2024 * NF2024
 LF2529 = PF2529 * NF2529
 LF3034 = PF3034 * NF3034

LF3539 = PF3539 * NF3539
 LF4044 = PF4044 * NF4044
 LF4549 = PF4549 * NF4549
 LF5054 = PF5054 * NF5054
 LF5559 = PF5559 * NF5559
 LF6064 = PF6064 * NF6064
 LF6569 = PF6569 * NF6569
 LF7074 = PF7074 * NF7074
 LF75O = PF75O * NF75O
 LF7579 = PF7579 * NF7579
 LF8084 = PF8084 * NF8084
 LF85O = PF85O * NF85O
 LCF = LF1617 + LF1819 + LF2024 + LF2529 + LF3034 + LF3539 + LF4044 + LF4549 + LF5054 + LF5559 + LF6064 + LF6569 + LF7074 + LF75O

LF2534 = LF2529 + LF3034
 LF3544 = LF3539 + LF4044
 LF4554 = LF4549 + LF5054
 LF5564 = LF5559 + LF6064
 LF65O = LF6569 + LF7074 + LF75O

LC = LCM + LCF

DLF1617_FE = (PF1617_FE - PF1617) * NF1617
 DLF1819_FE = (PF1819_FE - PF1819) * NF1819
 DLF2024_FE = (PF2024_FE - PF2024) * NF2024
 DLF2529_FE = (PF2529_FE - PF2529) * NF2529
 DLF3034_FE = (PF3034_FE - PF3034) * NF3034
 DLF3539_FE = (PF3539_FE - PF3539) * NF3539
 DLF4044_FE = (PF4044_FE - PF4044) * NF4044
 DLF4549_FE = (PF4549_FE - PF4549) * NF4549
 DLF5054_FE = (PF5054_FE - PF5054) * NF5054
 DLF5559_FE = (PF5559_FE - PF5559) * NF5559
 DLF6064_FE = (PF6064_FE - PF6064) * NF6064
 DLF6569_FE = (PF6569_FE - PF6569) * NF6569
 DLF7074_FE = (PF7074_FE - PF7074) * NF7074
 DLFCF_FE = DLF1617_FE + DLF1819_FE + DLF2024_FE + DLF2529_FE + DLF3034_FE + DLF3539_FE + DLF4044_FE + DLF4549_FE + DLF5054_FE + DLF5559_FE + DLF6064_FE + DLF6569_FE + DLF7074_FE

DLM1617_FE = (PM1617_FE - PM1617) * NM1617
 DLM1819_FE = (TPM1819_FE - TPM1819) * (NM1819 + NM1819M)
 DLM2024_FE = (TPM2024_FE - TPM2024) * (NM2024 + NM2024M)
 DLM2529_FE = (TPM2529_FE - TPM2529) * (NM2529 + NM2529M)
 DLM3034_FE = (TPM3034_FE - TPM3034) * (NM3034 + NM3034M)
 DLM3539_FE = (TPM3539_FE - TPM3539) * (NM3539 + NM3539M)
 DLM4044_FE = (TPM4044_FE - TPM4044) * (NM4044 + NM4044M)
 DLM4549_FE = (PM4549_FE - PM4549) * NM4549
 DLM5054_FE = (PM5054_FE - PM5054) * NM5054
 DLM5559_FE = (PM5559_FE - PM5559) * NM5559
 DLM6064_FE = (PM6064_FE - PM6064) * NM6064
 DLM6569_FE = (PM6569_FE - PM6569) * NM6569
 DLM7074_FE = (PM7074_FE - PM7074) * NM7074
 DLDCM_FE = DLM1617_FE + DLM1819_FE + DLM2024_FE + DLM2529_FE + DLM3034_FE + DLM3539_FE + DLM4044_FE + DLM4549_FE + DLM5054_FE + DLM5559_FE + DLM6064_FE + DLM6569_FE + DLM7074_FE

Unemployment Rates
Totals

MALES
 RM1617 = RM1617.1 + (-44.4261 * DIFF (RTP) - 2.1644 * DIFF (RTP.1) - 15.0910 * DIFF (RTP.2) - 27.8537 * DIFF (RTP.3)) * 50.00/44.05
 RM1819 = RM1819.1 + (-47.9124 * DIFF (RTP) - 31.9746 * DIFF (RTP.1) - 11.6764 * DIFF (RTP.2) + 2.8763 * DIFF (RTP.3)) * 50.00/44.05

RM2024 = RM2024.1 + (-58.0805 * DIFF (RTP) - 14.6504 * DIFF (RTP.1) - 17.3907 * DIFF (RTP.2) - 8.3721 * DIFF (RTP.3)) *
 50.00/44.05
 RM2529 = RM2529.1 + (-42.4466 * DIFF (RTP) - 16.4626 * DIFF (RTP.1) - 12.0763 * DIFF (RTP.2) - 2.6912 * DIFF (RTP.3)) *
 50.00/44.05
 RM3034 = RM3034.1 + (-21.9947 * DIFF (RTP) - 14.3423 * DIFF (RTP.1) - 7.7729 * DIFF (RTP.2) - 4.5343 * DIFF (RTP.3)) *
 50.00/44.05
 RM3539 = RM3539.1 + (-25.1969 * DIFF (RTP) - 4.8918 * DIFF (RTP.1) - 11.6627 * DIFF (RTP.2) - 9.4142 * DIFF (RTP.3)) *
 50.00/44.05
 RM4044 = RM4044.1 + (-14.7303 * DIFF (RTP) - 12.8348 * DIFF (RTP.1) - 7.4111 * DIFF (RTP.2) - 6.4487 * DIFF (RTP.3)) *
 50.00/44.05
 RM4549 = RM4549.1 + (-18.1162 * DIFF (RTP) - 11.8573 * DIFF (RTP.1) - 7.3805 * DIFF (RTP.2) + 0.9708 * DIFF (RTP.3)) *
 50.00/44.05
 RM5054 = RM5054.1 + (-18.8477 * DIFF (RTP) - 11.0262 * DIFF (RTP.1) - 6.6450 * DIFF (RTP.2) - 4.7406 * DIFF (RTP.3)) *
 50.00/44.05
 RM5559 = RM5559.1 + (-22.8478 * DIFF (RTP) - 11.8787 * DIFF (RTP.1) + 0.6517 * DIFF (RTP.2) - 3.0202 * DIFF (RTP.3)) *
 50.00/44.05
 RM6064 = RM6064.1 + (1.4997 * DIFF (RTP) - 9.8165 * DIFF (RTP.1) + 0.4070 * DIFF (RTP.2) - 15.0849 * DIFF (RTP.3)) *
 50.00/44.05
 RM6569 = RM6569.1 + (-9.7596 * DIFF (RTP) + 11.8572 * DIFF (RTP.1) - 18.3547 * DIFF (RTP.2) + 6.1962 * DIFF (RTP.3)) *
 50.00/44.05
 RM7074 = RM7074.1 + (19.6295 * DIFF (RTP) - 12.5956 * DIFF (RTP.1) - 27.9350 * DIFF (RTP.2) + 6.5664 * DIFF (RTP.3)) *
 50.00/44.05
 RM750 = RM750.1 + (- 5.5101 * DIFF (RTP) - 18.5248 * DIFF (RTP.1) - 17.9157 * DIFF (RTP.2) + 23.5855 * DIFF (RTP.3)) *
 50.00/44.05

FEMALES

RF1617 = RF1617.1 + (-20.9377 * DIFF (RTP) - 3.4543 * DIFF (RTP.1) - 32.5068 * DIFF (RTP.2) - 8.8148 * DIFF (RTP.3)) *
 50.00/44.05
 RF1819 = RF1819.1 + (-39.2646 * DIFF (RTP) - 5.1023 * DIFF (RTP.1) + 3.6786 * DIFF (RTP.2) - 27.3641 * DIFF (RTP.3)) *
 50.00/44.05
 RF2024 = RF2024.1 + (-26.8299 * DIFF (RTP) - 11.1330 * DIFF (RTP.1) - 11.4402 * DIFF (RTP.2) - 4.0048 * DIFF (RTP.3)) *
 50.00/44.05
 RF2529 = RF2529.1 + (-14.8770 * DIFF (RTP) - 8.8852 * DIFF (RTP.1) - 12.9537 * DIFF (RTP.2) - 0.6317 * DIFF (RTP.3)) *
 50.00/44.05
 RF3034 = RF3034.1 + (-10.3084 * DIFF (RTP) - 0.6837 * DIFF (RTP.1) - 18.4113 * DIFF (RTP.2) - 2.0406 * DIFF (RTP.3)) *
 50.00/44.05
 RF3539 = RF3539.1 + (-17.8370 * DIFF (RTP) - 2.3913 * DIFF (RTP.1) - 6.8006 * DIFF (RTP.2) - 5.0086 * DIFF (RTP.3)) * 50.00/44.05
 RF4044 = RF4044.1 + (-6.9101 * DIFF (RTP) - 9.4509 * DIFF (RTP.1) - 3.6043 * DIFF (RTP.2) - 1.8449 * DIFF (RTP.3)) * 50.00/44.05
 RF4549 = RF4549.1 + (-1.6363 * DIFF (RTP) - 19.4549 * DIFF (RTP.1) + 10.7519 * DIFF (RTP.2) - 8.6682 * DIFF (RTP.3)) *
 50.00/44.05
 RF5054 = RF5054.1 + (-12.0818 * DIFF (RTP) + 0.3229 * DIFF (RTP.1) - 15.0622 * DIFF (RTP.2) - 3.0337 * DIFF (RTP.3)) *
 50.00/44.05
 RF5559 = RF5559.1 + (2.7183 * DIFF (RTP) - 9.8009 * DIFF (RTP.1) - 6.1773 * DIFF (RTP.2) - 6.3238 * DIFF (RTP.3)) * 50.00/44.05
 RF6064 = RF6064.1 + (-11.7151 * DIFF (RTP) - 1.9990 * DIFF (RTP.1) - 3.1837 * DIFF (RTP.2) - 10.6383 * DIFF (RTP.3)) *
 50.00/44.05
 RF6569 = RF6569.1 + (-8.7042 * DIFF (RTP.2) - 4.6979 * DIFF (RTP.3)) * 50.00/44.05
 RF7074 = RF7074.1 + (0.2827 * DIFF (RTP.2) - 14.1285 * DIFF (RTP.3)) * 50.00/44.05
 RF750 = RF750.1 + (-11.6993 * DIFF (RTP) + 13.5790 * DIFF (RTP.1) + 8.9530 * DIFF (RTP.2) - 22.2054 * DIFF (RTP.3)) *
 50.00/44.05

EQ RUM = (RM1617 * LM1617 + RM1819 * LM1819 + RM2024 * LM2024 + RM2529 * LM2529 + RM3034 * LM3034 + RM3539 *
 LM3539 + RM4044 * LM4044 + RM4549 * LM4549
 + RM5054 * LM5054 + RM5559 * LM5559 + RM6064 * LM6064 + RM6569 * LM6569 + RM7074 * LM7074 + RM750 *
 LM750) / LCM
 EQ RUF = (RF1617 * LF1617 + RF1819 * LF1819 + RF2024 * LF2024 + RF2529 * LF2529 + RF3034 * LF3034 + RF3539 * LF3539 +
 RF4044 * LF4044 + RF4549 * LF4549 + RF5054
 * LF5054 + RF5559 * LF5559 + RF6064 * LF6064 + RF6569 * LF6569 + RF7074 * LF7074 + RF750 * LF750) / LCF
 EQ RU = (RUM * LCM + RUF * LCF) / LC

2.2 U.S. Earnings (USEAR)

Annual Employment Equations

Class of Worker

Nonagricultural

Wage Workers

Private Household Workers

$$\begin{aligned}
 EF1617NAWPH &= \text{MAX}(0.001, -0.20802 * RTP - 0.40988 * RTP.1 + 0.01015 + 61.2465 * 1/YEAR - 0.00965 * \text{MINW}/\text{CPIW_U} + 0.01561 \\
 &\quad * \text{NU10}/\text{NF1617} - 0.13398) * EF1617 \\
 EF1819NAWPH &= \text{MAX}(0.001, -0.03363 * RTP - 0.12989 * RTP.1 - 0.00661 + 8.44701 * 1/YEAR - 0.00539 * \text{MINW}/\text{CPIW_U} + 0.00345 \\
 &\quad * \text{NU10}/\text{NF1819} + 0.07597) * EF1819 \\
 EF2024NAWPH &= \text{MAX}(0.001, -0.18707 * \text{MOVAVG}(5, RTP) - 0.00223 + 2.12060 * 1/YEAR + 0.00820 * \text{NU10}/\text{NF2024} + 0.14537) * \\
 &\quad EF2024 \\
 EF2534NAWPH &= \text{MAX}(0.001, 0.01874 * RTP - 0.04167 * \text{MOVAVG}(5, RTP.1) - 0.00090 + 1.55167 * 1/YEAR + 0.01021 * \\
 &\quad \text{NU10}/\text{NF2534} - 0.00170) * EF2534 \\
 EF3544NAWPH &= (0.00622 * RTP - 0.06062 * \text{MOVAVG}(5, RTP.1) + 0.00008 + 0.29372 * \text{MOVAVG}(3, EF2534NAWPH.9/EF2534.9) + \\
 &\quad 0.06187) * EF3544 \\
 EF4554NAWPH &= (0.02788 * RTP - 0.10996 * \text{MOVAVG}(5, RTP.1) - 0.00349 + 0.53068 * \text{MOVAVG}(3, EF3544NAWPH.9/EF3544.9) + \\
 &\quad 0.08883) * EF4554 \\
 EF5564NAWPH &= (0.05939 * RTP - 0.10618 * \text{MOVAVG}(2, RTP.1) - 0.00579 + 0.66195 * \text{MOVAVG}(3, EF4554NAWPH.9/EF4554.9) + \\
 &\quad 0.05966) * EF5564 \\
 EF65ONAWPH &= (0.22642 * RTP - 0.02069 + 0.33505 * \text{MOVAVG}(3, EF5564NAWPH.9) - 0.19707) \\
 \\
 EM1617NAWPH &= \text{MAX}(0.001, -0.05284 * RTP - 0.17833 * RTP.1 - 0.00768 + 9.19738 * 1/YEAR - 0.00588 * \text{MINW}/\text{CPIW_U} + 0.16862) \\
 &\quad * EM1617 \\
 EM1819NAWPH &= \text{MAX}(0.001, -0.07122 * RTP - 0.03737 * RTP.1 - 0.00282 + 3.76796 * 1/YEAR - 0.00499 * \text{MINW}/\text{CPIW_U} + 0.08727) \\
 &\quad * EM1819 \\
 EM2024NAWPH &= \text{MAX}(0.001, -0.00450 * RTP - 0.02345 * RTP.1 - 0.00113 - 0.00057 * \text{MINW}/\text{CPIW_U} + 0.03265) * EM2024 \\
 EM2534NAWPH &= \text{MAX}(0.001, -0.00490 * RTP.1 - 0.00054 - 0.00051 * \text{MINW}/\text{CPIW_U} + 0.00789) * EM2534 \\
 EM3544NAWPH &= (-0.00446 * RTP.1 - 0.00041 - 0.00053 * \text{MINW}/\text{CPIW_U} + 0.00726) * EM3544 \\
 EM4554NAWPH &= (-0.00039 + 0.00129) * EM4554 \\
 EM5564NAWPH &= (-0.00015 + 0.00200) * EM5564 \\
 EM65ONAWPH &= (-0.00679 + 0.64405 * \text{MOVAVG}(3, EM5564NAWPH.9) + 0.00231)
 \end{aligned}$$

Government

Other

Self-employed

$$EF1617NAS = (0.12015 * RTP.1 - 0.10551) * EF1617$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\begin{aligned}
 ef1617nas/ef1617 &= & 0.12015 * rtp.1 & - & 0.10551 \\
 & & (1.96868) & & (1.73441)
 \end{aligned}$$

Sum Sq	0.0000
Std Error	0.0030
LHS Mean	0.0142
R-Squared	0.5637
R Bar Squared	0.4182
F-stat 1, 3	3.8757
D.W. (1)	1.5620
D.W. (2)	2.3626

$$EF1819NAS = (0.11184 * RTP.1 - 0.10241) * EF1819$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef1819nas/ef1819} = 0.11184 * \text{rtp.1} - 0.10241$$

(2.99537) (2.75170)

Sum Sq 0.0000
 Std Error 0.0018
 LHS Mean 0.0090
 R-Squared 0.7494
 R Bar Squared 0.6659
 F-STAT 1, 3 8.9722
 D.W. (1) 3.2586
 D.W. (2) 0.9766

$$\text{EF2024NAS} = (0.08908 * \text{RTP.1} - 0.07176) * \text{EF2024}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef2024nas/ef2024} = 0.08908 * \text{rtp.1} - 0.07176$$

(2.54605) (2.05763)

Sum Sq 0.0000
 Std Error 0.0017
 LHS Mean 0.0170
 R-Squared 0.6836
 R Bar Squared 0.5782
 F-STAT 1, 3 6.4824
 D.W. (1) 2.6600
 D.W. (2) 1.5247

$$\text{EF2534NAS} = (0.00906 * \text{RTP.1} + 0.03539) * \text{EF2534}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef2534nas/ef2534} = 0.00906 * \text{rtp.1} + 0.03539$$

(0.34277) (1.34366)

Sum Sq 0.0000
 Std Error 0.0013
 LHS Mean 0.0444
 R-Squared 0.0377
 R Bar Squared 0.2831
 F-STAT 1, 3 0.1175
 D.W. (1) 3.0818
 D.W. (2) 1.1094

$$\text{EF3544NAS} = (-0.01869 * \text{RTP.1} + 0.08087) * \text{EF3544}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef3544nas/ef3544} = -0.01869 * \text{rtp.1} + 0.08087$$

(0.70565) (3.06320)

Sum Sq 0.0000
 Std Error 0.0013
 LHS Mean 0.0622
 R-Squared 0.1424
 R Bar Squared 0.1435
 F-STAT 1, 3 0.4979
 D.W. (1) 2.2440
 D.W. (2) 2.1852

$$\text{EF4554NAS} = (0.07232 * \text{RTP.1} - 0.00701) * \text{EF4554}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef4554nas/ef4554} = 0.07232 * \text{rtp.1} - 0.00701 \\ (2.86756) \quad (0.27876)$$

Sum Sq 0.0000
Std Error 0.0012
LHS Mean 0.0651
R-Squared 0.7327
R Bar Squared 0.6436
F-STAT 1, 3 8.2229
D.W. (1) 1.7821
D.W. (2) 2.7029

$$\text{EF5564NAS} = (0.07872 * \text{RTP.1} + 0.00466) * \text{EF5564}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef5564nas/ef5564} = 0.07872 * \text{rtp.1} + 0.00466 \\ (1.38159) \quad (0.08196)$$

Sum Sq 0.0000
Std Error 0.0028
LHS Mean 0.0831
R-Squared 0.3889
R Bar Squared 0.1851
F-STAT 1, 3 1.9088
D.W. (1) 2.6092
D.W. (2) 2.2686

$$\text{EF65ONAS} = (0.10940 * \text{EF6569} + 0.12265 * \text{EF7074} + 0.14137 * \text{EF750})$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef6569nas/ef6569} = 0.10940 \\ (37.7493)$$

Sum Sq 0.0002
Std Error 0.0065
LHS Mean 0.1094
R-Squared 0.0000
R Bar Squared 0.0000
F 0, 4 NC
D.W. (1) 3.0431
D.W. (2) 1.2204

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef7074nas/ef7074} = 0.12265 \\ (16.4939)$$

Sum Sq 0.0011
Std Error 0.0166
LHS Mean 0.1226
R-Squared 0.0000
R Bar Squared 0.0000
F 0, 4 NC
D.W. (1) 1.0289
D.W. (2) 1.7188

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{ef75onas/ef75o} = \begin{array}{l} 0.14137 \\ (17.7500) \end{array}$$

Sum Sq 0.0013
Std Error 0.0178
LHS Mean 0.1414
R-Squared 0.0000
R Bar Squared 0.0000
F 0, 4 NC
D.W. (1) 1.6889
D.W. (2) 1.2345

$$\text{EM1617NAS} = (-0.23035 * \text{RTP.1} + 0.24985) * \text{EM1617}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em1617nas/em1617} = \begin{array}{l} -0.23035 * \text{rtp.1} + 0.24985 \\ (5.08538) \quad (5.53372) \end{array}$$

Sum Sq 0.0000
Std Error 0.0022
LHS Mean 0.0203
R-Squared 0.8961
R Bar Squared 0.8614
F-STAT 1, 3 25.8611
D.W. (1) 2.4658
D.W. (2) 1.6839

$$\text{EM1819NAS} = (-0.05782 * \text{RTP.1} + 0.07265) * \text{EM1819}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em1819nas/em1819} = \begin{array}{l} -0.05782 * \text{rtp.1} + 0.07265 \\ (3.43044) \quad (4.32458) \end{array}$$

Sum Sq 0.0000
Std Error 0.0008
LHS Mean 0.0150
R-Squared 0.7969
R Bar Squared 0.7291
F-STAT 1, 3 11.7679
D.W. (1) 3.3262
D.W. (2) 1.0399

$$\text{EM2024NAS} = (-0.09206 * \text{RTP.1} + 0.11567) * \text{EM2024}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em2024nas/em2024} = \begin{array}{l} -0.09206 * \text{rtp.1} + 0.11567 \\ (2.44839) \quad (3.08618) \end{array}$$

Sum Sq 0.0000
Std Error 0.0018
LHS Mean 0.0239
R-Squared 0.6665
R Bar Squared 0.5553
F-STAT 1, 3 5.9946
D.W. (1) 2.1493
D.W. (2) 1.7046

$$\text{EM2534NAS} = (-0.09661 * \text{RTP.1} + 0.14843) * \text{EM2534}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em2534nas/em2534} = -0.09661 * \text{rtp.1} + 0.14843 \\ (2.81478) \quad (4.33847)$$

Sum Sq 0.0000
Std Error 0.0017
LHS Mean 0.0522
R-Squared 0.7254
R Bar Squared 0.6338
F-STAT 1, 3 7.9230
D.W. (1) 1.8300
D.W. (2) 2.9632

$$\text{EM3544NAS} = (0.02739 * \text{RTP.1} + 0.05236) * \text{EM3544}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em3544nas/em3544} = 0.02739 * \text{rtp.1} + 0.05236 \\ (0.61129) \quad (1.17241)$$

Sum Sq 0.0000
Std Error 0.0022
LHS Mean 0.0797
R-Squared 0.1108
R Bar Squared 0.1857
F-STAT 1, 3 0.3737
D.W. (1) 2.5508
D.W. (2) 2.2676

$$\text{EM4554NAS} = (0.06217 * \text{RTP.1} + 0.03411) * \text{EM4554}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em4554nas/em4554} = 0.06217 * \text{rtp.1} + 0.03411 \\ (1.91738) \quad (1.05544)$$

Sum Sq 0.0000
Std Error 0.0016
LHS Mean 0.0961
R-Squared 0.5507
R Bar Squared 0.4009
F-STAT 1, 3 3.6764
D.W. (1) 2.5497
D.W. (2) 1.5554

$$\text{EM5564NAS} = (-0.04776 * \text{RTP.1} + 0.16626) * \text{EM5564}$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em5564nas/em5564} = -0.04776 * \text{rtp.1} + 0.16626 \\ (0.60480) \quad (2.11226)$$

Sum Sq 0.0000
Std Error 0.0039
LHS Mean 0.1187
R-Squared 0.1087
R Bar Squared 0.1884
F-STAT 1, 3 0.3658
D.W. (1) 2.9234
D.W. (2) 1.9432

$$EM65ONAS = (0.16527 * EM6569 + 0.17798 * EM7074 + 0.19058 * EM750)$$

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em6569nas/em6569} = \begin{matrix} 0.16527 \\ (53.9126) \end{matrix}$$

Sum Sq 0.0002
 Std Error 0.0069
 LHS Mean 0.1653
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 1.7716
 D.W. (2) 2.9645

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em7074nas/em7074} = \begin{matrix} 0.17798 \\ (22.9265) \end{matrix}$$

Sum Sq 0.0012
 Std Error 0.0174
 LHS Mean 0.1780
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 1.7116
 D.W. (2) 2.1991

Ordinary Least Squares

ANNUAL data for 5 periods from 2000 to 2004

Date: 9 NOV 2005

$$\text{em750nas/em750} = \begin{matrix} 0.19058 \\ (20.1892) \end{matrix}$$

Sum Sq 0.0018
 Std Error 0.0211
 LHS Mean 0.1906
 R-Squared 0.0000
 R Bar Squared 0.0000
 F-stat 0, 4 NC
 D.W. (1) 2.7330
 D.W. (2) 0.9992

Unpaid Family Workers

$$EF1617NAU = 0.00012 * ENAS$$

$$EF1819NAU = 0.00025 * ENAS$$

$$EF2024NAU = 0.00024 * ENAS$$

$$EF2534NAU = 0.00117 * ENAS$$

$$EF3544NAU = 0.00218 * ENAS$$

$$EF4554NAU = 0.00226 * ENAS$$

$$EF5564NAU = 0.00083 * ENAS$$

$$EF65ONAU = (0.00027 + 0.00021 + 0.00008) * ENAS$$

$$EM1617NAU = 0.00028 * ENAS$$

$$EM1819NAU = 0.00033 * ENAS$$

$$EM2024NAU = 0.00050 * ENAS$$

$$EM2534NAU = 0.00044 * ENAS$$

$$EM3544NAU = 0.00043 * ENAS$$

EM4554NAU = 0.00052 * ENAS
 EM5564NAU = 0.00037 * ENAS
 EM650NAU = (0.00023 + 0.00010 + 0.00011) * ENAS

Agricultural

Wage Workers

EM1617AW = EAW * (-0.00594 - 0.09353 * RTP + 5.28754 * EM1617/E + 0.08116)
 EM1819AW = EAW * (-0.00131 - 0.18120 * RTP + 3.87151 * EM1819/E + 0.16636)
 EM2024AW = EAW * (-0.00664 + 0.10493 * RTP + 2.00153 * EM2024/E - 0.08191)
 EM2534AW = EAW * (-0.02065 + 0.38358 * RTP - 0.98380 * EM2534/E + 0.00751)
 EM3544AW = EAW * (0.00402 - 0.15663 * RTP + 1.72119 * EM3544/E + 0.05679)
 EM4554AW = EAW * (0.00834 + 0.03746 * RTP + 0.46522 * EM4554/E + 0.00144)
 EM5564AW = EAW * (-0.00655 + 0.03521 * RTP + 0.46852 * EM5564/E - 0.00037)
 EM650AW = EAW * (-0.00114 + 0.07640 * RTP + 3.25911 * EM650/E - 0.10058)

EF1617AW = EAW * (-0.00055 - 0.05470 * RTP + 1.41760 * EF1617/E + 0.04979)
 EF1819AW = EAW * (0.00102 - 0.07375 * RTP + 0.78394 * EF1819/E + 0.07226)
 EF2024AW = EAW * (0.00112 - 0.05971 * RTP + 0.57256 * EF2024/E + 0.05907)
 EF2534AW = EAW * (0.00623 + 0.08868 * RTP + 1.00897 * EF2534/E - 0.15142)
 EF3544AW = EAW * (0.00687 - 0.00259 * RTP + 0.51319 * EF3544/E - 0.00937)
 EF4554AW = EAW * (0.00185 + 0.08747 * RTP + 0.28022 * EF4554/E - 0.08053)
 EF5564AW = EAW * (-0.00140 - 0.03001 * RTP - 0.59383 * EF5564/E + 0.07088)
 EF650AW = EAW * (0.00096 + 0.06768 * RTP + 1.04213 * EF650/E - 0.07359)

Self-employed

EM1617AS = NM1617 * (0.00528 + 0.00404)
 EM1819AS = NM1819 * (0.00309 + 0.28448 * EA / (NM160+ NF160) - 0.00165)
 EM2024AS = NM2024 * (-0.00181 + 0.97958 * EA / (NM160+ NF160) - 0.01093)
 EM2534AS = NM2534 * (-0.00263 + 1.23186 * EA / (NM160+ NF160) - 0.01021)
 EM3544AS = NM3544 * (-0.00151 + 1.66765 * EA / (NM160+ NF160) - 0.01450)
 EM4554AS = NM4554 * (-0.00381 + 2.86654 * EA / (NM160+ NF160) - 0.03175)
 EM5564AS = NM5564 * (-0.00460 + 2.78817 * EA / (NM160+ NF160) - 0.02398)
 EM650AS = NM650 * (0.00079 + 1.76904 * EA / (NM160+ NF160) - 0.01437)

EF1617AS = NF1617 * (0.00181 + 0.00030)
 EF1819AS = EM1819AS * (-0.02393 + 0.63672 * RTP + 0.98791 * EF1819/EM1819 - 1.43926)
 EF2024AS = EM2024AS * (0.07353 - 0.40207 * RTP + 0.57572 * EF2024/EM2024 - 0.01117)
 EF2534AS = EM2534AS * (0.16575 + 0.16967 * RTP + 0.55503 * EF2534/EM2534 - 0.43412)
 EF3544AS = EM3544AS * (0.15848 + 0.37839 * RTP + 0.37764 * EF3544/EM3544 - 0.45362)
 EF4554AS = EM4554AS * (0.21947 + 0.29497 * RTP + 0.58974 * EF4554/EM4554 - 0.51966)
 EF5564AS = EM5564AS * (0.20892 + 0.36294 * RTP + 0.65320 * EF5564/EM5564 - 0.66626)
 EF650AS = EM650AS * (0.16242 + 0.54916 * RTP + 0.06199 * EF650/EM650 - 0.47556)

Unpaid Family Workers

EF1617AU = 0.002
 EF1819AU = 0.001
 EF2024AU = 0.001
 EF2534AU = 0.003
 EF3544AU = 0.004
 EF4554AU = 0.005
 EF5564AU = 0.003
 EF650AU = 0.001

EM1617AU = 0.006
 EM1819AU = 0.005
 EM2024AU = 0.005
 EM2534AU = 0.002
 EM3544AU = 0.002
 EM4554AU = 0.001
 EM5564AU = 0.001
 EM650AU = 0.002

Quarterly Employment Equations
Class of Worker

Nonagricultural
ENA = E - EA

Wage Workers

Private Household Workers

EF1617NAWPH_R = MAX (0.001, -0.20802 * MOVAVG (4, RTP.1) - 0.40988 * MOVAVG (4, RTP.5) + 0.01015 + 61.2465 * 1/YEAR - 0.00965 * MINW/CPIW_U + 0.01561 * NU10/NF1617 - 0.13398) * EF1617 + EF1617NAWPH.ADJ
EF1819NAWPH_R = MAX (0.001, -0.03363 * MOVAVG (4, RTP.1) - 0.12989 * MOVAVG (4, RTP.5) - 0.00661 + 8.44701 * 1/YEAR - 0.00539 * MINW/CPIW_U + 0.00345 * NU10/NF1819 + 0.07597) * EF1819 + EF1819NAWPH.ADJ
EF2024NAWPH_R = MAX (0.001, -0.18707 * MOVAVG (20, RTP.1) - 0.00223 + 2.12060 * 1/YEAR + 0.00820 * NU10/NF2024 + 0.14537) * EF2024 + EF2024NAWPH.ADJ
EF2534NAWPH_R = MAX (0.001, 0.01874 * MOVAVG (4, RTP.1) - 0.04167 * MOVAVG (20, RTP.5) - 0.00090 + 1.55167 * 1/YEAR + 0.01021 * NU10/NF2534 - 0.00170) * EF2534 + EF2534NAWPH.ADJ
EF3544NAWPH_R = (0.00622 * MOVAVG (4, RTP.1) - 0.06062 * MOVAVG (20, RTP.5) + 0.00008 + 0.29372 * MOVAVG (12, EF2534NAWPH.36/EF2534.36) + 0.06187) * EF3544 + EF3544NAWPH.ADJ
EF4554NAWPH_R = (0.02788 * MOVAVG (4, RTP.1) - 0.10996 * MOVAVG (20, RTP.5) - 0.00349 + 0.53068 * MOVAVG (12, EF3544NAWPH.36/EF4554.36) + 0.08883) * EF4554 + EF4554NAWPH.ADJ
EF5564NAWPH_R = (0.05939 * MOVAVG (4, RTP.1) - 0.10618 * MOVAVG (8, RTP.5) - 0.00579 + 0.66195 * MOVAVG (12, EF4554NAWPH.36/EF5564.36) + 0.05966) * EF5564 + EF5564NAWPH.ADJ
EF650NAWPH_R = (0.22642 * MOVAVG (4, RTP.1) - 0.02069 + 0.33505 * MOVAVG (12, EF5564NAWPH.36) - 0.19707) + EF650NAWPH.ADJ
EM1617NAWPH_R = MAX (0.001, -0.05284 * MOVAVG (4, RTP.1) - 0.17833 * MOVAVG (4, RTP.5) - 0.00768 + 9.19738 * 1/YEAR - 0.00588 * MINW/CPIW_U + 0.16862) * EM1617 + EM1617NAWPH.ADJ
EM1819NAWPH_R = MAX (0.001, -0.07122 * MOVAVG (4, RTP.1) - 0.03737 * MOVAVG (4, RTP.5) - 0.00282 + 3.76796 * 1/YEAR - 0.00499 * MINW/CPIW_U + 0.08727) * EM1819 + EM1819NAWPH.ADJ
EM2024NAWPH_R = MAX (0.001, -0.00450 * MOVAVG (4, RTP.1) - 0.02345 * MOVAVG (4, RTP.5) - 0.00113 - 0.00057 * MINW/CPIW_U + 0.03265) * EM2024 + EM2024NAWPH.ADJ
EM2534NAWPH_R = MAX (0.001, -0.00490 * MOVAVG (4, RTP.5) - 0.00054 - 0.00051 * MINW/CPIW_U + 0.00789) * EM2534 + EM2534NAWPH.ADJ
EM3544NAWPH_R = (-0.00446 * MOVAVG (4, RTP.5) - 0.00041 - 0.00053 * MINW/CPIW_U + 0.00726) * EM3544 + EM3544NAWPH.ADJ
EM4554NAWPH_R = (-0.00039 + 0.00129) * EM4554 + EM4554NAWPH.ADJ
EM5564NAWPH_R = (-0.00015 + 0.00200) * EM5564 + EM5564NAWPH.ADJ
EM650NAWPH_R = (-0.00679 + 0.64405 * MOVAVG (12, EM5564NAWPH.36) + 0.00231) + EM650NAWPH.ADJ
ENAWPH_R = EF1617NAWPH_R + EF1819NAWPH_R + EF2024NAWPH_R + EF2534NAWPH_R + EF3544NAWPH_R + EF4554NAWPH_R + EF5564NAWPH_R + EF650NAWPH_R + EM1617NAWPH_R + EM1819NAWPH_R + EM2024NAWPH_R + EM2534NAWPH_R + EM3544NAWPH_R + EM4554NAWPH_R + EM5564NAWPH_R + EM650NAWPH_R

ENAWPH = IF LONGRANGE = 0
THEN ENAWPH_R
ELSE ENAWPH.1 * (E_FE/E_FE.1)

EM1617NAWPH = EM1617NAWPH_R * (ENAWPH/ENAWPH_R)
EM1819NAWPH = EM1819NAWPH_R * (ENAWPH/ENAWPH_R)
EM2024NAWPH = EM2024NAWPH_R * (ENAWPH/ENAWPH_R)
EM2534NAWPH = EM2534NAWPH_R * (ENAWPH/ENAWPH_R)
EM3544NAWPH = EM3544NAWPH_R * (ENAWPH/ENAWPH_R)
EM4554NAWPH = EM4554NAWPH_R * (ENAWPH/ENAWPH_R)
EM5564NAWPH = EM5564NAWPH_R * (ENAWPH/ENAWPH_R)
EM650NAWPH = EM650NAWPH_R * (ENAWPH/ENAWPH_R)

```

EF1617NAWPH = EF1617NAWPH_R * (ENAWPH/ENAWPH_R)
EF1819NAWPH = EF1819NAWPH_R * (ENAWPH/ENAWPH_R)
EF2024NAWPH = EF2024NAWPH_R * (ENAWPH/ENAWPH_R)
EF2534NAWPH = EF2534NAWPH_R * (ENAWPH/ENAWPH_R)
EF3544NAWPH = EF3544NAWPH_R * (ENAWPH/ENAWPH_R)
EF4554NAWPH = EF4554NAWPH_R * (ENAWPH/ENAWPH_R)
EF5564NAWPH = EF5564NAWPH_R * (ENAWPH/ENAWPH_R)
EF65ONAWPH = EF65ONAWPH_R * (ENAWPH/ENAWPH_R)

EFNAWPH = EF1617NAWPH + EF1819NAWPH + EF2024NAWPH + EF2534NAWPH + EF3544NAWPH + EF4554NAWPH +
EF5564NAWPH + EF65ONAWPH
EMNAWPH = EM1617NAWPH + EM1819NAWPH + EM2024NAWPH + EM2534NAWPH + EM3544NAWPH + EM4554NAWPH +
EM5564NAWPH + EM65ONAWPH

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Government

Other

Self-employed

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EF1617NAS_R = (0.12015 * RTP.1 - 0.10551) * EF1617 + EF1617NAS.ADJ
EF1819NAS_R = (0.11184 * RTP.1 - 0.10241) * EF1819 + EF1819NAS.ADJ
EF2024NAS_R = (0.08908 * RTP.1 - 0.07176) * EF2024 + EF2024NAS.ADJ
EF2534NAS_R = (0.00906 * RTP.1 + 0.03539) * EF2534 + EF2534NAS.ADJ
EF3544NAS_R = (-0.01869 * RTP.1 + 0.08087) * EF3544 + EF3544NAS.ADJ
EF4554NAS_R = (0.07232 * RTP.1 - 0.00701) * EF4554 + EF4554NAS.ADJ
EF5564NAS_R = (0.07872 * RTP.1 + 0.00466) * EF5564 + EF5564NAS.ADJ
EF65ONAS_R = (0.10940 * EF6569 + 0.12265 * EF7074 + 0.14137 * EF750) + EF65ONAS.ADJ

EM1617NAS_R = (-0.23035 * RTP.1 + 0.24985) * EM1617 + EM1617NAS.ADJ
EM1819NAS_R = (-0.05782 * RTP.1 + 0.07265) * EM1819 + EM1819NAS.ADJ
EM2024NAS_R = (-0.09206 * RTP.1 + 0.11567) * EM2024 + EM2024NAS.ADJ
EM2534NAS_R = (-0.09661 * RTP.1 + 0.14843) * EM2534 + EM2534NAS.ADJ
EM3544NAS_R = (0.02739 * RTP.1 + 0.05236) * EM3544 + EM3544NAS.ADJ
EM4554NAS_R = (0.06217 * RTP.1 + 0.03411) * EM4554 + EM4554NAS.ADJ
EM5564NAS_R = (-0.04776 * RTP.1 + 0.16626) * EM5564 + EM5564NAS.ADJ
EM65ONAS_R = (0.16527 * EM6569 + 0.17798 * EM7074 + 0.19058 * EM750) + EM65ONAS.ADJ

```

```

ENAS_R = EF1617NAS_R + EF1819NAS_R + EF2024NAS_R + EF2534NAS_R + EF3544NAS_R + EF4554NAS_R + EF5564NAS_R +
EF65ONAS_R + EM1617NAS_R + EM1819NAS_R
+ EM2024NAS_R + EM2534NAS_R + EM3544NAS_R + EM4554NAS_R + EM5564NAS_R + EM65ONAS_R

```

```

ENAS = IF LONGRANGE = 0
THEN ENAS_R
ELSE ENA * (ENAS.1/ENA.1)

```

```

EM1617NAS = EM1617NAS_R * (ENAS/ENAS_R)
EM1819NAS = EM1819NAS_R * (ENAS/ENAS_R)
EM2024NAS = EM2024NAS_R * (ENAS/ENAS_R)
EM2534NAS = EM2534NAS_R * (ENAS/ENAS_R)
EM3544NAS = EM3544NAS_R * (ENAS/ENAS_R)
EM4554NAS = EM4554NAS_R * (ENAS/ENAS_R)
EM5564NAS = EM5564NAS_R * (ENAS/ENAS_R)
EM65ONAS = EM65ONAS_R * (ENAS/ENAS_R)

```

```

EF1617NAS = EF1617NAS_R * (ENAS/ENAS_R)
EF1819NAS = EF1819NAS_R * (ENAS/ENAS_R)
EF2024NAS = EF2024NAS_R * (ENAS/ENAS_R)
EF2534NAS = EF2534NAS_R * (ENAS/ENAS_R)
EF3544NAS = EF3544NAS_R * (ENAS/ENAS_R)
EF4554NAS = EF4554NAS_R * (ENAS/ENAS_R)
EF5564NAS = EF5564NAS_R * (ENAS/ENAS_R)
EF65ONAS = EF65ONAS_R * (ENAS/ENAS_R)

```

```

EFNAS = EF1617NAS + EF1819NAS + EF2024NAS + EF2534NAS + EF3544NAS + EF4554NAS + EF5564NAS + EF65ONAS

```

EMNAS = EM1617NAS + EM1819NAS + EM2024NAS + EM2534NAS + EM3544NAS + EM4554NAS + EM5564NAS + EM65ONAS

Unpaid Family Workers

EF1617NAU_R = 0.00012 * ENAS + EF1617NAU.ADJ
EF1819NAU_R = 0.00025 * ENAS + EF1819NAU.ADJ
EF2024NAU_R = 0.00024 * ENAS + EF2024NAU.ADJ
EF2534NAU_R = 0.00117 * ENAS + EF2534NAU.ADJ
EF3544NAU_R = 0.00218 * ENAS + EF3544NAU.ADJ
EF4554NAU_R = 0.00226 * ENAS + EF4554NAU.ADJ
EF5564NAU_R = 0.00083 * ENAS + EF5564NAU.ADJ
EF65ONAU_R = (0.00027 + 0.00021 + 0.00008) * ENAS + EF65ONAU.ADJ

EM1617NAU_R = 0.00028 * ENAS + EM1617NAU.ADJ
EM1819NAU_R = 0.00033 * ENAS + EM1819NAU.ADJ
EM2024NAU_R = 0.00050 * ENAS + EM2024NAU.ADJ
EM2534NAU_R = 0.00044 * ENAS + EM2534NAU.ADJ
EM3544NAU_R = 0.00043 * ENAS + EM3544NAU.ADJ
EM4554NAU_R = 0.00052 * ENAS + EM4554NAU.ADJ
EM5564NAU_R = 0.00037 * ENAS + EM5564NAU.ADJ
EM65ONAU_R = (0.00023 + 0.00010 + 0.00011) * ENAS + EM65ONAU.ADJ

ENAU_R = EF1617NAU_R + EF1819NAU_R + EF2024NAU_R + EF2534NAU_R + EF3544NAU_R + EF4554NAU_R + EF5564NAU_R + EF65ONAU_R + EM1617NAU_R + EM1819NAU_R + EM2024NAU_R + EM2534NAU_R + EM3544NAU_R + EM4554NAU_R + EM5564NAU_R + EM65ONAU_R

ENAU = ENAU_R

EM1617NAU = EM1617NAU_R * (ENAU/ENAU_R)
EM1819NAU = EM1819NAU_R * (ENAU/ENAU_R)
EM2024NAU = EM2024NAU_R * (ENAU/ENAU_R)
EM2534NAU = EM2534NAU_R * (ENAU/ENAU_R)
EM3544NAU = EM3544NAU_R * (ENAU/ENAU_R)
EM4554NAU = EM4554NAU_R * (ENAU/ENAU_R)
EM5564NAU = EM5564NAU_R * (ENAU/ENAU_R)
EM65ONAU = EM65ONAU_R * (ENAU/ENAU_R)

EF1617NAU = EF1617NAU_R * (ENAU/ENAU_R)
EF1819NAU = EF1819NAU_R * (ENAU/ENAU_R)
EF2024NAU = EF2024NAU_R * (ENAU/ENAU_R)
EF2534NAU = EF2534NAU_R * (ENAU/ENAU_R)
EF3544NAU = EF3544NAU_R * (ENAU/ENAU_R)
EF4554NAU = EF4554NAU_R * (ENAU/ENAU_R)
EF5564NAU = EF5564NAU_R * (ENAU/ENAU_R)
EF65ONAU = EF65ONAU_R * (ENAU/ENAU_R)

EFNAU = EF1617NAU + EF1819NAU + EF2024NAU + EF2534NAU + EF3544NAU + EF4554NAU + EF5564NAU + EF65ONAU
EMNAU = EM1617NAU + EM1819NAU + EM2024NAU + EM2534NAU + EM3544NAU + EM4554NAU + EM5564NAU + EM65ONAU

Agricultural

EA = IF LONGRANGE = 0
THEN GDPFF00 / (EXP (-0.20541 + 0.03254 * YEAR - 0.07829 + 0.37854))
ELSE E * EA.1/E.1

Wage Workers

EAW = IF LONGRANGE = 0
THEN EA * (0.00893 * YEAR + 0.33159 * RTP - 0.67943)
ELSE EA * (EAW.1/EA.1)

EM1617AW_R = MAX (0, EAW * (-0.00594 - 0.09353 * MOVAVG (2, RTP.1) + 5.28754 * EM1617/E + 0.08116) + EM1617AW.ADJ)

EM1819AW_R = MAX (0, EAW * (-0.00131 - 0.18120 * MOVAVG (2, RTP.1) + 3.87151 * EM1819/E + 0.16636) + EM1819AW.ADJ)
 EM2024AW_R = MAX (0, EAW * (-0.00664 + 0.10493 * MOVAVG (2, RTP.1) + 2.00153 * EM2024/E - 0.08191) + EM2024AW.ADJ)
 EM2534AW_R = MAX (0, EAW * (-0.02065 + 0.38358 * MOVAVG (2, RTP.1) - 0.98380 * EM2534/E + 0.00751) + EM2534AW.ADJ)
 EM3544AW_R = MAX (0, EAW * (0.00402 - 0.15663 * MOVAVG (2, RTP.1) + 1.72119 * EM3544/E + 0.05679) + EM3544AW.ADJ)
 EM4554AW_R = MAX (0, EAW * (0.00834 + 0.03746 * MOVAVG (2, RTP.1) + 0.46522 * EM4554/E + 0.00144) + EM4554AW.ADJ)
 EM5564AW_R = MAX (0, EAW * (-0.00655 + 0.03521 * MOVAVG (2, RTP.1) + 0.46852 * EM5564/E - 0.00037) + EM5564AW.ADJ)
 EM65OAW_R = MAX (0, EAW * (-0.00114 + 0.07640 * MOVAVG (2, RTP.1) + 3.25911 * EM65O/E - 0.10058) + EM65OAW.ADJ)

EF1617AW_R = MAX (0, EAW * (-0.00055 - 0.05470 * MOVAVG (2, RTP.1) + 1.41760 * EF1617/E + 0.04979) + EF1617AW.ADJ)
 EF1819AW_R = MAX (0, EAW * (0.00102 - 0.07375 * MOVAVG (2, RTP.1) + 0.78394 * EF1819/E + 0.07226) + EF1819AW.ADJ)
 EF2024AW_R = MAX (0, EAW * (0.00112 - 0.05971 * MOVAVG (2, RTP.1) + 0.57256 * EF2024/E + 0.05907) + EF2024AW.ADJ)
 EF2534AW_R = MAX (0, EAW * (0.00623 + 0.08868 * MOVAVG (2, RTP.1) + 1.00897 * EF2534/E - 0.15142) + EF2534AW.ADJ)
 EF3544AW_R = MAX (0, EAW * (0.00687 - 0.00259 * MOVAVG (2, RTP.1) + 0.51319 * EF3544/E - 0.00937) + EF3544AW.ADJ)
 EF4554AW_R = MAX (0, EAW * (0.00185 + 0.08747 * MOVAVG (2, RTP.1) + 0.28022 * EF4554/E - 0.08053) + EF4554AW.ADJ)
 EF5564AW_R = MAX (0, EAW * (-0.00140 - 0.03001 * MOVAVG (2, RTP.1) - 0.59383 * EF5564/E + 0.07088) + EF5564AW.ADJ)
 EF65OAW_R = MAX (0, EAW * (0.00096 + 0.06768 * MOVAVG (2, RTP.1) + 1.04213 * EF65O/E - 0.07359) + EF65OAW.ADJ)

EAW_R = EF1617AW_R + EF1819AW_R + EF2024AW_R + EF2534AW_R + EF3544AW_R + EF4554AW_R +
 EF5564AW_R + EF65OAW_R + EM1617AW_R + EM1819AW_R
 + EM2024AW_R + EM2534AW_R + EM3544AW_R + EM4554AW_R + EM5564AW_R + EM65OAW_R

EM1617AW = EM1617AW_R * (EAW/EAW_R)
 EM1819AW = EM1819AW_R * (EAW/EAW_R)
 EM2024AW = EM2024AW_R * (EAW/EAW_R)
 EM2534AW = EM2534AW_R * (EAW/EAW_R)
 EM3544AW = EM3544AW_R * (EAW/EAW_R)
 EM4554AW = EM4554AW_R * (EAW/EAW_R)
 EM5564AW = EM5564AW_R * (EAW/EAW_R)
 EM65OAW = EM65OAW_R * (EAW/EAW_R)

EF1617AW = EF1617AW_R * (EAW/EAW_R)
 EF1819AW = EF1819AW_R * (EAW/EAW_R)
 EF2024AW = EF2024AW_R * (EAW/EAW_R)
 EF2534AW = EF2534AW_R * (EAW/EAW_R)
 EF3544AW = EF3544AW_R * (EAW/EAW_R)
 EF4554AW = EF4554AW_R * (EAW/EAW_R)
 EF5564AW = EF5564AW_R * (EAW/EAW_R)
 EF65OAW = EF65OAW_R * (EAW/EAW_R)

EFAW = EF1617AW + EF1819AW + EF2024AW + EF2534AW + EF3544AW + EF4554AW + EF5564AW + EF65OAW
 EMAW = EM1617AW + EM1819AW + EM2024AW + EM2534AW + EM3544AW + EM4554AW + EM5564AW + EM65OAW

Self-employed
 EAS = EA - EAU - EAW

EM1617AS_R = MAX (0, NM1617 * (0.00528 + 0.00404) + EM1617AS.ADJ)
 EM1819AS_R = MAX (0, NM1819 * (0.00309 + 0.28448 * EA / (NM16O + NF16O) - 0.00165) + EM1819AS.ADJ)
 EM2024AS_R = MAX (0, NM2024 * (-0.00181 + 0.97958 * EA / (NM16O + NF16O) - 0.01093) + EM2024AS.ADJ)
 EM2534AS_R = MAX (0, NM2534 * (-0.00263 + 1.23186 * EA / (NM16O + NF16O) - 0.01021) + EM2534AS.ADJ)
 EM3544AS_R = MAX (0, NM3544 * (-0.00151 + 1.66765 * EA / (NM16O + NF16O) - 0.01450) + EM3544AS.ADJ)
 EM4554AS_R = MAX (0, NM4554 * (-0.00381 + 2.86654 * EA / (NM16O + NF16O) - 0.03175) + EM4554AS.ADJ)
 EM5564AS_R = MAX (0, NM5564 * (-0.00460 + 2.78817 * EA / (NM16O + NF16O) - 0.02398) + EM5564AS.ADJ)
 EM65OAS_R = MAX (0, NM65O * (0.00079 + 1.76904 * EA / (NM16O + NF16O) - 0.01437) + EM65OAS.ADJ)

EF1617AS_R = MAX (0, NF1617 * (0.00181 + 0.00030) + EF1617AS.ADJ)
 EF1819AS_R = MAX (0, EM1819AS.1 * (-0.02393 + 0.63672 * MOVAVG (2, RTP.1) + 0.98791 * EF1819/EM1819 - 4.43926) +
 EF1819AS.ADJ)
 EF2024AS_R = MAX (0, EM2024AS.1 * (0.07353 - 0.40207 * MOVAVG (2, RTP.1) + 0.57572 * EF2024/EM2024 - 0.01117) +
 EF2024AS.ADJ)
 EF2534AS_R = MAX (0, EM2534AS.1 * (0.16575 + 0.16967 * MOVAVG (2, RTP.1) + 0.55503 * EF2534/EM2534 - 0.43412) +
 EF2534AS.ADJ)
 EF3544AS_R = MAX (0, EM3544AS.1 * (0.15848 + 0.37839 * MOVAVG (2, RTP.1) + 0.37764 * EF3544/EM3544 - 0.45362) +
 EF3544AS.ADJ)
 EF4554AS_R = MAX (0, EM4554AS.1 * (0.21947 + 0.29497 * MOVAVG (2, RTP.1) + 0.58974 * EF4554/EM4554 - 0.51966) +
 EF4554AS.ADJ)

EF5564AS_R = MAX (0, EM5564AS.1 * (0.20892 + 0.36294 * MOVAVG (2, RTP.1) + 0.65320 * EF5564/EM5564 - 0.66626) +
 EF5564AS.ADJ)
 EF65OAS_R = MAX (0, EM65OAS.1 * (0.16242 + 0.54916 * MOVAVG (2, RTP.1) + 0.06199 * EF65O/EM65O - 0.47556) +
 EF65OAS.ADJ)

EAS_R = EF1617AS_R + EF1819AS_R + EF2024AS_R + EF2534AS_R + EF3544AS_R + EF4554AS_R + EF5564AS_R +
 EF65OAS_R + EM1617AS_R + EM1819AS_R
 + EM2024AS_R + EM2534AS_R + EM3544AS_R + EM4554AS_R + EM5564AS_R + EM65OAS_R

EM1617AS = EM1617AS_R * (EAS/EAS_R)
 EM1819AS = EM1819AS_R * (EAS/EAS_R)
 EM2024AS = EM2024AS_R * (EAS/EAS_R)
 EM2534AS = EM2534AS_R * (EAS/EAS_R)
 EM3544AS = EM3544AS_R * (EAS/EAS_R)
 EM4554AS = EM4554AS_R * (EAS/EAS_R)
 EM5564AS = EM5564AS_R * (EAS/EAS_R)
 EM65OAS = EM65OAS_R * (EAS/EAS_R)

EF1617AS = EF1617AS_R * (EAS/EAS_R)
 EF1819AS = EF1819AS_R * (EAS/EAS_R)
 EF2024AS = EF2024AS_R * (EAS/EAS_R)
 EF2534AS = EF2534AS_R * (EAS/EAS_R)
 EF3544AS = EF3544AS_R * (EAS/EAS_R)
 EF4554AS = EF4554AS_R * (EAS/EAS_R)
 EF5564AS = EF5564AS_R * (EAS/EAS_R)
 EF65OAS = EF65OAS_R * (EAS/EAS_R)

EFAS = EF1617AS + EF1819AS + EF2024AS + EF2534AS + EF3544AS + EF4554AS + EF5564AS + EF65OAS
 EMAS = EM1617AS + EM1819AS + EM2024AS + EM2534AS + EM3544AS + EM4554AS + EM5564AS + EM65OAS

Unpaid Family Workers

EM1617AU_R = MAX (0, 0.002 + EM1617AU.ADJ)
 EM1819AU_R = MAX (0, 0.001 + EM1819AU.ADJ)
 EM2024AU_R = MAX (0, 0.001 + EM2024AU.ADJ)
 EM2534AU_R = MAX (0, 0.003 + EM2534AU.ADJ)
 EM3544AU_R = MAX (0, 0.004 + EM3544AU.ADJ)
 EM4554AU_R = MAX (0, 0.005 + EM4554AU.ADJ)
 EM5564AU_R = MAX (0, 0.003 + EM5564AU.ADJ)
 EM65OAU_R = MAX (0, 0.001 + EM65OAU.ADJ)

EF1617AU_R = MAX (0, 0.006 + EM1617AU.ADJ)
 EF1819AU_R = MAX (0, 0.005 + EF1819AU.ADJ)
 EF2024AU_R = MAX (0, 0.005 + EF2024AU.ADJ)
 EF2534AU_R = MAX (0, 0.002 + EF2534AU.ADJ)
 EF3544AU_R = MAX (0, 0.002 + EF3544AU.ADJ)
 EF4554AU_R = MAX (0, 0.001 + EF4554AU.ADJ)
 EF5564AU_R = MAX (0, 0.001 + EF5564AU.ADJ)
 EF65OAU_R = MAX (0, 0.002 + EF65OAU.ADJ)

EAU_R = EF1617AU_R + EF1819AU_R + EF2024AU_R + EF2534AU_R + EF3544AU_R + EF4554AU_R + EF5564AU_R +
 EF65OAU_R + EM1617AU_R + EM1819AU_R
 + EM2024AU_R + EM2534AU_R + EM3544AU_R + EM4554AU_R + EM5564AU_R + EM65OAU_R

EAU = IF LONGRANGE = 0
 THEN EAU_R
 ELSE EAU.1/EA.1 * EA

EM1617AU = EM1617AU_R * (EAU/EAU_R)
 EM1819AU = EM1819AU_R * (EAU/EAU_R)
 EM2024AU = EM2024AU_R * (EAU/EAU_R)
 EM2534AU = EM2534AU_R * (EAU/EAU_R)
 EM3544AU = EM3544AU_R * (EAU/EAU_R)
 EM4554AU = EM4554AU_R * (EAU/EAU_R)
 EM5564AU = EM5564AU_R * (EAU/EAU_R)

$$EM65OAU = EM65OAU_R * (EAU/EAU_R)$$

$$EF1617AU = EF1617AU_R * (EAU/EAU_R)$$

$$EF1819AU = EF1819AU_R * (EAU/EAU_R)$$

$$EF2024AU = EF2024AU_R * (EAU/EAU_R)$$

$$EF2534AU = EF2534AU_R * (EAU/EAU_R)$$

$$EF3544AU = EF3544AU_R * (EAU/EAU_R)$$

$$EF4554AU = EF4554AU_R * (EAU/EAU_R)$$

$$EF5564AU = EF5564AU_R * (EAU/EAU_R)$$

$$EF65OAU = EF65OAU_R * (EAU/EAU_R)$$

$$EFAU = EF1617AU + EF1819AU + EF2024AU + EF2534AU + EF3544AU + EF4554AU + EF5564AU + EF65OAU$$

$$EMAU = EM1617AU + EM1819AU + EM2024AU + EM2534AU + EM3544AU + EM4554AU + EM5564AU + EM65OAU$$

ANNUAL "AVERAGE HOURS WORKED" EQUATIONS

Total

Nonagricultural sector

Wage Workers

Males

$$AHWM1617NAW = 34.4953 * RTP + 1.03247 * MINW/CPIW_U - 31.4229 * RNM1617S - 0.12369 * 1 + 12.1981$$

$$AHWM1819NAW = 32.4361 * RTP + 12.8742 * RTP.1 + 1.28624 * MINW/CPIW_U - 16.0989 * RNM1819S - 0.27834 * 1 - 8.58664$$

$$AHWM2024NAW = 20.1161 * RTP - 10.2292 * RNM2024S - 2.28628 * (PM2024NM * NM2024NM / (PM2024NM * NM2024NM + PM2024MS * NM2024MS + PM2024MA * NM2024MA)) + 23.2252$$

$$AHWM2534NAW = 17.0559 * RTP - 0.29076 * 1 + 0.04542 * YEAR + 22.5121$$

$$AHWM3544NAW = 18.3314 * RTP - 0.30475 * 1 + 0.04409 * YEAR + 22.3275$$

$$AHWM4554NAW = 16.0678 * RTP - 0.12289 * 1 + 0.07446 * YEAR + 21.4366$$

$$AHWM5564NAW = 10.8277 * RTP + 0.23715 * 1 + 30.8975$$

$$AHWM65ONAW = -0.28038 * 1 + 5.40682 * RTP + 25.4797$$

Females

$$AHWF1617NAW = 17.2598 * RTP + 7.31262 * RTP.1 - 24.9241 * RNF1617S + 16.9218$$

$$AHWF1819NAW = 18.7922 * RTP + 13.1066 * RTP.1 - 18.9417 * RNF1819S - 16.2182 * PF1819 + 0.81920 * 1 + 16.7826$$

$$AHWF2024NAW = -219.154 * PF2024 + 12.4226 * RTP - 0.98470 * 1 + 158.354 * (PF2024)^2 + 98.6034$$

$$AHWF2534NAW = -39.2904 * PF2534 + 39.6515 * (PF2534)^2 + 9.10839 * RTP - 0.33503 * 1 + 35.8525$$

$$AHWF3544NAW = -39.2904 * PF3544 + 39.6515 * (PF3544)^2 + 7.40115 * RTP - 0.38214 * 1 + 36.9133$$

$$AHWF4554NAW = 8.64511 * RTP - 63.8042 * PF4554 + 59.9107 * (PF4554)^2 - 0.58355 * 1 + 44.0723$$

$$AHWF5564NAW = 6.61506 * RTP - 0.28969 * 1 + 8.36882 * PF5564 + 24.8288$$

$$AHWF65ONAW = -0.43916 * 1 + 26.5228 + (8.36882/2) * (PF65O - 0.086)$$

Self-employed

$$AHWNAS = 24.9592 * RTP - 17.2194 * EFNAS/ENAS + 22.0120$$

Unpaid Family Workers

$$AHWNAU = 25.5124 * (EF2534NAU + EF3544NAU + EF4554NAU)/ENAU + 19.2730$$

Agricultural sector

Wage Workers

$$AHWAW = 6.58073 * RTP + 14.9130 * RTP.1 + 19.7800$$

Self-employed

$$AHWAS = 2.45830 * PGDPAF/PGDP - 3.61107 * 1 + 44.5318$$

Unpaid Family Workers

$$AHWAW = 39.3563$$

QUARTERLY "AVERAGE HOURS WORKED" EQUATIONS

Total

$$AHW = (AHWNA * ENA + AHWA * EA) / E$$

Nonagricultural sector

$$AHWNA = (AHWNAW * (E-ENAS-ENAU-EA) + AHWNAS * ENAS + AHWNAU * ENAU) / ENA$$

Wage Workers

$$AHWNAW = (AHWMNAW * (EM - EMNAS - EMNAU - EMAW - EMAS - EMAU) + AHWFNAW * (EF - EFNAS - EFNAU - EFAW - EFAS - EFAU)) / (E - ENAS - ENAU - EA)$$

Males

$$AHWMNAW1 = AHWM1617NAW * (EM1617 - EM1617NAS - EM1617NAU - EM1617AW - EM1617AS - EM1617AU)$$

$$AHWMNAW2 = AHWM1819NAW * (EM1819 - EM1819NAS - EM1819NAU - EM1819AW - EM1819AS - EM1819AU)$$

$$AHWMNAW3 = AHWM2024NAW * (EM2024 - EM2024NAS - EM2024NAU - EM2024AW - EM2024AS - EM2024AU)$$

$$AHWMNAW4 = AHWM2534NAW * (EM2534 - EM2534NAS - EM2534NAU - EM2534AW - EM2534AS - EM2534AU)$$

$$AHWMNAW5 = AHWM3544NAW * (EM3544 - EM3544NAS - EM3544NAU - EM3544AW - EM3544AS - EM3544AU)$$

$$AHWMNAW6 = AHWM4554NAW * (EM4554 - EM4554NAS - EM4554NAU - EM4554AW - EM4554AS - EM4554AU)$$

$$AHWMNAW7 = AHWM5564NAW * (EM5564 - EM5564NAS - EM5564NAU - EM5564AW - EM5564AS - EM5564AU)$$

$$AHWMNAW8 = AHWM65ONAW * (EM65O - EM65ONAS - EM65ONAU - EM65OAW - EM65OAS - EM65OAU)$$

$$AHWMNAW = (AHWMNAW1 + AHWMNAW2 + AHWMNAW3 + AHWMNAW4 + AHWMNAW5 + AHWMNAW6 + AHWMNAW7 + AHWMNAW8) / (EM - EMNAS - EMNAU - EMAW - EMAS - EMAU)$$

$$AHWM1617NAW = 34.4953 * MOVAVG(2, RTP.1) + 1.03247 * MINW/CPIW_U - 31.4229 * RNM1617S - 0.12369 * 1 + 12.1981 + AHWM1617NAW.ADJ$$

$$AHWM1819NAW = 32.4361 * MOVAVG(2, RTP.1) + 12.8742 * MOVAVG(2, RTP.5) + 1.28624 * MINW/CPIW_U - 16.0989 * RNM1819S - 0.27834 * 1 - 8.58664 + AHWM1819NAW.ADJ$$

$$AHWM2024NAW = 20.1161 * MOVAVG(2, RTP.1) - 10.2292 * RNM2024S - 2.28628 * (PM2024NM * NM2024NM / (PM2024NM * NM2024NM + PM2024MS * NM2024MS + PM2024MA * NM2024MA)) + 23.2252 + AHWM2024NAW.ADJ$$

$$AHWM2534NAW = MIN(45, 17.0559 * MOVAVG(2, RTP.1) - 0.29076 * 1 + 0.04542 * YEAR + 22.5121) + AHWM2534NAW.ADJ$$

$$AHWM3544NAW = MIN(45, 18.3314 * MOVAVG(2, RTP.1) - 0.30475 * 1 + 0.04409 * YEAR + 22.3275) + AHWM3544NAW.ADJ$$

$$AHWM4554NAW = MIN(45, 16.0678 * MOVAVG(2, RTP.1) - 0.12289 * 1 + 0.07446 * YEAR + 21.4366) + AHWM4554NAW.ADJ$$

$$AHWM5564NAW = 10.8277 * MOVAVG(2, RTP.1) + 0.23715 * 1 + 30.8975 + AHWM5564NAW.ADJ$$

$$AHWM65ONAW = -0.28038 * 1 + 5.40682 * MOVAVG(2, RTP.1) + 25.4797 + AHWM65ONAW.ADJ$$

Females

$$AHWFNAW1 = AHWF1617NAW * (EF1617 - EF1617NAS - EF1617NAU - EF1617AW - EF1617AS - EF1617AU)$$

$$AHWFNAW2 = AHWF1819NAW * (EF1819 - EF1819NAS - EF1819NAU - EF1819AW - EF1819AS - EF1819AU)$$

$$AHWFNAW3 = AHWF2024NAW * (EF2024 - EF2024NAS - EF2024NAU - EF2024AW - EF2024AS - EF2024AU)$$

$$AHWFNAW4 = AHWF2534NAW * (EF2534 - EF2534NAS - EF2534NAU - EF2534AW - EF2534AS - EF2534AU)$$

$$AHWFNAW5 = AHWF3544NAW * (EF3544 - EF3544NAS - EF3544NAU - EF3544AW - EF3544AS - EF3544AU)$$

$$AHWFNAW6 = AHWF4554NAW * (EF4554 - EF4554NAS - EF4554NAU - EF4554AW - EF4554AS - EF4554AU)$$

$$AHWFNAW7 = AHWF5564NAW * (EF5564 - EF5564NAS - EF5564NAU - EF5564AW - EF5564AS - EF5564AU)$$

$$AHWFNAW8 = AHWF65ONAW * (EF65O - EF65ONAS - EF65ONAU - EF65OAW - EF65OAS - EF65OAU)$$

$$AHWFNAW = (AHWFNAW1 + AHWFNAW2 + AHWFNAW3 + AHWFNAW4 + AHWFNAW5 + AHWFNAW6 + AHWFNAW7 + AHWFNAW8) / (E - EFAS - EFAU)$$

$$AHWF1617NAW = 17.2598 * MOVAVG(2, RTP.1) + 7.31262 * MOVAVG(2, RTP.5) - 24.9241 * RNF1617S + 16.9218 + AHWF1617NAW.ADJ$$

$$AHWF1819NAW = 18.7922 * MOVAVG(2, RTP.1) + 13.1066 * MOVAVG(2, RTP.5) - 18.9417 * RNF1819S - 16.2182 * PF1819 + 0.81920 * 1 + 16.7826 + AHWF1819NAW.ADJ$$

$$AHWF2024NAW = -219.154 * PF2024 + 12.4226 * MOVAVG(2, RTP.1) - 0.98470 * 1 + 158.354 * (PF2024)^2 + 98.6034 + AHWF2024NAW.ADJ$$

$$AHWF2534NAW = -39.2904 * PF2534 + 39.6515 * (PF2534)^2 + 9.10839 * MOVAVG(2, RTP.1) - 0.33503 * 1 + 35.8525 + AHWF2534NAW.ADJ$$

$$AHWF3544NAW = -39.2904 * PF3544 + 39.6515 * (PF3544)^2 + 7.40115 * MOVAVG(2, RTP.1) - 0.38214 * 1 + 36.9133 + AHWF3544NAW.ADJ$$

$$AHWF4554NAW = 8.64511 * MOVAVG(2, RTP.1) - 63.8042 * PF4554 + 59.9107 * (PF4554)^2 - 0.58355 * 1 + 44.0723 + AHWF4554NAW.ADJ$$

$$AHWF5564NAW = 6.61506 * MOVAVG(2, RTP.1) - 0.28969 * 1 + 8.36882 * PF5564 + 24.8288 + AHWF5564NAW.ADJ$$

$$AHWF65ONAW = -0.43916 * 1 + 26.5228 + (8.36882/2) * (PF65O - 0.086) + AHWF65ONAW.ADJ$$

Self-employed

AHWNAS = 24.9592 * MOVAVG (2, RTP.1) - 17.2194 * EFNAS/ENAS + 22.0120 + AHWNAS.ADJ

Unpaid Family Workers

AHWNAU = 25.5124 * (EF2534NAU + EF3544NAU + EF4554NAU) / ENAU + 19.2730 + AHWNAU.ADJ

Agricultural sector

AHWA = (AHWAW * EAW + AHWAS * EAS + AHWAU * EAU) / EA

Wage Workers

AHWAW = 6.58073 * MOVAVG (2, RTP.1) + 14.9130 * MOVAVG (2, RTP.5) + 19.7800 + AHWAW.ADJ

Self-employed

AHWAS = 2.45830 * PGDPAF/PGDP - 3.61107 * 1 + 44.5318 + AHWAS.ADJ

Unpaid Family Workers

AHWAU = 39.3563 + AHWAU.ADJ

OTHER EMPLOYMENT MEASURES

Federal Civilian Government and Government Enterprises

EGFC = IF LONGRANGE = 0
THEN (EGFC.1 * 1.0094^{0.25})
ELSE (EGFC.1 * (E_FE/E_FE.1))

EGEFCPS = IF LONGRANGE = 0
THEN (EGEFCPS.1 * 1.0075^{0.25})
ELSE (EGEFCPS.1 * (E_FE/E_FE.1))

EGGEFC = EGFC + EGEFCPS

State and Local Government and Government Enterprises

EGGESL = IF LONGRANGE = 0
THEN EGGESL.1 * (LC_FE/LC_FE.4)^{0.25}
ELSE EGGESL.1 * (E_FE/E_FE.1)

Other Employment Measures

DNEDMIL = IF (EDMIL-EDMIL.4) < 0
THEN (EDMIL-EDMIL.4)
ELSE 0

EP = E-EGGESL - EGGEFC - EAS - ENAS

Compensation and Output Sectors

Price Deflator for Medical Services

CPIWMS = CPIWMS.1 * (1 + ((CPIW_U/CPIW_U.4)^{0.25} - 1) * CPIWMSWT)

UI and WC Effective Tax Rates

TMAXUI_SL = TMAXUI_SL.1 * AWSUI.1/AWSUI.2

RELMAX_UI = TMAXUI_SL/AWSUI.1/1000

CR_UI = 0.775

TRATIO_UI = 0.96996 * RELMAX_UI - 0.13744 * MOVAVG (4, RTP.1) + 0.10368 * MOVAVG (4, RTP.5) + 0.04887

TRATE_UI = 0.00143 * MOVAVG (4, RU.5) + 0.00128 * MOVAVG (4, RU.9) + 0.00057 * MOVAVG (4, RU.13) + 0.00356

RUIWS1 = CR_UI * TRATIO_UI * TRATE_UI

RUIWS2 = 0.32476 * MOVAVG (4, RUIWS1.8 * (WSP.8 - WSPRR.8 + WSGGESL.8)) / (WSP.1 - WSPRR.2.1 + WSGGESL.1)

Workers' Compensation

RWCWS = RWCWS.1 - (RWCWS.1 - 0.0144)/12

State and Local Government and Government Enterprises

TAXMAX = IF QTR = 1
THEN 300 * ROUND (MOVAVG (4, AWSE.5)/MOVAVG (4, AWSE.9) * 1000 * TAXMAX.1/300 + 0.5)/1000
ELSE TAXMAX.1

Wages
 AWSPL = MOVAVG (8, AWSP.1)
 AWSSPL = MOVAVG (8, AWSSP.1)
 AWSGGESL = IF LONGRANGE = 0
 THEN AWSGGESL.1 * AWSPL/AWSPL.1
 ELSE AWSGGESL.1 * AVG_GDP/AVG_GDP.1 * (1 + WS_TO_WSS_D/100)^{0.25}
 WSGGESL = AWSGGESL * EGGESL

Employer Contribution for Government Social Insurance
 OASDISL_L = (EMPTROASI + EMPTRDI) * 0.978 * CSLA * WSGGESL
 HISL_L = EMPTRHI * 1.0 * CSLHI * WSGGESL
 SOC_UISL = (-0.02821 * MOVAVG (4, RTP.2) + 0.03145) * WSGGESL
 RSOCSL_WC = RSOCSL_WC.1 - (RSOCSL_WC.1 - 0.176)/12
 SOC_WCSL = RSOCSL_WC * RWCWS * WSGGESL
 SOC_SL = (OASDISL_L + HISL_L + SOC_UISL + SOC_WCSL)

Employer Contributions for Employee Pension and Insurance funds

Workers' Compensation - employees and annuitants
 OLI_WCSL = (1 - RSOCSL_WC) * RWCWS * WSGGESL

Pensions
 OLI_RETSL = WSGGESL * (OLI_RETSL.1/WSGGESL.1)

Life Insurance - employees and annuitants
 OLI_GLI_SL = 2.0 * EGGESL * ((WSGGESL/EGGESL) + 2.0) * 0.075 * 26/1000

Health Insurance - employees and annuitants
 OLI_GHI_SL = (OLI_GHI_SL.1 / EGGESL.1) * CPIWMS/CPIWMS.1 * EGGESL * RGR_GHI

Total
 OLI_SL = (OLI_GLI_SL + OLI_GHI_SL + OLI_WCSL + OLI_RETSL)
 RCWSSL = (1 + (SOC_SL + OLI_SL)/WSGGESL)
 WSSGGESL = IF LONGRANGE = 0
 THEN RCWSSL*WSGGESL
 ELSE (WSSGGESL.1/EGGESL.1) * AVG_GDP/AVG_GDP.1 * EGGESL
 WSSGSL = WSSGGESL * WSSGSL.1/WSSGGESL.1
 WSSGESL = WSSGGESL - WSSGSL
 CFCGSL = IF LONGRANGE = 0
 THEN WSSGSL * RCFCGSL
 ELSE CFCGSL.1 * WSSGGESL/WSSGGESL.1
 GDPGSL = WSSGSL + CFCGSL
 CFCGESL = IF LONGRANGE = 0
 THEN WSSGESL * RCFCGESL
 ELSE CFCGESL.1 * WSSGGESL/WSSGGESL.1
 GDPGESL = WSSGESL + CFCGESL
 GDPGGESL = GDPGSL + GDPGESL

Federal Civilian General Government and Government Enterprises

Wages

General Government and Government Enterprises
 CRAZI = IF LONGRANGE = 0
 THEN ((IF QTR = 1 THEN (0.82429 * (AWSP.6/AWSP.10 - 1) - 0.005) ELSE 0))
 ELSE (IF QTR = 1 THEN (AWSP.6/AWSP.10 - 1) ELSE 0)
 MRAZ = IF LONGRANGE = 0
 THEN ((IF QTR = 1 THEN (0.82429 * (AWSP.6/AWSP.10 - 1) - 0.005) ELSE 0))
 ELSE (IF QTR = 1 THEN (AWSP.6/AWSP.10 - 1) ELSE 0)

AMSGGEFC = IF LONGRANGE = 0
 THEN (AMSGGEFC.1 * (1 + 1.0 * CRAZ1 + 0.0015))
 ELSE AMSGGEFC.1 * AVG_GDP / AVG_GDP.1 * (1 + WS_TO_WSS_D / 100)^{0.25}
 WSGGEFC = AMSGGEFC * EGGEFC

CSRS workers

AWEFC_N = IF LONGRANGE = 0
 THEN (AWEFC_N.1 * (1 + 1.0 * CRAZ1 + 0.00082))
 ELSE AWEFC_N.1 * AVG_GDP / AVG_GDP.1 * (1 + WS_TO_WSS_D / 100)^{0.25}
 WEFC_N = AWEFC_N * TEFC_N

Government Enterprises (Mostly U.S. Postal Service)

AWSGFEFC = IF LONGRANGE = 0
 THEN (AWSGFEFC.1 * (1 + 1.0 * CRAZ1 + 0.0015))
 ELSE AWSGFEFC.1 * AVG_GDP / AVG_GDP.1 * (1 + WS_TO_WSS_D / 100)^{0.25}
 WSGFEFC = AWSGFEFC * EGEFCPS

General Government

WSGFC = WSGGEFC - WSGEFC
 AWSGFC = WSGFC / EGFC

Employer Contribution for Government Social Insurance

General Government and Government Enterprises

OASDIFC_L = (EMPTRD1 + EMPTRD1) * 1.04 * (WSGGEFC - WEFC_N) * ADJ_FSA_FC
 HIFC_L = EMPTRH1 * 1.055 * WSGGEFC * ADJ_FSA_FC
 SOCF_UIFC = (-0.05934 * RTP + 0.06165) * WSGGEFC
 SOCF_WC = 0.0159 * WSGGEFC
 SOC_FC = (SOCF_UIFC + SOCF_WC + OASDIFC_L + HIFC_L)

Employer Contributions for Employee Pension and Insurance funds

General Government and Government Enterprises

Pensions

OLI_CSRS1 = ((0.174 * WSGEFC + 0.07 * WSGFC) / WSGGEFC) * WEFC_N
 OLI_FERS1 = 0.107 * (WSGGEFC * 0.9 - WEFC_N)
 OLI_FERSFC = 0.048 * (WSGGEFC * 0.9 - WEFC_N)
 OLI_RETFC = OLI_CSRS1 + OLI_FERS1 + OLI_FERSFC + OLIF_RETFCO

Life Insurance - employees and annuitants

OLI_GLI_FC = 2.0 * EGGEFC * ((WSGGEFC / EGGEFC) + 2.0) * 0.075 * 26 / 1000

Health Insurance - employees and annuitants

OLI_GHI_FC = (OLI_GHI_FC.1 / EGGEFC.1) * CPIWMS / CPIWMS.1 * EGGEFC * RGR_GHI
 OLI_FC = (OLI_GHI_FC + OLI_GLI_FC + OLI_RETFC)

Compensation

General Government and Government Enterprises

RCWSF = (1 + (SOC_FC + OLI_FC) / WSGGEFC)
 WSSGGEFC = IF LONGRANGE = 0
 THEN RCWSF * WSGGEFC
 ELSE (WSSGGEFC.1 / EGGEFC.1) * AVG_GDP / AVG_GDP.1 * EGGEFC
 WSSGFC = IF LONGRANGE = 0
 THEN RCWSF * WSGFC
 ELSE (WSSGFC.1 / (EGGEFC.1 - EGEFCPS.1)) * AVG_GDP / AVG_GDP.1 * (EGGEFC - EGEFCPS)
 WSSGEFC = IF LONGRANGE = 0

THEN RCWSF * WSGEFC
ELSE (WSSGEFC.1/EGEFCPS.1) * AVG_GDP/AVG_GDP.1 * EGEFCPS

Consumption of Fixed Capital

General Government and Government Enterprises

CFCGFC = IF LONGRANGE = 0
THEN WSSGFC * RCFCGFC
ELSE CFCGFC.1 * WSSGGEFC / WSSGGEFC.1
CFCGEFC = IF LONGRANGE = 0
THEN WSSGEFC * RCFCGEFC
ELSE CFCGEFC.1 * WSSGGEFC / WSSGGEFC.1

Gross Domestic Product

General Government and Government Enterprises

GDPGFC = WSSGFC + CFCGFC
GDPGEFC = WSSGEFC + CFCGEFC
GDPGGEFC = GDPGFC + GDPGEFC

Federal Government Military

Wages

AWSGFM = IF LONGRANGE = 0
THEN (AWSGFM.1 * (1.0027 + 1.0 * MRAZ))
ELSE AWSGFM.1 * AVG_GDP/AVG_GDP.1 * (1 + WS_TO_WSS_D/100)^{0.25}
WSGFM = AWSGFM * (EDMIL + EDMIL_R)

Employer Contribution for Government Social Insurance

OASDIFM_L = (EMPTRASI + EMPTRDI) * 0.9975 * CML * WSGFM
HIFM_L = EMPTRHI * 1.0 * CML * WSGFM
SOCF_UIFM = MAX (0.001, (-0.05263 * DIFF (EDMIL + EDMIL_R) - 0.03079 * RTP + 0.03310)) * WSGFM
SOCF_MIFM = 0.30 * CPIWMS * (EDMIL + EDMIL_R)
SOC_FM = (SOCF_UIFM + SOCF_MIFM + OASDIFM_L + HIFM_L)

Employer Contributions for Employee Pension and Insurance funds

OLI_RETFM = (OLI_RETFM.1/WSGFM.1 - (OLI_RETFM.1/WSGFM.1 - 0.472)/12) * WSGFM

Compensation

RCWSM = (1 + (OLI_RETFM + SOC_FM)/WSGFM)
WSSGFM = IF LONGRANGE = 0
THEN RCWSM * WSGFM
ELSE (WSSGFM.1/EDMIL.1) * AVG_GDP/AVG_GDP.1 * EDMIL

Consumption of Fixed Capital

CFCGFM = IF LONGRANGE = 0
THEN WSSGFM * RCFCGFM
ELSE CFCGFM.1 * WSSGFM/WSSGFM.1

Gross Domestic Product

GDPGFM = WSSGFM + CFCGFM
GDPGF = GDPGFC + GDPGFM
GDPGGE = GDPGGEFC + GDPGGEGL + GDPGFM

Total (Civilian and Military) Federal General Government and Government Enterprises

WSSGF = WSSGFC + WSSGFM
WSSGE = WSSGEFC + WSSGESL
WSSG = WSSGF + WSSGSL

GDPGE = GDPGEFC + GDPGESL
 GDPG = GDPGF + GDPGSL

NIPA Farm Output and Earnings

GDPPF00 = IF LONGRANGE = 0
 THEN EXP (- 3.52340 + 0.02055 * YEAR) * N_SSA
 ELSE GDPPF00.1 * GDP00/GDP00.1
 PGDPAF = IF LONGRANGE = 0
 THEN PGDPAF.1 * ((PGDP/PGDP.1)⁴ - 0.01)^{0.25}
 ELSE PGDPAF.1 * ((PGDP/PGDP.1)⁴)^{0.25}
 GDPPF = GDPPF00 * PGDPAF
 AYF_K = ((YF.1 / EAS.1) / (WSSPF.1/EAW.1) - 1.594) * 0.8 + 1.594
 WSSPF = IF LONGRANGE = 0
 THEN EAW * MOVAVG (4, WSSP.2/EP.2) * (3.15749 / (YEAR - 65) - 0.43419 * RTP + 0.68725)
 ELSE (WSSPF.1/EAW.1) * AVG_GDP/AVG_GDP.1 * EAW
 WSPF = IF LONGRANGE = 0
 THEN WSSPF * (MOVAVG (12, (WSP.1/WSSP.1)) + 0.015)
 ELSE (WSPF.1 / WSSPF.1) * (WSP.1/WSSP.1) / (WSP.2/WSSP.2) * WSSPF
 AWSPF = WSPF/EAW
 YF = AYF_K * (WSSPF/EAW) * EAS

GDP, WSS and WS, Private Households & Nonprofit Institutions

Private Households

Compensation & Wages

WSSPH = IF LONGRANGE = 0
 THEN (((WSSPH.1/ENAWPH.1)/MOVAVG (4, WSSP.3/EP.3) - 0.41) * 0.875 + 0.41) * MOVAVG (4, WSSP.2/EP.2) *
 ENAWPH
 ELSE (AVG_GDP/AVG_GDP.1) * ENAWPH * (WSSPH.1/ENAWPH.1)
 WSPH = IF LONGRANGE = 0
 THEN WSSPH / (1 + CPH * 1 * (EMPTROASI + EMPTRDI + EMPTRHI))
 ELSE (AWSPH.1 * ENAWPH.1/WSSPH.1) * (1 + WS_TO_WSS_D/100)^{0.25} * WSSPH
 AWSPH = WSPH / ENAWPH

Owner Occupied Housing

OOH = OOH.1 * (KGDP00 * PGDP) / (KGDP00.1 * PGDP.1)

Gross Value Added

GDPPH = IF LONGRANGE = 0
 THEN WSSPH + OOH
 ELSE (AVG_GDP/AVG_GDP.1) * ENAWPH * (GDPPH.1/ENAWPH.1)

Nonprofit Institutions

Health Services

EPHS_EST = IF LONGRANGE = 0
 THEN EPHS_EST.1 + 0.275/4
 ELSE EPHS_EST.1 * (E_FE/E_FE.1)
 AWSSPHS = IF LONGRANGE = 0
 THEN AWSSPHS.1 * AWSSPL/AWSSPL.1
 ELSE AWSSPHS.1 * AVG_GDP/AVG_GDP.1
 WSSPHS = AWSSPHS * EPHS_EST

Educational Services

EPES_EST = IF LONGRANGE = 0
 THEN EPES_EST.1 + 0.075/4
 ELSE EPES_EST.1 * (E_FE/E_FE.1)
 AWSSPES = IF LONGRANGE = 0
 THEN AWSSPES.1 * AWSSPL/AWSSPL.1

WSSPES = ELSE AWSSPES.1 * AVG_GDP/AVG_GDP.1
 AWSSPES* EPES_EST

Social Services

EPSS_EST = IF LONGRANGE = 0
 THEN EPSS_EST.1 + 0.075/4
 ELSE EPSS_EST.1 * (E_FE/E_FE.1)
 AWSSPSS = IF LONGRANGE = 0
 THEN AWSSPSS.1 * AWSSPL/AWSSPL.1
 ELSE AWSSPSS.1 * AVG_GDP/AVG_GDP.1
 WSSPSS = AWSSPSS* EPSS_EST

Gross Value Added

WSSPNI = WSSPNI.1 * (WSSPHS + WSSPES + WSSPSS) / (WSSPHS.1 + WSSPES.1 + WSSPSS.1)
 WSPNI = IF LONGRANGE = 0
 THEN WSSPNI * (WSPNI.1/WSSPNI.1) * ((WSP.1/WSSP.1) / (WSP.9/WSSP.9))^(1/8)
 ELSE WSSPNI * (WSPNI.1/WSSPNI.1) * (1 + WS_TO_WSS_D/100)^{0.25}
 GDPPNI = IF LONGRANGE = 0
 THEN WSSPNI / ((WSSPNI.1/GDPPNI.1 - 0.866) * 0.8 + 0.866)
 ELSE WSSPNI / 0.866

Private Output and Compensation

SOC

ROADIP_L = (EMPTRDIP + EMPTRDI) * TXRP * CP
 RHIP_L = EMPTRHI * 1.0 * CP
 RSOC_UIP = 0.00109 * MOVAVG (4, RU.2) + 0.00045 * MOVAVG (4, RU.10) + 0.00048 * MOVAVG (4, RU.18) - 0.00331
 RSOC_WCP = RWCWS * RSOC_SL_WC
 RSOCF_PBG = 0.00022

OLI

ROLI_WCP = RWCWS * (1 - RSOC_SL_WC)
 ROLI_SU = 0.0005
 OLI_GLI_P = 0.0025 * EP * AWSP.1
 OLI_GHI_P = (OLI_GHI_P.1 / EP.1) * CPIWMS/CPIWMS.1 * EP * RGR_GHI
 ROLI_PPPS = MAX (ROLI_PPPS.1, 0.00031 * YEAR + 0.00866)

WSS and YNF

WSSGGE = (WSSGGESL + WSSGGEFC + WSSGFM)
 GDPPBNFXGE = (GDP - GDPGGE - GDPPF - GDPPH - GDPPNI)
 RWSSPBNFXGE = (-0.35334 * DRTTP - 0.16925 * DRTTP.1 + 0.20074 * DRTPN.1 + 0.16114 * DRTPN.2 + 0.59442)
 ENAW = ENA - ENAS - ENAU
 ENAWPBXGE = ENAW - (ENAWPH + EGGEFC + EGGESL + WSSPNI / (WSSPHS + WSSPES + WSSPSS) * (EPHS_EST + EPES_EST + EPSS_EST))
 ENAWSPBXGE = ENAWPBXGE + ENAS
 AYNF_K = ((YNF.1/ENAS.1 / (WSSPBNFXGE.1/ENAWPBXGE.1)) - 1.65) * 0.9 + 1.65
 AYF = YF/EAS
 AWSSPF = WSSPF/EAW
 AYNF = YNF/ENAS
 AWSSPBNFXGE = WSSPBNFXGE/ENAWPBXGE
 YNF = YNF.1 * (GDPPBNFXGE / GDPPBNFXGE.1) * (ENAS / (ENAS + ENAWPBXGE)) / (ENAS.1 / (ENAS.1 + ENAWPBXGE.1))
 WSSPBNFXGE = (GDPPBNFXGE - YNF) * ((WSSPBNFXGE.1 / (GDPPBNFXGE.1 - YNF.1) - 0.621) * (0.8) + 0.621)
 WSSP = WSSPBNFXGE + WSSPF + WSSPH + WSSPNI
 RCWSP = WSSP / (WSSP - SOCF_RETRR - OLI_GLI_P - OLI_GHI_P) * (1 + ROADIP_L + RHIP_L + RSOC_UIP + RSOC_WCP + RSOCF_PBG + ROLI_WCP + ROLI_SU + ROLI_PPPS)
 WS = IF WS_TO_WSS_DYR = 0
 THEN (WSSGGE + WSSGGEFC + WSSGFM + WSSP/RCWSP)
 ELSE WSS * WS.1/WSS.1 * (1 + WS_TO_WSS_D/100)^{0.25}
 WSP = (WS - WSSGGE - WSSGGEFC - WSSGFM)
 AWSP = WSP/EP
 AWSSP = WSSP/EP

Other Variables

WSD = WS
 WSDP = (WSD - WSGGESL - WSGGEFC - WSGFM)
 AWSE = WS / (E + EDMIL - EAS - ENAS)
 AWSUI = (WS - WSGGEFC - WSGFM) / (E - EGGEFC - EAS - ENAS)
 WSS = (WSSP + WSSGGE)
 OLI = OLI_GGE + OLI_P
 SOC = SOC_GGE + SOC_P
 OLI_GGE = OLI_FC + OLI_SL + OLI_RETFM
 SOC_GGE = SOC_FC + SOC_FM + SOC_SL
 SOC_UIP = RSOC_UIP * WSP
 SOC_WCP = RSOC_WCP * WSP
 OASDIP_L = ROADIP_L * WSP
 HIP_L = RHIP_L * WSP
 SOCF_PBG = RSOCF_PBG * WSP
 SOCF_RETRR = 0.20 * WSPRRB
 SOC_P = SOC_UIP + SOC_WCP + OASDIP_L + HIP_L + SOCF_PBG + SOCF_RETRR
 OLI_WCP = ROLI_WCP * WSP
 OLI_SU = ROLI_SU * WSP
 OLI_PPPS = ROLI_PPPS * WSP
 OLI_P = OLI_WCP + OLI_SU + OLI_GHI_P + OLI_GLI_P + OLI_PPPS
 OLI_PPS = OLI_PPPS + OLI_RETFC + OLI_RETFM + OLI_RETSL
 OLI_GHI = OLI_GHI_P + OLI_GHI_FC + OLI_GHI_SL
 OLI_GLI = OLI_GLI_P + OLI_GLI_FC + OLI_GLI_SL
 OLI_WC = OLI_WCP + OLI_WCSL
 SOCSL_WC = SOC_WCSL + SOC_WCP
 SOCF_UIFED = SOCF_UIFC + SOCF_UIFM
 SOCF_UIS = (SOC_UIP + SOC_UISL) * RUIWS1 / (RUIWS1 + RUIWS2)
 SOCF_UIF = (SOC_UIP + SOC_UISL) - SOCF_UIS
 SOCF_OASDI = OASDIP_L + OASDISL_L + OASDIFC_L + OASDIFM_L
 SOCF_HI = HIP_L + HISL_L + HIFC_L + HIFM_L

2.3 OASDI Covered Employment and Earnings (COV)

Total At-Any-Time Employment

Males

Aged 1 through 9

CEM1 = 0.00026 * NM1
 CEM2 = 0.00031 * NM2
 CEM3 = 0.00025 * NM3
 CEM4 = 0.00019 * NM4
 CEM5 = 0.00036 * NM5
 CEM6 = 0.00042 * NM6
 CEM7 = 0.00165 * NM7
 CEM8 = 0.00239 * NM8
 CEM9 = 0.00279 * NM9

Aged 10 through 15

TEM1013 = 0.01969 * (NM10 + NM11 + NM12 + NM13 + NM10[+] + NM11[+] + NM12[+] + NM13[+])/2
 TEM1415 = (0.42767 * RTP - 1.02624 * NM1415 / (NM1415 + NM1617) + 0.23102) * ((NM14 + NM15 + NM14[+] + NM15[+])/2)

Females

Aged 1 through 9

CEF1 = 0.00028 * NF1
 CEF2 = 0.00023 * NF2
 CEF3 = 0.00020 * NF3
 CEF4 = 0.00031 * NF4
 CEF5 = 0.00041 * NF5
 CEF6 = 0.00055 * NF6
 CEF7 = 0.00129 * NF7
 CEF8 = 0.00156 * NF8
 CEF9 = 0.00212 * NF9

Aged 10 through 15

TEF1013 = IF YEAR <= 110
 THEN (0.04615 * RTP - 0.00014 * YEAR - 0.02268) * (NF10 + NF11 + NF12 + NF13 + NF10[+] + NF11[+] + NF12[+] +
 NF13[+])/2
 ELSE (0.04615 * RTP - 0.00014 * 110 - 0.02268) * (NF10 + NF11 + NF12 + NF13 + NF10[+] + NF11[+] + NF12[+] +
 NF13[+])/2
 TEF1415 = (0.48500 * RTP - 0.79002 * NF1415 / (NF1415 + NF1617) + 0.02205) * ((NF14 + NF15 + NF14[+] + NF15[+])/2)

Total Males, Total Females and Combined Total

TEM = CEM1 + CEM2 + CEM3 + CEM4 + CEM5 + CEM6 + CEM7 + CEM8 + CEM9 + TEM1013 + TEM1415 + TEM160
 TEF = CEF1 + CEF2 + CEF3 + CEF4 + CEF5 + CEF6 + CEF7 + CEF8 + CEF9 + TEF1013 + TEF1415 + TEF160
 TE = TEM + TEF

Male Disaggregates Aged 16 and Over

TEM1617 = (-0.0337 - 1.32778 * REM1617_A + 1.11118 + RNM1617MAX-0.5 * 0.0637 - 0.161 * RILEM1519) * (EM1617
 * REM1617_M * REM2_1617 - EM1617AU - EM1617NAU + NM1617M)
 TEM1819 = (-0.0267 - 0.60094 * REM1819_A + 0.89703 + RNM1819MAX-0.5 * 0.0637 - 0.265 * RILEM1519) * (EM1819
 * REM1819_M * REM2_1819 - EM1819AU - EM1819NAU + NM1819M)
 TEM2024 = (-0.0196 + 0.02072 + 0.82020 + RNM2024MAX-0.50 * 0.0176 - 0.357 * RILEM2024 - 0.80 * (REM2024_A + RM2024D))
 * (EM2024 * REM2024_M * REM2_2024 - EM2024AU - EM2024NAU + NM2024M)
 TEM2529 = (-0.0113 + 0.02898 + 0.81312 + RNM2529MAX-0.50 * 0.0214 - 0.416 * RILEM2529 - 0.80 * (REM2529_A + RM2529D))
 * (EM2529 * REM2529_M * REM2_2529 - EM2529AU - EM2529NAU + NM2529M)
 TEM3034 = (0.0062 - 0.00445 + 0.55577 + RNM3034MAX-0.50 * 0.0416 - 0.424 * RILEM3034 - 0.50 * (REM3034_A + RM3034D))
 * (EM3034 * REM3034_M * REM2_3034 - EM3034AU - EM3034NAU + NM3034M)
 TEM3539 = (-0.0092 + 0.00162 + 0.50480 + RNM3539MAX-0.50 * 0.0279 - 0.423 * RILEM3539 - 0.45 * (REM3539_A + RM3539D))
 * (EM3539 * REM3539_M * REM2_3539 - EM3539AU - EM3539NAU + NM3539M)
 TEM4044 = (-0.0083 + 0.01717 + 0.41973 + RNM4044MAX-0.50 * 0.0062 - 0.418 * RILEM4044 - 0.40 * (REM4044_A + RM4044D))
 * (EM4044 * REM4044_M * REM2_4044 - EM4044AU - EM4044NAU + NM4044M)
 TEM4549 = (-0.0026 - 0.00292 + 0.52707 + RNM4549MAX-0.50 * 0.0010 - 0.410 * RILEM4549 - 0.50 * (REM4549_A + RM4549D))

* (EM4549 * REM4549_M * REM2_4549 - EM4549AU-EM4549NAU + NM4549M)
 TEM5054 = (-0.0008 + 0.02301 + 0.53226 + RNM5054MAX - 0.50 * 0.0025 - 0.395 * RILEM5054 - 0.50 * (REM5054_A + RM5054D))
 * (EM5054 * REM5054_M * REM2_5054 - EM5054AU-EM5054NAU + NM5054M)
 TEM5559 = (-0.0010 - 0.00842 + 0.46888 + RNM5559MAX - 0.50 * 0.0042 - 0.356 * RILEM5559 - 0.40 * (REM5559_A + RM5559D))
 * (EM5559 * REM5559_M * REM2_5559 - EM5559AU-EM5559NAU + NM5559M)
 TEM6064 = (-0.0030 + 0.04966 + 0.59056 + RNM6064MAX - 0.50 * 0.0051 - 0.248 * RILEM6064 - 0.50 * (REM6064_A + RM6064D))
 * (EM6064 * REM6064_M * REM2_6064 - EM6064AU-EM6064NAU)
 TEM6569 = (-0.82949 * REM6569_A + 0.10116 + 1.55172) * (EM6569 * REM6569_M * REM2_6569 - EM6569AU - EM6569NAU)
 TEM700 = (0.01973 * 96 - 0.34903) * (EM700 * REM700_M * REM2_700 - EM700AU-EM700NAU)
 TEM160 = TEM1617 + TEM1819 + TEM2024 + TEM2529 + TEM3034 + TEM3539 + TEM4044 + TEM4549 + TEM5054 + TEM5559 +
 TEM6064 + TEM6569 + TEM700

Female Disaggregates Aged 16 and Over

TEF1617 = (0.0106 - 1.52661 * REF1617_A - 0.02280 + 1.17229 + RNF1617MAX + 0.5 * 0.0631 - 0.161 * RILEF1519) * (EF1617
 * REF1617_M * REF2_1617 - EF1617AU - EF1617NAU + NF1617M)
 TEF1819 = (0.0166 - 0.56521 * REF1819_A + 0.04837 + 0.85527 + RNF1819MAX - 0.5 * 0.0631 - 0.252 * RILEF1519) * (EF1819
 * REF1819_M * REF2_1819 - EF1819AU - EF1819NAU + NF1819M)
 TEF2024 = (0.0202 + 0.02418 + 0.89686 + RNF2024MAX - 0.50 * 0.0349 - 0.318 * RILEF2024 - 0.90 * (REF2024_A + RF2024D))
 * (EF2024 * REF2024_M * REF2_2024 - EF2024AU - EF2024NAU + NF2024M)
 TEF2529 = (0.0156 + 0.03554 + 0.68431 + RNF2529MAX - 0.50 * 0.0378 - 0.342 * RILEF2529 - 0.70 * (REF2529_A + RF2529D))
 * (EF2529 * REF2529_M * REF2_2529 - EF2529AU - EF2529NAU + NF2529M)
 TEF3034 = (0.0156 + 0.00488 + 0.47083 + RNF3034MAX - 0.50 * 0.0465 - 0.340 * RILEF3034 - 0.45 * (REF3034_A + RF3034D))
 * (EF3034 * REF3034_M * REF2_3034 - EF3034AU - EF3034NAU + NF3034M)
 TEF3539 = (0.0156 + 0.000796 + 0.41423 + RNF3539MAX - 0.50 * 0.0318 - 0.343 * RILEF3539 - 0.40 * (REF3539_A + RF3539D))
 * (EF3539 * REF3539_M * REF2_3539 - EF3539AU - EF3539NAU + NF3539M)
 TEF4044 = (0.0127 + 0.00312 + 0.35298 + RNF4044MAX - 0.50 * 0.0135 - 0.353 * RILEF4044 - 0.35 * (REF4044_A + RF4044D))
 * (EF4044 * REF4044_M * REF2_4044 - EF4044AU - EF4044NAU + NF4044M)
 TEF4549 = (0.0064 + 0.00138 + 0.40080 + RNF4549MAX - 0.50 * 0.0086 - 0.358 * RILEF4549 - 0.40 * (REF4549_A + RF4549D))
 * (EF4549 * REF4549_M * REF2_4549 - EF4549AU - EF4549NAU + NF4549M)
 TEF5054 = (0.0037 + 0.01202 + 0.39021 + RNF5054MAX - 0.50 * 0.0089 - 0.336 * RILEF5054 - 0.40 * (REF5054_A + RF5054D))
 * (EF5054 * REF5054_M * REF2_5054 - EF5054AU - EF5054NAU + NF5054M)
 TEF5559 = (0.0034 + 0.03036 + 0.32074 + RNF5559MAX - 0.50 * 0.0094 - 0.280 * RILEF5559 - 0.30 * (REF5559_A + RF5559D))
 * (EF5559 * REF5559_M * REF2_5559 - EF5559AU - EF5559NAU + NF5559M)
 TEF6064 = (0.0030 + 0.02068 + 0.49318 + RNF6064MAX - 0.50 * 0.0115 - 0.176 * RILEF6064 - 0.45 * (REF6064_A + RF6064D))
 * (EF6064 * REF6064_M * REF2_6064 - EF6064AU - EF6064NAU)
 TEF6569 = 1.42 * (EF6569 * REF6569_M * REF2_6569 - EF6569AU - EF6569NAU)
 TEF700 = (0.92002 * TEM700 / (EM700 * REM700_M * REM2_700 - EM700AU - EM700NAU) + 0.12035) * (EF700 * REF700_M
 * REF2_700 - EF700AU - EF700NAU)
 TEF160 = TEF1617 + TEF1819 + TEF2024 + TEF2529 + TEF3034 + TEF3539 + TEF4044 + TEF4549 + TEF5054 + TEF5559 +
 TEF6064 + TEF6569 + TEF700

Self-Employed Only

SEOCMB = 0.039 * (TEFC_N_N + TESL_N_N + EPRRB)
 SEOCMBL1 = 0.039 * (TEFC_N_N.1 + TESL_N_N.1 + EPRRB.1)
 SEO = (SEO.1 * (EAS + ENAS) / (EAS.1 + ENAS.1) + (SEOCMB - SEOCMBL1)) * MULTSEO

Combination Workers

CMB_TOT = ((-0.01468 + 0.06227 * RTP.1 - 0.0008) * WSWA - SEOCMB) * MULTCMB
 CSW_TOT = SEO + CMB_TOT
 AW_CMBTOT = 1.4953 * ACWA
 W_CMBTOT = AW_CMBTOT * CMB_TOT
 CMB_WRELMAX = TAXMAX/AW_CMBTOT

CMB Wage Andover Curve

CMB_WAO1 = IF (CMB_WRELMAX < 0.0543009)
 THEN 1 - 0.722659 * CMB_WRELMAX^{0.65} - 0.461913 * CMB_WRELMAX^{0.8}
 ELSE IF (CMB_WRELMAX < 0.1086018)
 THEN -1.02884 * CMB_WRELMAX^{0.6} + 0.324761 * CMB_WRELMAX^{1.6} + 1.02015
 ELSE IF (CMB_WRELMAX < 0.1629027)
 THEN -0.906607 * CMB_WRELMAX^{0.7} + 0.947662
 ELSE IF (CMB_WRELMAX < 0.2172037)
 THEN -0.813951 * CMB_WRELMAX^{0.55} + 0.991722
 ELSE IF (CMB_WRELMAX < 0.3258055)

```

        THEN -0.755135 * CMB_WRELMAX0.55 + 0.964593
        ELSE 0

CMB_WAO2 = IF (CMB_WRELMAX < 0.5430091)
            THEN -0.649755 * CMB_WRELMAX0.6 + 0.886467
            ELSE IF (CMB_WRELMAX < 0.7059119)
                THEN -0.573205 * CMB_WRELMAX0.7 + 0.810122
                ELSE IF (CMB_WRELMAX < 0.9231155)
                    THEN - 5.22264 * CMB_WRELMAX0.06 + 5.47514
                    ELSE IF (CMB_WRELMAX < 1.0860183)
                        THEN - 2.02619 * CMB_WRELMAX0.15 + 2.27963
                        ELSE IF (CMB_WRELMAX < 1.5204256)
                            THEN 0.605192 * EXP (-0.2 * CMB_WRELMAX) - 0.827158 * EXP (-0.8 * CMB_WRELMAX)
                            ELSE 0
+ 1.52918 * EXP (- 1.5 * CMB_WRELMAX) - 0.212269
        ELSE 0

CMB_WAO3 = IF (CMB_WRELMAX < 1.8462311)
            THEN 0.19139 * EXP (-0.6 * CMB_WRELMAX) + 0.764408 * EXP (- 1.8 * CMB_WRELMAX) + 0.0194903
            ELSE IF (CMB_WRELMAX < 2.3077888)
                THEN 0.12964 * EXP (-0.5 * CMB_WRELMAX) + 0.644861 * EXP (- 1.5 * CMB_WRELMAX) + 0.0183343
                ELSE IF (CMB_WRELMAX < 2.9865502)
                    THEN 0.361318 * EXP (-0.8 * CMB_WRELMAX) + 0.0219491
                    ELSE IF (CMB_WRELMAX < 4.3440731)
                        THEN 0.193202 * EXP (-0.45 * CMB_WRELMAX) + 0.00425171
                        ELSE IF (CMB_WRELMAX < 5.4300913)
                            THEN 0.0560412 * EXP (-0.25 * CMB_WRELMAX) + 0.311286 * EXP (-0.8 *
CMB_WRELMAX) + 0.00297316
        ELSE 0

CMB_WAO4 = IF (CMB_WRELMAX < 13.5752283)
            THEN 0.0995677 * EXP (-0.32 * CMB_WRELMAX) + 0.00355234
            ELSE IF (CMB_WRELMAX < 21.7203653)
                THEN 0.041159 * EXP (-0.19 * CMB_WRELMAX) + 0.00156765
                ELSE IF (CMB_WRELMAX < 678.7614168)
                    THEN 0.265022 * CMB_WRELMAX(-1.555)
                    ELSE 0

CMB_WAO = IF (CMB_WRELMAX < 0.3258055)
            THEN CMB_WAO1
            ELSE IF (CMB_WRELMAX < 1.5204256)
                THEN CMB_WAO2
            ELSE IF (CMB_WRELMAX < 5.4300913)
                THEN CMB_WAO3
            ELSE CMB_WAO4

CMB = (1 - (CMB_WAO - 0.019)) * CMB_TOT
CSW = SEO + CMB
SEOCMB_HI = 0.039 * (TEFC_N_N + TESL_N_N_HI)
SEO_HI = SEO - SEOCMB_HI
CMB_HI = CMB_TOT + SEOCMB_HI
CSW_HI = SEO_HI + CMB_HI

```

NIPA Wages

Private Residual Sector

```

WSDPB = WSDP - WSPH - WSPF - WSPRRB - TIPS_SR
TIPS_SR = (0.000508328 * RTP - 0.000481700) * GDP * 1.26393

```

OASDI Wages

Covered Employment and Wages – Federal Civilian Government

TEFC = (TEFC.1 / EGGEFC.1) * EGGEFC
 TEFC_N = IF (CSRS.1 > 0) THEN TEFC_N.1/CSRS.1 * CSRS - TEFC_N_SW ELSE 0
 TEFC_N_N = TEFC_N * 0.86
 TEFC_N_O = (TEFC_N - TEFC_N_N)
 TEFC_O = (TEFC - TEFC_N)
 WEFC = (WEFC.1 / WSGGEFC.1) * WSGGEFC
 WEFC_O = (WEFC - WEFC_N) * ADJ_FSA_FC

Covered Employment and Wages - State and Local Govt.

TESL = (TESL.1/EGGESLMAX.1) * EGGESLMAX
 TESL_O = (TESL_O.1/TESL.1) * TESL
 TESL_N = (TESL-TESL_O)
 TESL_N_O = (TESL_N_O_HI + TESL_N_O_NHI)
 TESL_N_O_HI = (TESL_N - TESL_N_O_NHI - TESL_N_N_NHI) * CER_MQGE_O
 TESL_N_O_NHI = (TESL_N_O_NHI_S + TESL_N_O_NHI_E + TESL_N_O_NHI_NS)
 TESL_N_S = TESL_N_S.1 * (NF1819 + NF2024 + NM1819 + NM2024) / (NF1819.1 + NF2024.1 + NM1819.1 + NM2024.1)
 TESL_N_E = TESL_N_E.1 * (TESL / TESL.1)
 TESL_N_O_NHI_S = TESL_N_S * (TESL_N_O_NHI_S.1/TESL_N_S.1)
 TESL_N_O_NHI_E = TESL_N_E * 0.6
 TESL_N_O_NHI_NS = TESL_N_O_NHI_NS.1 * ESR_NS
 TESL_N_N = (TESL_N - TESL_N_O)
 TESL_N_N_HI = (TESL_N_N - TESL_N_N_NHI)
 TESL_N_N_NHI = (TESL_N_N_NHI_S + TESL_N_N_NHI_E + TESL_N_N_NHI_NS)
 TESL_N_N_NHI_S = (TESL_N_S - TESL_N_O_NHI_S)
 TESL_N_N_NHI_E = (TESL_N_E - TESL_N_O_NHI_E)
 TESL_N_N_NHI_NS = TESL_N_N_NHI_NS.1 * ESR_NS
 WESL = (WESL.1/WSGGESL.1) * WSGGESL
 WESL_O = (WESL_O.1/WSGGESL.1) * WSGGESL
 WESL_N = (WESL - WESL_O)
 WESL_N_HI = (WESL_N - WESL_N_NHI)
 WESL_N_NHI = (WESL_N_NHI_S + WESL_N_NHI_E + WESL_N_NHI_NS)
 WESL_N_NHI_S = WESL_N_NHI_S.1 * (TESL_N_S/TESL_N_S.1) * (AWSGGESL/AWSGGESL.1)
 WESL_N_NHI_E = WESL_N_NHI_E.1 * (TESL_N_E/TESL_N_E.1) * (AWSGGESL/AWSGGESL.1)
 RAWR_NS = IF (AWR_NS = 0) THEN 0 ELSE AWR_NS/AWR_NS.1
 WESL_N_NHI_NS = IF (ESR_NS = 0) THEN 0 ELSE WESL_N_NHI_NS.1 * (TESL_N_O_NHI_NS + TESL_N_N_NHI_NS) / (TESL_N_O_NHI_NS.1 + TESL_N_N_NHI_NS.1) * (AWSGGESL/AWSGGESL.1) * RAWR_NS

Self-Employed Earnings Sector

Covered SENE

CSE_TOT = (YF + YNF) / (YF.1 + YNF.1) * CSE_TOT.1
 CSE_CMB_N = (CSE_TOT / (CMB_TOT + SEO)) / (CSE_TOT.1 / (CMB_TOT.1 + SEO.1)) * (CSE_CMB_N.1 / (CMB_TOT.1 - CMB.1)) * (CMB_TOT - CMB)
 CSE = CSE_TOT - CSE_CMB_N
 ACSE_SEO = (CSE_TOT / (SEO + 0.416488 * CMB_TOT))
 ACSE_CMB_TOT = 0.416488 * ACSE_SEO
 CSE_SEO = ACSE_SEO * SEO
 CSE_CMB_TOT = ACSE_CMB_TOT * CMB_TOT
 CSE_CMB = CSE_CMB_TOT - CSE_CMB_N
 ACSE_CMB = CSE_CMB/CMB

Present Law OASDI and HI Covered Wages and Earnings

WSGMLC = CML * WSGFM
 WSGFCA = WEFC_O
 CFCA = WSGFCA/WSGGEFC
 CSLHI = (WESL_O+WESL_N_HI)/WSGGESL
 WSGSLCA = WESL_O
 WSPH_O = CPH * WSPH
 WSPF_O = WSPF_O.1 * WSPF/WSPF.1
 CPF = WSPF_O/WSPF
 WSPRR_O = CPRR * WSPRRB
 CPB_ILL = (CPB) * (WSWA * 0.86 - LOST_MF) / (WSWA * 0.86 + LOST_MF) + (1.5 * 0.5) * (2.0 * LOST_MF) / (WSWA * 0.86 + LOST_MF)

CPB_ADJ = IF REAL_ACW_DYR = 0
 THEN CPB_ILL
 ELSE ((REAL_ACW_D + PCH (CPIW_U))/100 * ACWA.1 + ACWA.1) * WSWA - WSPH_O - WSPF_O - WSPRR_O -
 TIPS_SR - WSGSLCA - WSGFCA - WSGMLC)/WSDPB
 WSPB_O = CPB_ADJ * WSDPB
 WSPC = WSPH_O + WSPF_O + WSPRR_O + TIPS_SR + WSPB_O
 CP = WSPC/WSDP
 WSCA = (WSPC + WSGSLCA + WSGFCA + WSGMLC)
 COVERNA = (WSCA + CSE)
 ACWA = WSCA/WSWA
 ASE = CSE/CSW
 ASEHI = CSE_TOT/CSW_HI
 ACEA = COVERNA/TCEA
 ACSLW = WESL_O/TESL_O * MULTACSLW
 ACMW = ACMW.1 * AWSGFM/AWSGFM.1 * CML/CML.1
 ACFCW = WEFC_O/TEFC_O
 ACFMW = ACFMW.1 * (AIW.1/AIW.3)^{0.5}
 TEPH_N = ENAWPH * (1 - CPH)
 TEP_N_N_S = TEP_N_N_S.1 * (NF1819 + NF2024 + NM1819 + NM2024) / (NF1819.1 + NF2024.1 + NM1819.1 + NM2024.1)
 TCEA = (TE - TEPH_N - EPRRB - TEP_N_N_S - TEPO_N - TESL_N_N_NHI_S - TESL_N_N_NHI_E - TESL_N_N_NHI_NS -
 TESL_N_N_HI - TEFC_N_N)
 WSWA = (TCEA - SEO)

Present Law HI Covered Wages and Earnings

WSCAHI_ADD = WSCA * WSCAHI_ADD.1/WSCA.1
 TCEAHI = (TCEA + TEFC_N_N + TESL_N_N_HI)
 WSWAHI = TCEAHI - SEO_HI
 WSCAHI = WSCA + WEFC_N + WESL_N_HI + WSCAHI_ADD
 ACWAHI = WSCAHI/WSWAHI
 COVERNHI = WSCAHI + CSE_TOT
 ACEAHI = COVERNHI/TCEAHI

Complete Coverage concepts

WSWC = (WSWAHI + TEPH_N + EPRRB + TEP_N_N_S + TEPO_N + TESL_N_N_NHI) + LOST_MF
 ACWC = WSD/WSWC
 AIW = IF AIW_GR_YR = 0
 THEN AIW.1 * ACWC/ACWC.1 * MULTAIW
 ELSE AIW.1 * (1 + AIW_GR/100)

Taxable Maximums

RAIW = AIW.2/AIWBASE
 TAXMAXB1 = RAIW * TMAXBASE * 1000/300
 TAXMAXB2 = IF TAXMAXB1 - ROUND (TAXMAXB1) >= 0.5
 THEN ROUND (TAXMAXB1) + 1
 ELSE ROUND (TAXMAXB1)
 TAXMAXB3 = IF TAXMAXB2 < TAXMAX.1
 THEN TAXMAX.1 * 1000/300
 ELSE TAXMAXB2
 TAXMAX = IF BENINC.1 <= 0.001
 THEN TAXMAX.1
 ELSE 300 * TAXMAXB3/1000

Deemed Military Wage Credits

EDMILAF = EDMIL * 1.1
 EDMILT = (2.00303 - 50.7517/YEAR) * EDMILAF
 EDMILR = EDMILT - EDMILAF
 MWC_ED_O = 1.2 * EDMILAF * 0.997
 MWC_ED_HI = 1.2 * EDMILAF
 AMWC_G02 = MIN (1.2, AWSGFM * (2/52) * (1/3))

$MWC_EDR_O = AMWC_GO2 * EDMILR * (1 - 0.017)$
 $MWC_EDR_HI = MWC_EDR_O + ((1.2 + AMWC_GO2) * 0.5) * EDMILR * 0.017$
 $MWC_O = MWC_ED_O + MWC_EDR_O$
 $MWC_HI = MWC_ED_HI + MWC_EDR_HI$

2-4 Effective Taxable Payroll (TAXPAY)

2-4.1 Ratio of taxable employee to total covered OASDI wages (RWTEE)

IF(RM.LT.0.439103091)THEN
 $RWTEE = RM - (0.26651/1.5) * RM^{**}1.5 - (0.388274/1.9) * RM^{**}1.9$
 ELSE IF(RM.LT.1.313896269)THEN
 $RWTEE = (1.17877/1.5) * RM^{**}1.5 - (2.9291/0.68) * DEXP(-0.68 * RM) - 2.31401 * RM + 4.327630386$
 ELSE IF(RM.LT.2.570433485)THEN
 $RWTEE = -(0.0202216/0.25) * DEXP(-0.25 * RM) - (1.329/1.4) * DEXP(-1.4 * RM) + 0.00944271 * RM + 0.904661863$
 ELSE IF(RM.LT.4.418783378)THEN
 $RWTEE = -(0.0654817/0.35) * DEXP(-0.35 * RM) - (1.415/1.51) * DEXP(-1.51 * RM) + 0.000676576 * RM + 0.954098019$
 ELSE IF(RM.LT.14.73231225)THEN
 $RWTEE = -(0.0110626/0.15) * DEXP(-0.15 * RM) - (0.0472319/0.45) * DEXP(-0.45 * RM) - (0.257195/0.95) * DEXP(-0.95 * RM) + 0.000367645 * RM + 0.970880621$
 ELSE
 $RWTEE = -(0.179935/0.75) * RM^{**}(-0.75) + 0.999970738$
 END IF

Where

$RM =$ OASDI taxable maximum / average covered OASDI wage
 $RWTEE =$ Ratio if OASDI taxable employee to covered wages

2-4.2 Taxable employee OASDI wages (WTEE)

$WTEE = RWTEE * WSC$

Where

$RWTEE =$ Ratio of OASDI taxable employee to covered wages
 $WSC =$ OASDI total covered wages
 $WTEE =$ OASDI taxable employee wages

2-4.3 Ratio of multi-employer refund wages to total OASDI covered wages (RMER)

$RMER = (MER(-1) / WSC(-1)) - 0.03217 * (RWTEE - RWTEE(-1)) - 0.00024 * (RU - RU(-1))$

Where

$MER(-1) =$ Multi-employer refund wages in prior year
 $RMER =$ Ratio of multi-employer refund wages to total OASDI covered wages
 $RU =$ Annual average civilian unemployment rate
 $RWTEE =$ Ratio of OASDI taxable employee to covered wages
 $WSC(-1) =$ OASDI total covered wages in prior year

2-4.4 Multi-employer refund wages (MER)

$MER = RMER * WSC$

Where

$MER =$ OASDI multi-employer refund wages
 $RMER =$ Ratio of multi-employer refund wages to total OASDI covered wages
 $WSC =$ OASDI total covered wages

2-4.5 Taxable employer OASDI wages (WTER)

$WTER = WTEE + MER$

Where

MER = OASDI multi-employer refund wages
WTEE = OASDI taxable employee wages
WTER = OASDI taxable employer wages

2-4.6 Ratio of taxable to covered self-employment earnings (RSET)

Preliminary

BASECT = 47831.98
BASECW = 36831.79
BASEO = 23448.56

Self-employed only

SECSEO = CSE - SECCMB
ASESEO = SECSEO / SEO
ASEO96 = ASESEO(1996)
ASESEO = ASESEO * BASEO / ASEO96
O = TAXMAX / ASESEO
IF(O.LT.0.021323273)THEN
 OTR=O-(401.8/3.8)*O**3.8
ELSE IF(O.LT.0.170586184)THEN
 OTR=(12.6861/1.1)*O**1.1-(10.7855/1.15)*O**1.15-(.208585/1.9)*O**1.9-1.58102*O-.0000212479
ELSE IF(O.LT.0.938224014)THEN
 OTR=-(.203924/2.5)*DEXP(-2.5*O)+(2.52113/1.6)*O**1.6-(1.11979/2.2)*O**2.2-1.33547*O+.825500132
ELSE IF(O.LT.1.705861843)THEN
 OTR=-(.375957/3)*DEXP(-.3*O)-(.642976/2.3)*DEXP(-2.3*O)-.108863*O+1.555087525
ELSE IF(O.LT.2.302913488)THEN
 OTR=-(.248596/2.5)*DEXP(-.25*O)-(1.003/2.2)*DEXP(-2.2*O)-.0565126*O+1.368881905
ELSE IF(O.LT.3.624956417)THEN
 OTR=-(.207896/1.15)*DEXP(-.15*O)-(3.99112/2.5)*DEXP(-2.5*O)-.0699402*O+1.823982598
ELSE IF(O.LT.5.970516451)THEN
 OTR=-(.110423/2)*DEXP(-.2*O)-(.571353/1.25)*DEXP(-1.25*O)-.00842494*O+1.068478626
ELSE IF(O.LT.12.79396382)THEN
 OTR=-(.0631844/2)*DEXP(-.2*O)-(.242063/65)*DEXP(-.65*O)+.0011163*O+.947373543
ELSE IF(O.LT.21.32327304)THEN
 OTR=-(.0218297/13)*DEXP(-.13*O)-(.0876571/32)*DEXP(-.32*O)+.000522742*O+.966817185
ELSE IF(O.LT.213.2327304)THEN
 OTR=-(.286725/1.36)*O**(-1.36)+1.00002612
ELSE
 OTR=1D0
END IF
SETSEO=OTR*SECSEO

OASDI taxable wages of workers with both wages and self-employment earnings

AWSCMB=WSCMB/CMBNT
AWSCMB96=AWSCMB(1996)
AWSCMB=AWSCMB*BASECW/AWSCMB96
CW=TAXMAX/AWSCMB
IF(CW.LT.0.0543009)THEN
 CWTR=CW-(.722659/1.65)*CW**1.65-(.461913/1.8)*CW**1.8
ELSE IF(CW.LT.0.1086018)THEN
 CWTR=-(.102884/1.6)*CW**1.6+(.324761/2.6)*CW**2.6+1.02015*CW-.0000130669
ELSE IF(CW.LT.0.1629027)THEN
 CWTR=-(.906607/1.7)*CW**1.7+.947662*CW+.002059472
ELSE IF(CW.LT.0.2172037)THEN
 CWTR=-(.813951/1.55)*CW**1.55+.991722*CW+.002022215
ELSE IF(CW.LT.0.3258055)THEN
 CWTR=-(.755135/1.55)*CW**1.55+.964593*CW+.004355898
ELSE IF(CW.LT.0.5430091)THEN
 CWTR=-(.649755/1.6)*CW**1.6+.886467*CW+.011658928
ELSE IF(CW.LT.0.7059119)THEN
 CWTR=-(.573205/1.7)*CW**1.7+.810122*CW+.019653316
ELSE IF(CW.LT.0.9231155)THEN

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CWTR=(-5.22264/1.06)*CW**1.06+5.47514*CW-.053844798
ELSE IF(CW.LT.1.0860183)THEN
CWTR=(-2.02619/1.15)*CW**1.15+2.27963*CW-.023407805
ELSE IF(CW.LT.1.5204256)THEN
CWTR=(-.605192/.2)*DEXP(-.2*CW)+(.827158/.8)*DEXP(-.8*CW)-(1.52918/1.5)*DEXP(-1.5*CW)-.212269*CW+2.946985922
ELSE IF(CW.LT.1.8462311)THEN
CWTR=(-.191389/.6)*DEXP(-.6*CW)-(.764408/1.8)*DEXP(-1.8*CW)+.0194903*CW+.719848486
ELSE IF(CW.LT.2.3077888)THEN
CWTR=(-.12964/.5)*DEXP(-.5*CW)-(.644861/1.5)*DEXP(-1.5*CW)+.0183343*CW+.731280763
ELSE IF(CW.LT.2.9865502)THEN
CWTR=(-.361318/.8)*DEXP(-.8*CW)+.0219491*CW+.698954858
ELSE IF(CW.LT.4.3440731)THEN
CWTR=(-.193202/.45)*DEXP(-.45*CW)+.00425171*CW+.82237055
ELSE IF(CW.LT.5.4300913)THEN
CWTR=(-.0560412/.25)*DEXP(-.25*CW)-(.311286/.8)*DEXP(-.8*CW)+.00297316*CW+.854848493
ELSE IF(CW.LT.13.5752283)THEN
CWTR=(-.0995677/.32)*DEXP(-.32*CW)+.00355234*CW+.843717127
ELSE IF(CW.LT.21.7203653)THEN
CWTR=(-.041159/.19)*DEXP(-.19*CW)+.00156765*CW+.883046178
ELSE IF(CW.LT.678.7614168)THEN
CWTR=(-.265022/.555)*CW**(-.555)+1.000103517
ELSE
CWTR=1D0
END IF
WSTCMB=CWTR*WSCCMB

```

OASDI taxable earnings of workers with both wages and self-employment earnings

```

TECCMB=SECCMB+WSCCMB
ATECMB=TECCMB/CMBNT
ATECMB96=ATECMB(1996)
ATECMB=ATECMB*BASECT/ATECMB96
CT=TAXMAX/ATECMB
IF(CT.LT.0.0209065)THEN
CTTR=CT-(58.6063/3.5)*CT**3.5
ELSE IF(CT.LT.0.1254391)THEN
CTTR=(.320825/1.4)*CT**1.4-(1.90732/2)*CT**2+.967979*CT+.000050526-.00000622
ELSE IF(CT.LT.0.3345042)THEN
CTTR=(-(1.77251/1.9)*CT**1.9+(1.18333/2.8)*CT**2.8+1.11299*CT-.003813108-.00000622
ELSE IF(CT.LT.0.7108215)THEN
CTTR=(-.246427/1.35)*CT**1.35-(1.24155/1.8)*CT**1.8+ (.55025/2.3)*CT**2.3+1.16953*CT-.001078375-.00000622
ELSE IF(CT.LT.0.7944476)THEN
CTTR=(3.15997/1.4)*CT**1.4-(4.21829/1D0)*DEXP(-1*CT)-4.46998*CT+4.300993015-.00000622
ELSE IF(CT.LT.1.0453257)THEN
CTTR=(-.55343/1.6)*CT**1.6-(1.76791/1.2)*DEXP(-1.2*CT)-.845578*CT+1.479639989-.00000622
ELSE IF(CT.LT.1.8815863)THEN
CTTR=(-.360108/.3)*DEXP(-.3*CT)-(1.24485/2.7)*DEXP(-2.7*CT)-.110244*CT+1.566702582-.00000622
ELSE IF(CT.LT.3.7631726)THEN
CTTR=(-.262138/.6)*DEXP(-.6*CT)+(.208323/1.9)*DEXP(-1.9*CT)-(2.27562/2.7)*DEXP(-2.7*CT)+.00933849*CT+.799685739-.00000622
ELSE IF(CT.LT.4.1813029)THEN
CTTR=(-.26942/.6)*DEXP(-.6*CT)+(.266252/1.7)*DEXP(-1.7*CT)+.00905196*CT+.80182568-.00000622
ELSE IF(CT.LT.10.4532574)THEN
CTTR=(-.0592625/.25)*DEXP(-.25*CT)-(.541514/1)*DEXP(-1*CT)+.00174723*CT+.88757547-.00000622
ELSE IF(CT.LT.18.8158632)THEN
CTTR=(-.0610012/.25)*DEXP(-.25*CT)+.00174344*CT+.888109189-.00000622
ELSE IF(CT.LT.522.6628676)THEN
CTTR=(-.21993/.5455)*CT**(-.5455)+1.000030393-.00000622
ELSE
CTTR=1D0
END IF
TETCMB=CTTR*TECCMB
SETCMB=TETCMB-WSTCMB

```

Ratio OASDI taxable to covered self-employment earnings

$$RSET=(SETCMB+SETSEO)/CSE$$

Where

ASEO96	=	Average self-employment earnings of workers with no OASDI taxable wages in 1996
ATECMB	=	Average OASDI covered earnings of workers with both OASDI covered wages and self-employment earnings
ATECMB96	=	Average OASDI covered earnings of workers with both OASDI covered wages and self-employment earnings in 1996
AWSCMB	=	Average OASDI covered wage of workers with both wages and self-employment earnings
AWSCMB96	=	Average OASDI covered wage of workers with both wages and self-employment earnings in 1996
ASESEO	=	Average self-employment earnings of workers with no OASDI taxable wages
AWSCMB	=	Average OASDI covered wage of workers with both wages and self-employment earnings
BASECT	=	Average total earnings of workers with both self-employment earnings and wages in 1% sample data for 1996 used to produce equations
BASECW	=	Average OASDI covered wages of workers with both self-employment earnings and wages in 1% sample data for 1996 used to produce equations
BASEO	=	Average self-employment earnings of workers with no OASDI taxable wages in 1% sample data for 1996 used to produce equations
CMBNT	=	Number of workers with both OASDI taxable wages and self-employment earnings
CSE	=	OASDI covered self-employment earnings
CT	=	Ratio OASDI taxable maximum to average earnings of workers with both self-employment earnings and OASDI taxable wages
CW	=	Ratio OASDI taxable maximum to average self-employment earnings of workers with both self-employment earnings and OASDI taxable wages
CTTR	=	Ratio of OASDI taxable to covered earnings for workers with both wages and self-employment earnings
CWTR	=	Ratio of OASDI taxable to covered wages for workers with both wages and self-employment earnings
O	=	Ratio OASDI taxable maximum to average self-employment earnings of workers with no OASDI taxable wages
OTR	=	Ratio of OASDI taxable self-employment to covered earnings for workers with no OASDI taxable wages
SECCMB	=	OASDI covered self-employment earnings of workers with both self-employment earnings and OASDI taxable wages
SECSEO	=	OASDI covered self-employment earnings of workers with no OASDI taxable wages
SEO	=	Number of workers with OASDI covered self-employment earnings and no OASDI taxable wages
SETCMB	=	OASDI taxable self-employment earnings of workers with both OASDI taxable wages and self-employment earnings
SETSEO	=	OASDI taxable self-employment earnings of workers with no OASDI taxable wages
TAXMAX	=	OASDI taxable maximum
TECCMB	=	OASDI covered earnings of workers with both wages and self-employed earnings
TETCMB	=	OASDI taxable earnings of workers with both wages and self-employed earnings
WSCCMB	=	OASDI covered wages of workers with both wages and self-employed earnings
WSTCMB	=	OASDI taxable wages of workers with both wages and self-employed earnings

2-4.7 OASDI taxable self-employment earnings (SET)

$$SET = SETR * CSE$$

Where

CSE	=	OASDI covered self-employment earnings
SET	=	OASDI taxable self-employment earnings
SETR	=	Ratio of OASDI taxable to covered self-employment earnings

2-4.8 OASDI effective taxable payroll (ETP)

$$ETP=WTER+SET-0.5*MER$$

Where

ETP	=	OASDI effective taxable payroll
MER	=	OASDI multi-employer refund wages
SET	=	OASDI taxable self-employment earnings
WTER	=	Annual OASDI taxable employer wages

2-4.9 OASDI taxable wage liability (WTL)

$$WTL = WTER * TRW$$

Where

TRW	=	OASDI combined employee-employer tax rate
-----	---	---

WTL = Annual OASDI taxable wage liabilities
 WTER = Annual OASDI taxable employer wages

2-4.10 OASDI taxable self-employment liability (SEL)

SEL = SET * TRSE

Where

SEL = OASDI taxable self-employment earnings liabilities
 SET = OASDI taxable self-employment earnings
 TRSE = OASDI self-employment tax rate

2-4.11 OASDI quarterly taxable wage liability (WTLQ)

Federal Civilian

Annual total wages (OASDI + MQGE)

BAFCW = 34198.84
 AWCFC = WCFC / ECFC * BAFCW / AWCFCOT97
 T=MAX/AWCFC

```
IF(T.LT.0.014620379)THEN
  FCTR=T-(1.04262/1.73)*T**1.73
ELSE IF(T.LT.0.292407578)THEN
  FCTR=(-1.22471/1.6)*T**1.6+(.826746/1.8)*DEXP(-1.8*T)+1.8535*T-.459368449
ELSE IF(T.LT.0.760259704)THEN
  FCTR=(-.635082/2D0)*T**2+(.604884/2.9)*T**2.9-(.403213/4.6)*T**4.6+.910343*T+.002291358
ELSE IF(T.LT.1.228111829)THEN
  FCTR=(-.162181/1.7)*T**1.7+(.143632/2.7)*T**2.7-(.312012/3.4)*T**3.4+.841165*T+.011332647
ELSE IF(T.LT.1.520519407)THEN
  FCTR=(-1.34084/3.5)*T**3.5+(1.09868/5D0)*T**5-(.404253/5.8)*T**5.8+1.17397*T-.222555715
ELSE IF(T.LT.2.339260627)THEN
  FCTR=(.671304/.5)*DEXP(-.5*T)-(3.27076/1.4)*DEXP(-1.4*T)+.126626*T+.353367869
ELSE IF(T.LT.3.50889094)THEN
  FCTR=(.0571643/.95)*DEXP(-.95*T)-(3.17633/1.8)*DEXP(-1.8*T)+.000623031*T+.996284293
ELSE IF(T.LT.4.970928832)THEN
  FCTR=(-12.3148/2.25)*DEXP(-2.25*T)+.0000698013*T+.999222265
ELSE
  FCTR=(-.0285502/2D0)*T**(-2D0)+1.00007094
END IF
```

WTFCTOT=FCTR*WCFC

Where

AWCFC = Average covered Federal Civilian wages (OASDI plus MQGE)
 AWCFCOT97 = Average covered Federal Civilian wages (OASDI plus MQGE) for 1997
 BAFCW = Average Federal Civilian wages (OASDI plus MQGE) in 1% sample data for 1997 used to produce equations
 ECFC = Covered Federal Civilian employment (OASDI plus MQGE)
 FCTR = Ratio of taxable to covered Federal Civilian wages (OASDI plus MQGE)
 MAX = OASDI taxable maximum
 T = Ratio of the OASDI taxable maximum to average covered Federal Civilian wages (OASDI plus MQGE)
 WCFC = Covered Federal Civilian wages (OASDI plus MQGE)
 WTFCTOT = Taxable Federal Civilian wages (OASDI plus MQGE)

Annual MQGE wages

BAFCW = 50147.72
 AWCFC = WCFC / ECFC * BAFCW / AWCFCO97
 T = MAX / AWCFC

```
IF(T.LT.0.019941085)THEN
  FCTR=T-(0.0450661/1.47)*T**1.47
ELSE IF(T.LT.0.099705424)THEN
  FCTR=(-.0518044/1.9)*T**1.9-(.0368056/2.3)*T**2.3+.99479*T+.0000248091
ELSE IF(T.LT.0.358939528)THEN
```

```

FCTR=(-.05907/1.25)*T**1.25-(.0746657/2.9)*T**2.9+1.02092*T-.00032173
ELSE IF(T.LT.0.558350377)THEN
FCTR=(-2.4664/1.4)*T**1.4+(4.82919/2.3)*T**2.3-(3.97473/3)*T**3+1.83998*T-.026694932
LSE IF(T.LT.0.797643395)THEN
FCTR=(.609091/2.1)*T**2.1-(1.16086/4)*T**4+.788373*T+.043208139
ELSE IF(T.LT.1.196465093)THEN
FCTR=(2.35647/.4)*DEXP(-.4*T)-(3.87811/1.2)*DEXP(-1.2*T)-(1.1179/2.5)*DEXP(-2.5*T)+.738296*T-2.83402534
ELSE IF(T.LT.1.694992215)THEN
FCTR=(-.422884/1.3)*DEXP(-1.3*T)-(6.90241/3D0)*DEXP(-3*T)-.0229917*T+1.068147457
ELSE IF(T.LT.2.592341034)THEN
FCTR=(.557032/1.2)*DEXP(-1.2*T)-(5.40739/2.2)*DEXP(-2.2*T)+.0102014*T+.960037325
ELSE
FCTR=(-(32.3187/3.5)*DEXP(-3.5*T)+1.000030482
END IF

```

WTFCHO=FCTR*WCFC

Where

AWCFC	=	Average covered Federal Civilian MQGE wages
AWFCCHO97	=	Average covered Federal Civilian MQGE wages for 1997
BAFCW	=	Average Federal Civilian MQGE wages in 1% sample data for 1997 used to produce equations
ECFC	=	Covered Federal Civilian MQGE employment
FCTR	=	Ratio of taxable to covered Federal Civilian MQGE wages
MAX	=	OASDI taxable maximum
T	=	Ratio of the OASDI taxable maximum to average covered Federal Civilian MQGE wages
WCFC	=	Covered Federal Civilian MQGE wages
WTFCHO	=	Taxable Federal Civilian MQGE wages

Annual OASDI taxable wages

WTFC = WTFCTOT - WTFCHO

Where

WTFC	=	Annual OASDI taxable Federal Civilian wages
WTFCHO	=	Taxable Federal Civilian MQGE wages
WTFCTOT	=	Taxable Federal Civilian wages (OASDI plus MQGE)

Quarterly OASDI covered wages

```

CFCQD(1) = .98357 * TCFCD(I,1) + FCPD(I,1)
CFCQD(2) = .98909 * TCFCD(I,2) + FCPD(I,2)
CFCQD(3) = 1.01833 * TCFCD(I,3) + FCPD(I,3)
CFCQD(4) = 1.00814 * TCFCD(I,4) + FCPD(I,4)

```

QWFCOD(J) = CFCQD(J) * WTFC

Where

CFCQD	=	Proportion of annual OASDI covered Federal Civilian wages paid in each quarter
FCPD	=	Payday variable for Federal Civilian wages based on calendar
I	=	Calendar year
J	=	Quarter
TCFCD	=	Proportion of annual NIPA Federal Civilian wages paid in each quarter
QWFCOD	=	Quarterly OASDI covered Federal Civilian wages
WTFC	=	Annual OASDI taxable Federal Civilian wages

Quarterly OASDI taxable wages

```

IF(FCTR.LE.0.928)FCQD(2)=CFCQD(2)+.27522*(1.-FCTR)-.15127*(1.-FCTR)**2+.35146*(1.-FCTR)**3
IF(FCTR.LE.0.993)THEN
FCQD(3)=CFCQD(3)+.28047*(1.-FCTR)-4.73021*(1.-FCTR)**2+.253606*(1.-FCTR)**3-58.1741*(1.-FCTR)**4+45.1465*(1.-FCTR)**5
FCQD(4)=CFCQD(4)-.75095*(1.-FCTR)+3.65109*(1.-FCTR)**2-16.9355*(1.-FCTR)**3+23.9578*(1.-FCTR)**4
END IF
c First quarter is always 100 percent taxable.
QWTFC(I,1)=QWFC(I,1)
IF(FCTR.LE.0.928)THEN

```

c Compute taxable for 2nd-4th quarter.
 $FCQ=FCQD(2)+FCQD(3)+FCQD(4)$
 $WTFC2=WTFC-QWTF(1,1)$
 $FCQD(2:4)=FCQD(2:4)/FCQ$
 $QWTF(1,2:4)=FCQD(2:4)*WTFC2$
ELSE IF(FCTR.LE.0.993)THEN
c Second quarter covered is completely taxable.
 $QWTF(1,2)=QWCF(1,2)$
 $QWTF(1,3)=FCQD(3)*WTFC$
 $QWTF(1,4)=WTFC-QWTF(1,1)-QWTF(1,2)-QWTF(1,3)$
ELSE
c Second and third quarter covered is completely taxable.
 $QWTF(1,2)=QWCF(1,2)$
 $QWTF(1,3)=QWCF(1,3)$
 $QWTF(1,4)=WTFC-QWTF(1,1)-QWTF(1,2)-QWTF(1,3)$
END IF

Where

CFCQD = Proportion of annual OASDI covered Federal Civilian wages paid in each quarter
FCQ = Sum of proportions of annual OASDI covered Federal Civilian wages paid in each quarter for quarters two to four
FCQD = Proportion of annual OASDI taxable Federal Civilian wages paid in each quarter
FCTR = Ratio annual OASDI taxable to covered Federal Civilian wages
I = Calendar year
TCFCD = Proportion of annual NIPA Federal Civilian wages paid in each quarter
QWCF = Quarterly OASDI covered Federal Civilian wages
QWTF = Quarterly OASDI taxable Federal Civilian wages
WTFC = Annual OASDI taxable Federal Civilian wages
WTFC2 = Total OASDI taxable Federal Civilian wages paid in quarters two to four

Quarterly OASDI taxable wage liabilities

$WTLQFCEE(I, J) = QWTF(I, J) * TRWEE(I)$
 $WTLQFCER(I, J) = QWTF(I, J) * TRWER(I)$
 $WTLQFC(I, J) = WTLQFCEE(I, J) + WTLQFCER(I, J)$

Where

I = Calendar year
J = Quarter
TRWEE = OASDI employee tax rate
TRWER = OASDI employer tax rate
WTLQFC = Quarterly OASDI taxable Federal Civilian combined employee-employer wage liabilities
WTLQFCEE = Quarterly OASDI taxable Federal Civilian employee wage liabilities
WTLQFCER = Quarterly OASDI taxable Federal Civilian employer wage liabilities

Military wages

Annual OASDI taxable wages

$BACMW = 16439.95$
 $ACMW = AWCML * BACMW / AWCML97$
 $T = MAX / ACMW$

IF(T.LT.0.060827432)THEN
 $MTR=T-(.712875/2)*T**2$
ELSE IF(T.LT.0.182482295)THEN
 $MTR=(.71197/1.8)*T**1.8-(1.59752/2D0)*T**2+.97587*T+0.000542413$
ELSE IF(T.LT.0.608274315)THEN
 $MTR=-(1.75026/2D0)*T**2+(2.86837/3D0)*T**3-(1.90346/4D0)*T**4+1.10056*T-.006441373$
ELSE IF(T.LT.1.094893767)THEN
 $MTR=-(.700864/1.4)*T**1.4-(.40042/3.3)*T**3.3+(.197091/4.1)*T**4.1+1.33615*T-.056637087$
ELSE IF(T.LT.1.703168082)THEN
 $MTR=(21.3527/.3)*DEXP(-.3*T)-(21.1277/0.5)*DEXP(-.5*T)+(2.73027/1.1)*DEXP(-1.1*T)+4.34833*T-31.56802874$
ELSE IF(T.LT.2.311442397)THEN
 $MTR=-(33.3894/1.2)*T**1.2+(14.9436/1.6)*T**1.6-(2.58041/2.1)*T**2.1+21.3365*T-.872981629$
ELSE IF(T.LT.3.163026438)THEN
 $MTR=-(.076094/.3)*DEXP(-.3*T)-(1.59668/1.4)*DEXP(-1.4*T)-.0271355*T+1.182946986$

```

ELSE IF(T.LT.4.257920205)THEN
  MTR=(.482918/1.5)*T**1.5-(9.21141/.9)*DEXP(-.9*T)+(25.93/1.5)*DEXP(-1.5*T)-1.14706*T+3.246003821
ELSE
  MTR=(-.9.00723/1.8)*DEXP(-1.8*T)+1.000285789
END IF
WTML=MTR*WCML

```

Where

```

ACMW      = Average OASDI covered military wages adjusted for level used to produce equations
AWCML     = Average OASDI covered military wages
AWCML97   = Average OASDI covered military wages in 1997
BACMW     = Average OASDI covered military wages in 1% sample data for 1997 used to produce equations
MAX       = OASDI taxable maximum
MTR       = Ratio of OASDI taxable to covered military wages
T         = Ratio of the OASDI taxable maximum to average covered military wages
WCML      = Annual OASDI covered military wages
WTML      = Annual OASDI taxable military wages

```

Quarterly OASDI covered wages

```

CMLQD(1)=.97978*TCMLD(I,1)*MLPD(I,1)
CMLQD(2)=1.002*TCMLD(I,2)*MLPD(I,2)
CMLQD(3)=1.02145*TCMLD(I,3)*MLPD(I,3)
CMLQD(4)=.99689*TCMLD(I,4)*MLPD(I,4)
QWCML=CMLQD(J)*WCML

```

Where

```

CMLQD     = Proportion of annual OASDI covered military wages paid in each quarter
I         = Calendar year
J         = Quarter
MLPD     = Payday variable for military wages based on calendar
QWCML    = Quarterly OASDI covered military wages
TCMLD    = Proportion of annual NIPA military wages paid in each quarter
WCML     = Annual OASDI covered military wages

```

Quarterly OASDI taxable wages

```

T=MAX/AWCML
IF(MLTR.LT.0.776)QML(1)=CMLQD(1)+.393565-.018307*T-3.44641/T+15.6381/T**2-40.0168/T**3+62.0449/T**4-
57.525/T**5+30.2498/T**6-7.8664/T**7+.674629/T**8
IF(MLTR.LT.0.952)QML(2)=CMLQD(2)+.844748-.0401062*T-7.24247/T+32.4957/T**2-83.3328/T**3+129.374/T**4-
122.526/T**5+68.2737/T**6-20.1479/T**7+2.34289/T**8
IF(MLTR.LT.0.985)QML(3)=CMLQD(3)-2.62266+.125592*T+22.5832/T-105.727/T**2+300.027/T**3-540.915/T**4+622.304/T**5-
441.658/T**6+175.722/T**7-29.8987/T**8
IF(MLTR.LT.1.)QML(4)=CMLQD(4)+2.37295-.111565*T-21.1954/T+106.049/T**2-330.637/T**3+658.869/T**4-
835.626/T**5+648.641/T**6-279.392/T**7+50.9246/T**8
IF(MLTR.LT.0.776)THEN
  QWTML(I,1:4)=QML(1:4)*WTML
ELSE IF(MLTR.LT.0.952)THEN
  QWTML(I,1)=QWCML(I,1)
  TOTWG1=WTML-QWTML(I,1)
  Q1=QML(2)+QML(3)+QML(4)
  QML(2:4)=QML(2:4)/Q1
  QWTML(I,2:4)=QML(2:4)*TOTWG1
ELSE IF(MLTR.LT.0.985)THEN
  QWTML(I,1)=QWCML(I,1)
  QWTML(I,2)=QWCML(I,2)
  TOTWG1=WTML-QWTML(I,1)-QWTML(I,2)
  Q1=QML(3)+QML(4)
  QML(2:4)=QML(2:4)/Q1
  QWTML(I,2:4)=QML(2:4)*TOTWG1
ELSE IF(MLTR.LT.1.)THEN
  QWTML(I,1)=QWCML(I,1)
  QWTML(I,2)=QWCML(I,2)
  QWTML(I,3)=QWCML(I,3)
  QWTML(I,4)=WTML-QWTML(I,1)-QWTML(I,2)-QWTML(I,3)

```

END IF

Where

- AWCML = Average OASDI covered military wages
- CMLQD = Proportion of annual OASDI covered military wages paid in each quarter
- MLTR = Ratio of OASDI taxable to covered military wages
- MAX = OASDI taxable maximum
- I = Calendar year
- Q1 = Sum of proportions of annual OASDI taxable military wages paid in each quarter for last three or two quarters

in year

- QML = Proportion of annual OASDI taxable military wages paid in each quarter
- QWCML = Quarterly OASDI covered military wages
- QWTML = Quarterly OASDI taxable military wages
- T = Ratio of the OASDI taxable maximum to average covered military wages
- TOTWG1 = Annual OASDI taxable military wages for all quarters except first or first and second
- WTML = Annual OASDI taxable military wages

Quarterly OASDI taxable wage liabilities

$$\begin{aligned} \text{WTLQMLEE}(I, J) &= \text{QWTML}(I, J) * \text{TRWEE}(I) \\ \text{WTLQMLER}(I, J) &= \text{QWTML}(I, J) * \text{TRWER}(I) \\ \text{WTLQML}(I, J) &= \text{WTLQMLEE}(I, J) + \text{WTLQMLER}(I, J) \end{aligned}$$

Where

- I = Calendar year
- J = Quarter
- TRWEE = OASDI employee tax rate
- TRWER = OASDI employer tax rate
- WTLQML = Quarterly OASDI taxable military combined employee-employer wage liabilities
- WTLQMLEE = Quarterly OASDI taxable military employee wage liabilities
- WTLQMLER = Quarterly OASDI taxable military employer wage liabilities

Federal

$$\begin{aligned} \text{WCF} &= \text{WCFC} + \text{WCML} \\ \text{QWCF} &= \text{QWCFC} + \text{QWCML} \\ \text{WTF} &= \text{WTFC} + \text{WTML} \\ \text{QWTF} &= \text{QWTFC} + \text{QWTML} \\ \text{WTLQFEE}(I, J) &= \text{QWTF}(I, J) * \text{TRWEE}(I) \\ \text{WTLQFER}(I, J) &= \text{QWTF}(I, J) * \text{TRWER}(I) \\ \text{WTLQF}(I, J) &= \text{WTLQFEE}(I, J) + \text{WTLQFER}(I, J) \end{aligned}$$

Where

- I = Calendar year
- J = Quarter
- QWCF = Quarterly OASDI covered Federal wages
- QWCF C = Quarterly OASDI covered Federal Civilian wages
- QWCML = Quarterly OASDI covered military wages
- QWTF = Quarterly OASDI taxable Federal wages
- QWTFC = Quarterly OASDI taxable Federal Civilian wages
- QWTML = Quarterly OASDI taxable military wages
- WCF = Annual OASDI covered Federal wages
- WCFC = Annual OASDI covered Federal Civilian wages
- WCML = Annual OASDI covered military wages
- WTF = Annual OASDI taxable Federal wages
- WTFC = Annual OASDI taxable Federal Civilian wages
- WTLQF = Quarterly OASDI taxable Federal combined employee-employer wage liabilities
- WTLQFEE = Quarterly OASDI taxable Federal employee wage liabilities
- WTLQFER = Quarterly OASDI taxable Federal employer wage liabilities
- WTML = Annual OASDI taxable military wages

State and Local wages

Annual OASDI taxable wages

$$\text{BACW} = 21583.61$$

```

AWCSL = WCSL / ESLC * BACW / AWCSLOD97
S = MAX / ASLC
IF(S.LT.0.02316573)THEN
  SLTR=S-(1.1803/1.71)*S**1.71
ELSE IF(S.LT.0.463314609)THEN
  SLTR=(-1.54738/1.6)*S**1.6-(-.421147/2.5)*S**2.5+(3.34881/.5)*DEXP(-.5*S)+4.39012*S-6.697774474
ELSE IF(S.LT.0.833966296)THEN
  SLTR=(-.756943/1.8)*S**1.8+(-.485982/2.3)*S**2.3-(-.175681/3.2)*S**3.2+.88749*S+.004652169
ELSE IF(S.LT.1.945921357)THEN
  SLTR=(3.4167/.3)*DEXP(-.3*S)-(.7.26467/.9)*DEXP(-.9*S)+(4.57049/1.5)*DEXP(-1.5*S)+1.0378*S-6.245057503
ELSE IF(S.LT.3.243202261)THEN
  SLTR=(-2.40293/.2)*DEXP(-.2*S)+(6.44952/.4)*DEXP(-.4*S)-(-5.64852/.6)*DEXP(-.6*S)-.278204*S+5.099074279
ELSE IF(S.LT.5.559775305)THEN
  SLTR=(-.0434955/.6)*DEXP(-.6*S)-(-4.00403/1.7)*DEXP(-1.7*S)+.00006219*S+.997065459
ELSE IF(S.LT.18.53258435)THEN
  SLTR=(-.0272758/.5)*DEXP(-.5*S)+.0000671826*S+.997657785
ELSE
  SLTR=(-.00861948/.7)*S**(-.7)+1.000492941
END IF
WTSL=SLTR*WCSL

```

Where

AWCSL	=	Average OASDI covered State and Local wages adjusted for average wage used to produce equations
AWCSLOD97	=	Average OASDI covered State and Local wages for 1997
BACW	=	Average OASDI covered State and Local wages in 1% sample data for 1997 used to produce equations
ESLC	=	OASDI covered State and Local employment
MAX	=	OASDI taxable maximum
S	=	Ratio of the OASDI taxable maximum to average covered State and Local wages
SLTR	=	Ratio of OASDI taxable to covered State and Local wages
WCSL	=	OASDI covered State and Local wages
WTSL	=	OASDI taxable State and Local wages

Quarterly OASDI covered wages

```

CSLQD(1)=1.0131455*TCSLD(1,1)+SLPD(1,1)
CSLQD(2)=1.0431906*TCSLD(1,2)+SLPD(1,2)
CSLQD(3)=.9060524*TCSLD(1,3)+SLPD(1,3)
CSLQD(4)=1.0365866*TCSLD(1,4)+SLPD(1,4)
QWCSL=CSLQD(1:4)*WCSL

```

Where

CSLQD	=	Proportion of annual OASDI covered State and Local wages paid in each quarter
I	=	Calendar year
QWCSL	=	Quarterly OASDI covered State and Local wages
SLPD	=	Payday variable for State and Local wages based on calendar
TCSLD	=	Proportion of annual NIPA State and Local wages paid in each quarter
WCSL	=	Annual OASDI covered State and Local wages

Quarterly OASDI taxable wages

```

QSL(1)=(CSLQD(1)-.24087*(1.-1./SLTR))
QSL(2)=(CSLQD(2)-1.0492*(1.-1./SLTR)+.51259*(1.-1./SLTR**2)-.07643*(1.-1./SLTR**3))
QSL(3)=(CSLQD(3)-5.99032*(1.-SLTR**2)+13.238*(1.-SLTR**3)-11.3291*(1.-SLTR**4)+3.52237*(1.-SLTR**5))
QSL(4)=(CSLQD(4)+8.99897*(1.-SLTR**.25)-5.48866*(1.-SLTR**.5))
TQSL=QSL(2)+QSL(3)+QSL(4)
QSL(2:4)=QSL(2:4)/TQSL
QWTSL(I,1)=QSL(1)*WTSL
QWTSL(I,2:4)=QSL(2:4)*(WTSL- QWTSL(I,1))

```

Where

CSLQD	=	Proportion of annual OASDI covered State and Local wages paid in each quarter
I	=	Calendar year
QSL	=	Proportion of annual OASDI taxable State and Local wages paid in each quarter
QWTSL	=	Quarterly OASDI taxable State and Local wages
SLTR	=	Ratio of OASDI taxable to covered State and Local wages
WTSL	=	OASDI taxable State and Local wages

Quarterly OASDI taxable wage liabilities

$$\text{WTLQSL}(I,J) = \text{QWTSL}(I,J) * \text{TRW}(I)$$

Where

I = Calendar year
J = Quarter
TRW = OASDI combined employee-employer tax rate
WTLQSL = Quarterly OASDI taxable State and Local combined employee-employer wage liabilities

Private household quarterly OASDI taxable wages and liabilities

$$\text{QWTPHH}(I,J) = \text{WCPHH}(I) * \text{QDPHH}(J)$$

$$\text{WTLQPHH}(I,J) = \text{QWTPHH}(I,J) * \text{TRW}(I)$$

Where

I = Calendar year
J = Quarter
QDPHH = Proportion of annual OASDI taxable private household wages paid in each quarter
QWTPHH = Quarterly OASDI taxable private household wages
TRW = OASDI combined employee-employer tax rate
WCPHH = Annual OASDI covered private household wages
WTLQPHH = Quarterly OASDI taxable private household combined employee-employer wage liabilities

Farm taxable wages

Annual OASDI

$$\text{BAFMW} = 7467.91$$

$$\text{AWCFM97} = \text{ACFMW}(1997)$$

$$F = \text{MAX} / (\text{ACFMW} * \text{BAFMW} / \text{AWCFM97})$$

IF(F.LT.0.066953142)THEN

$$\text{FMTR} = F - (1.30211/1.75) * F ** 1.75$$

ELSE IF(F.LT.0.401718855)THEN

$$\text{FMTR} = -(1.18244/1.35) * F ** 1.35 + (25412/1.75) * F ** 1.75 + 1.24681 * F - .001598087$$

ELSE IF(F.LT.0.669531425)THEN

$$\text{FMTR} = -(508764/.6) * \text{DEXP}(-.6 * F) - (.300083/2.8) * \text{DEXP}(-2.8 * F) + .0188542 * F + .966550312$$

ELSE IF(F.LT.1.87468799)THEN

$$\text{FMTR} = -(638146/.6) * \text{DEXP}(-.6 * F) - (.0322774/1.5) * \text{DEXP}(-1.5 * F) - .033706 * F + 1.133974442$$

ELSE IF(F.LT.2.41031313)THEN

$$\text{FMTR} = -(2.64644/1.1) * \text{DEXP}(-1.1 * F) + (17.4638/2) * \text{DEXP}(-2 * F) - (26.4191/2.5) * \text{DEXP}(-2.5 * F) + .00686748 * F + .909154345$$

ELSE IF(F.LT.4.82062626)THEN

$$\text{FMTR} = -(1.06567/1.3) * F ** 1.3 + (.073837/2.1) * F ** 2.1 + 1.31021 * F - .007628879$$

ELSE IF(F.LT.6.427501679)THEN

$$\text{FMTR} = -(1.78355/.5) * \text{DEXP}(-.5 * F) - (1.70356/1.3) * \text{DEXP}(-1.3 * F) + .00115171 * F + .959096096$$

ELSE IF(F.LT.10.7125028)THEN

$$\text{FMTR} = -(0.0474377/0.35) * \text{DEXP}(-.35 * F) - (1.32456/1) * \text{DEXP}(-1 * F) + .0016146 * F + .957903052$$

ELSE IF(F.LT.11.38203422)THEN

$$\text{FMTR} = -(0.0581938/.35) * \text{DEXP}(-.35 * F) + .00130453 * F + .961918378$$

ELSE IF(F.LT.24.1031313)THEN

$$\text{FMTR} = -(0.0492564/.3) * \text{DEXP}(-.3 * F) + .000761577 * F + .97040299$$

ELSE

$$\text{FMTR} = -(0.00304904/.06) * \text{DEXP}(-.06 * F) + 1.000606299$$

END IF

$$\text{TFMW} = \text{FMTR} * \text{WCFM}$$

Where

ACFMW = Annual average OASDI covered farm wages

AWCFM97 = Annual average OASDI covered farm wages for 1997

BAFMW = Average farm wage in 1% sample data for 1997 used to produce equations

F = Ratio of taxable maximum to annual average OASDI covered farm wages adjusted for average wage used in

equations

FMTR = Ratio of OASDI taxable to covered farm wages

MAX = OASDI taxable maximum

TFMW = Annual OASDI taxable farm wages

Quarterly OASDI wages and liabilities

QWTFM(I,J) = TTFMD(I,J) * TFMW
WTLQFM(I,J) = QWTFM(I,J) * TRW(I)

Where

I = Calendar year
J = Quarter
QWTFM = Quarterly OASDI taxable farm wages
TFMW = Annual OASDI taxable farm wages
TRW = OASDI com
TTFMD = Proportion of annual OASDI taxable farm wages paid in each quarter
WTLQFM = Quarterly OASDI taxable farm combined employee-employer wage liabilities

Quarterly OASDI taxable employee tips

QWTTIPSEE(I,J) = QDTIP(J) * WTTIPSEE(I)
QWTTIPSEE(I,2) = QWTTIPSEE(I,2) + WTTIPSSR(I)
WTLQTIPSEE(I,J) = QWTTIPSEE(I,J) * TRW(I)

Where

I = Calendar year
J = Quarter
QDTIP = Proportion of annual OASDI taxable tips received in each quarter
QWTTIPSEE = Quarterly OASDI taxable tips received by employees
WTLQTIPSEE = Quarterly OASDI combined employee-employer wage liabilities on taxable tips received by employees
TRW = OASDI combined employee-employer tax rate
WTTIPSEE = Annual OASDI taxable tips received by employees reported by employers
WTTIPSSR = Annual OASDI taxable tips received by employees self-reported on income tax returns

Private non-farm OASDI taxable wages and liabilities

Annual

WTPNF = WTER - WTFC - WTML - WTSL - TFMW - WTTIPSEE - WTTIPSSR

Where

TFMW = Annual OASDI taxable farm wages
WTSL = Annual OASDI taxable State and Local wages
WTFC = Annual OASDI taxable Federal Civilian wages
WTPNF = Annual OASDI taxable private non-farm wages excluding tips
WTTIPSEE = Annual OASDI taxable tips received by employees reported by employers
WTTIPSSR = Annual OASDI taxable tips received by employees self-reported on income tax returns
WTER = Annual OASDI taxable employer wages

Quarterly

BACW93 = 21912.00
NACW = BACW93 / ACW93 * AWC
X = MAX / NACW
IF(X.LT.0.91274)THEN
TWTR=1D0+.990751*DEXP(X)**(-1)/(-1)-.013904602
ELSE IF(X.LT.2.05367)THEN
TWTR=1D0+(-.003129*X+(1.167562*DEXP(X)**(-1.17))/(-1.17))-0.065747345
ELSE IF(X.LT.4.791895)THEN
TWTR=1D0+(.003962*X+(.770093*X**(-1.85053))/(-1.85053))-0.06071106
ELSE
TWTR=1D0+(.267708*X**(-.94))/(-.94)+.00066
END IF
IF(TWTR.LT.0.70)THEN
QP(1)=-(-0.000575+0.18692*DLOG(TWTR)-0.23133*DLOG(TWTR)**2-
0.10453*DLOG(TWTR)**3+0.04306*DLOG(TWTR)**4+0.01906*DLOG(TWTR)**5)-0.0325201+PD(1)+TCPD(I,1)
QP(2)=-(-0.00657+1.7015*TWTR-8.60615*TWTR**2+14.444*TWTR**3-9.97171*TWTR**4+2.42519*TWTR**5)-
0.0080956+PD(2)+TCPD(I,2)

$QP(3) = -(0.12167 + 1.31142 * TWTR^{**3} - 6.31672 * TWTR^{**4} + 8.03785 * TWTR^{**5} - 3.15412 * TWTR^{**6}) + 0.019325 + PD(3) + TCPD(I,3)$
 $QP(4) = -(0.1548 - 0.41354 * TWTR^{**5} + 0.25874 * TWTR^{**7}) + 0.0197767 + PD(4) + TCPD(I,4)$
ELSE IF(TWTR.LT.0.88)THEN
 $QP(1) = 0.224763 - 0.237056 * TWTR + PD(1) + TCPD(I,1)$
 $QP(2) = 0.190385 - 0.209676 * TWTR + 0.00176 * (TWTR - 0.7) / (0.88 - 0.7) + PD(2) + TCPD(I,2)$
 $QP(3) = -0.052523 + 0.05309 * TWTR + PD(3) + TCPD(I,3)$
 $QP(4) = -0.354571 + 0.38249 * TWTR + PD(4) + TCPD(I,4)$
ELSE
 $QP(1) = 0.968092 - 1.877574 * TWTR + 0.904348 * TWTR^{**2} + PD(1) + TCPD(I,1)$
 $QP(2) = -0.468266 + 1.148107 * TWTR - 0.690132 * TWTR^{**2} + PD(2) + TCPD(I,2)$
 $QP(3) = -0.850885 + 1.824094 * TWTR - 0.981557 * TWTR^{**2} + PD(3) + TCPD(I,3)$
 $QP(4) = 0.350767 - 1.093966 * TWTR + 0.766972 * TWTR^{**2} + PD(4) + TCPD(I,4)$
END IF
IF(PTR.LT.0.86)THEN
 $QP(J) = QP(J) + ADJTP(J)$
ELSE
 $IF((ADJCP(J) - ADJTP(J)).NE.0D0)QP(J) = QP(J) + ADJTP(J) + ((PTR - BPTR) / (1. - BPTR))^{**4} * (ADJCP(J) - ADJTP(J))$
END IF
 $QWTPNF(I, J) = QP(J) * WTPNF(I) + QWTTIPSEE(I, J) + QWTPHH(I, J)$
 $QWTPNF(I, 2) = QWTPNF(I, 2) + WTTIPSSR(I)$

Where

ACW93 = Annual average OASDI covered wage for 1993
AWC = Annual average OASDI covered wage for current year
BACW93 = Annual average OASDI covered wage for 1993 from actual data used to determine taxable to covered wage equations
I = Calendar year
J = Quarter
MAX = Annual OASDI taxable maximum
NACW = Annual average OASDI covered wage for current year adjusted for average from actual data used to determine equations
PD = Payday variable for private non-farm based on calendar
QP = Proportion of annual OASDI taxable private non-farm wages excluding taxable tips paid in each quarter
QWTPNF = Quarterly OASDI taxable private non-farm wages including tips
TCPD = Proportion of annual NIPA private wages paid in each quarter
TWTR = Ratio of OASDI taxable to covered wages computed using equations based on data for 1993
X = Ratio of annual OASDI taxable maximum to adjusted annual average OASDI covered wage (NACW)

Quarterly OASDI wage liabilities

$WTLQPNF(I,J) = (QWTPNF(I,J) - QWTPHH(I,J)) * TRW(I)$

Where

QWTPHH = Quarterly OASDI taxable private household wages
QWTPNF = Quarterly OASDI taxable private non-farm wages including tips
TRW = OASDI combined employee-employer tax rate
WTLQPNF = Quarterly OASDI tax liabilities from taxable private non-farm wages including tips, excluding private household taxable wages

Total quarterly OASDI taxable wages and wage liabilities

$QWT(I,J) = QWTPNF(I,J) + QWTF(I,J) + QWTSL(I,J) + QWTFM(I,J)$
 $WTLQ(I,J) = QWT(I,J) * TRW(I)$

Where

I = Calendar year
J = Quarter
QWT = Quarterly OASDI taxable wages
QWTF = Quarterly OASDI taxable Federal wages
QWTFM = Quarterly OASDI taxable farm wages
QWTPNF = Quarterly OASDI taxable private non-farm wages including tips
QWTSL = Quarterly OASDI taxable State and Local wages
WTLQ = Quarterly OASDI taxable wage liabilities

2-4.12 OASDI quarterly taxable wage liability collections (WTLQC)

OASDI taxable private non-farm wages by sub-quarterly periods

PTR=WTP/WCP
 MR=MAR(I)-.04346*(1.-PTR)+.08497*(1.-PTR)**2
 JR=JUN(I)-.02627*(1.-PTR)-.26844*(1.-PTR)**2
 SR=SEP(I)-.12321*(1.-PTR)-.02344*(1.-PTR)**2
 DR=DEC(I)-.12468*(1.-PTR)-.20710*(1.-PTR)**2
 MWTP(1)=QWTP(I,1)*MR
 MWTP(2)=QWTP(I,1)-MWTP(1)
 MWTP(3)=QWTP(I,2)*JR
 MWTP(4)=QWTP(I,2)-MWTP(3)
 MWTP(5)=QWTP(I,3)*SR
 MWTP(6)=QWTP(I,3)-MWTP(5)
 MWTP(7)=QWTP(I,4)*DR
 MWTP(8)=QWTP(I,4)-MWTP(7)

Where

DEC = Proportion of fourth quarter OASDI covered private non-farm wages (excluding tips and household) paid in December
 DR = Proportion of fourth quarter OASDI taxable private non-farm wages (excluding tips and household) paid in December
 I = Calendar year
 JR = Proportion of second quarter OASDI taxable private non-farm wages (excluding tips and household) paid in June
 JUN = Proportion of second quarter OASDI covered private non-farm wages (excluding tips and household) paid in June
 MAR = Proportion of first quarter OASDI covered private non-farm wages (excluding tips and household) paid in March
 MR = Proportion of first quarter OASDI taxable private non-farm wages (excluding tips and household) paid in March
 MWTP = OASDI taxable private non-farm wages (excluding tips and household) paid in last month and in first two months of quarter
 PTR = Ratio of annual OASDI taxable private non-farm wages (excluding tips and household) to covered private non-farm wages
 QWTP = Quarterly OASDI taxable private non-farm wages (excluding tips and household)
 SEP = Proportion of third quarter OASDI covered private non-farm wages (excluding tips and household) paid in September
 SR = Proportion of third quarter OASDI taxable private non-farm wages (excluding tips and household) paid in September
 WCP = Annual OASDI covered private non-farm wages
 WTP = Annual OASDI taxable private non-farm wages (excluding tips and household)

OASDI taxable private non-farm wages collected on in same quarter wages are paid

TRAT=RATEE(I,5)
 CA=.95
 MWCP(1)=QWSCPWF(I,1)*MAR(I)
 MWCP(2)=QWSCPWF(I,1)-MWCP(1)
 MWCP(3)=QWSCPWF(I,2)*JUN(I)
 MWCP(4)=QWSCPWF(I,2)-MWCP(3)
 MWCP(5)=QWSCPWF(I,3)*SEP(I)
 MWCP(6)=QWSCPWF(I,3)-MWCP(5)
 MWCP(7)=QWSCPWF(I,4)*DEC(I)
 MWCP(8)=QWSCPWF(I,4)-MWCP(7)
 RCSM=.80
 QRMREQ=750.
 QRWREQ=11250.
 RMF=70786.*WSP(I)/1001400.
 CALL ITERNU(QRMREQ,MWTP(2),MWCP(2),TRAT,RMF,PWCS(1))
 CALL ITERNU(QRWREQ,MWTP(1),MWCP(1),TRAT,RMF,PWCE(1))
 CALL ITERNU(QRMREQ,MWTP(4),MWCP(4),TRAT,RMF,PWCS(2))
 CALL ITERNU(QRWREQ,MWTP(3),MWCP(3),TRAT,RMF,PWCE(2))
 CALL ITERNU(QRMREQ,MWTP(6),MWCP(6),TRAT,RMF,PWCS(3))
 CALL ITERNU(QRWREQ,MWTP(5),MWCP(5),TRAT,RMF,PWCE(3))
 CALL ITERNU(QRMREQ,MWTP(8),MWCP(8),TRAT,RMF,PWCS(4))
 CALL ITERNU(QRWREQ,MWTP(7),MWCP(7),TRAT,RMF,PWCE(4))
 DO J=1,4
 QWTPC(I,J)=PWCS(J)+PWCE(J)*RCSM*CA
 QWTPF(I,J)=QWSTXPHH(I,J)-QWTPC(I,J)
 END DO

Where

AWSCODXSRT = Annual average OASDI covered private non-farm wages (excluding household)
 CA = Compliance allowance

DEC = Proportion of fourth quarter OASDI covered private non-farm wages (excluding tips and household) paid in
 December
 I = Calendar year
 J = Quarter
 JUN = Proportion of second quarter OASDI covered private non-farm wages (excluding tips and household) paid in June
 MAR = Proportion of first quarter OASDI covered private non-farm wages (excluding tips and household) paid in
 March
 MWCP = OASDI covered private non-farm wages paid in third month and in first two months of each quarter
 MWTP = OASDI taxable private non-farm wages paid in third month and in first two months of each quarter
 PWCE = OASDI taxable private non-farm wages paid in the third month of each quarter
 PWCS = OASDI taxable private non-farm wages paid in the first two months of each quarter on which taxes are
 collected in that quarter
 QWSCPWF = Quarterly OASDI covered private non-farm wages
 QRMREQ = Monthly deposit requirement
 QRWREQ = Quarterly deposit requirement
 QWSTXPHH = Quarterly OASDI taxable private non-farm wages (excluding household)
 QWTPC = Quarterly OASDI taxable private non-farm wages on which employers deposit taxes in the quarter the wages
 were paid
 QWTPF = Quarterly OASDI taxable private non-farm wages on which employers deposit taxes in the quarter after the
 wages were paid
 RATEE(1,5) = OASDHI employee tax rate
 RCSM = Proportion of OASDI taxable private non-farm wages wages paid in same quarter in which taxes are collected
 RMF = Current year average wage size of firm
 SEP = Proportion of third quarter OASDI covered private non-farm wages (excluding tips and household) paid in
 September
 TRAT = OASDHI employee tax rate
 WSP = Economy-wide (NIPA) private wages

SUBROUTINE ITERNU(A11,QPAR,QTOT,T,RMF,AMTOUT)

R=QPAR/QTOT

X=A11/(T*2+.10)

DO

IWH=X*(.16011+.01998*LOG(X/RMF)-.01)

FWH=T*2.*X*((-1.4402*LOG(1.+X/RMF)+1.)*(1.-R)+R)

A1=IWH+FWH

D=A1/A1

N1=D*1000.

IF(N1.EQ.999.OR.N1.EQ.1000)THEN

RTAX=R+(1.-R)*(-1.07115*X/RMF+.38633*(X/RMF)**2+1)

TOD=177.16+1142.7*DEXP(-(X/RMF))+1181.26*DEXP(-3.*(X/RMF))-907.88*DEXP(-4.*(X/RMF))+646.49*DEXP(-5.*(X/RMF))-
165.09*DEXP(-6.*(X/RMF))-20.92*X/RMF-2906.07/(X/RMF+1.))**2+831.44/(X/RMF+1.))**3

AMTOUT=QPAR-RTAX*TOD*QTOT

RETURN

END IF

X=X*D

END DO

END SUBROUTINE ITERNU

Where

A1 = Total (income plus FICA) taxes withheld

A11 = Deposit requirement

AMTOUT = OASDI taxable private non-farm wages paid in sub-quarterly period and collected on in same quarter

D = Ratio of deposit requirement to total taxes withheld

FWH = FICA taxes withheld

IWH = Income taxes withheld

N1 = Ratio of deposit requirement to total taxes withheld times 1000 (used to see how close we are to target)

QPAR = OASDI taxable private non-farm wages paid in sub-quarterly period

QTOT = OASDI covered private non-farm wages paid in sub-quarterly period

R = Initial ratio of OASDI taxable to covered private non-farm wages paid in sub-quarterly period

RMF = Current year average wage size of firm

RTAX = Ratio of OASDI taxable to covered private non-farm wages paid in sub-quarterly period

T = OASDHI employee tax rate

TOD = Proportion of liabilities to be deposited in quarter after that in which wages paid

X = Taxable wage amount needed to meet deposit requirement

OASDI taxable private wages collected on in same quarter wages paid and in following quarter

QWTPCQ(I,J)=QWTPC(I,J)+QWTPHHCQ(I,J)+QWTFM(I,J)
 QWTPFQ(I,J)=QWTPF(I,J)+QWTPHHFQ(I,J)

OASDI taxable State and Local wages collected on in same quarter wages paid and in following quarter

SLTR=WTSL/WCSL
 LMPW(1)=MARSL(I)-.00329*(1.-SLTR**2)
 LMPW(2)=JUNSL(I)-.68187*(1.-SLTR**3)+.52206*(1.-SLTR**4)
 LMPW(3)=SEPSL(I)-1.33596*(1.-SLTR)+1.51187*(1.-SLTR**2)-.63523*(1.-SLTR**3)
 LMPW(4)=DECSL(I)-2.03892*(1.-SLTR)+1.90430*(1.-SLTR**2)-.6633*(1.-SLTR**3)
 DO J=1,4
 SLCR(J)=(1.-LMPW(J))+LMPW(J)*LMCRPR(I-16,J)
 QWTS LC(I,J)=SLCR(J)*QWTS L(I,J)
 QWTS LF(I,J)=QWTS L(I,J)-QWTS LC(I,J)
 END DO

Where

DECSL = Proportion of OASDI taxable State and Local wages paid in fourth quarter which are paid in December
 I = Calendar year
 J = Quarter
 JUNSL = Proportion of OASDI taxable State and Local wages paid in second quarter which are paid in June
 LMCRPR = Proportion of OASDI taxable State and Local wages paid in final month of quarter on which employers are to deposit taxes
 in the same quarter
 LMPW = Proportion of quarterly OASDI taxable State and Local wages paid in final month of quarter
 MARSL = Proportion of OASDI taxable State and Local wages paid in first quarter which are paid in March
 QWTS L = Quarterly OASDI taxable State and Local wages paid in quarter
 QWTS LC = Quarterly OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in the same
 quarter
 QWTS LF = Quarterly OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in the
 following quarter
 SEPSL = Proportion of OASDI taxable State and Local wages paid in third quarter which are paid in September
 SLCR = Proportion of OASDI taxable State and Local wages paid in quarter on which taxes are deposited by the employer in
 the same quarter
 SLTR = Ratio of OASDI taxable to covered State and Local wages
 WCSL = Annual OASDI covered State and Local wages
 WTSL = Annual OASDI taxable State and Local wages

OASDI taxable wages collected on in same quarter wages paid and in following quarter

WTQCQ(I,J)= QWTPCQ(I,J)+ QWTS LC(I,J)+QWTF(I,J)
 WTQFQ(I,J)= QWTPFQ(I,J)+ QWTS LF(I,J)

Where

I = Calendar year
 J = Quarter
 QWTF = Quarterly OASDI taxable Federal wages
 QWTPCQ = Quarterly OASDI taxable private wages collected on in same quarter wages paid
 QWTPFQ = Quarterly OASDI taxable private wages collected on quarter following that in which wages paid
 QWTS LCQ = Quarterly OASDI taxable State and Local wages collected on in same quarter wages paid
 QWTS LFIQ = Quarterly OASDI taxable State and Local wages collected on in quarter following that in which wages paid
 WTQCQ = Quarterly OASDI taxable wages collected on in same quarter wages paid
 WTQFQ = Quarterly OASDI taxable wages collected on in quarter following that in which wages paid

Quarterly OASDI wage tax collections

WTLQC(I,1) = TRW(I-1) * WTQFQ(I-1,4) + TRW(I) * WTQCQ(I,1)
 DO J = 2, 4
 WTLQC(I,J) = TRW(I) * (WTQFQ(I,J-1) + WTQCQ(I,J))
 END DO

Where

I = Calendar year
 J = Quarter
 TRW = OASDI combined employee-employer tax rate
 WTLQC = Quarterly OASDI wage tax collections

WTQCQ = Quarterly OASDI taxable wages collected on in same quarter wages paid
WTQFQ = Quarterly OASDI taxable wages collected on in quarter following that in which wages paid

2-4.13 Quarterly Self-Employed Net Income Tax Collections (SELQC)

DO J = 1, 4
SELQC(I,J) = SECRCY(I,J) * SEL(I) + SECRPY(I,J) * SEL(I-1)
END DO

Where

I = Calendar year
J = Quarter
SECRCY = Proportion of OASDI taxable self-employment earnings collected on in same year earned
SECRPY = Proportion of OASDI taxable self-employment earnings collected on in year following that in which earned
SEL = OASDI taxable self-employment earnings liabilities
SELQC = Quarterly OASDI self-employed net income tax collections

2-4.14 SECA Appropriation Adjustments (MSEC AAA)

MSEC AAA = 0
DO L = I-2, I-9, -1
MSEC AAA = MSEC AAA + SEAACO(I,J) * SELIAC(L)
END DO

Where

I = Calendar year
J = Quarter
L = Liability year
MSEC AAA = OASDI SECA appropriation adjustment for quarter (assigned to last month in quarter)
SEAACO = Proportion of past year's OASDI self-employment tax liability which will be reported in current quarter
SELIAC = Prior years' OASDI self-employment tax liability

Appendix 2-2

Economic Acronyms

AA	Appropriation adjustments
ACE	Average OASDI covered earnings
ACSE	Average OASDI covered self-employed income
ACW	Average OASDI covered wage
ACWC	Average economy-wide wage
AWI	Average wage index calculated by SSA; based on the average wage of all workers with wages from Forms W-2
BEA	The Bureau of Economic Analysis
BLS	The Bureau of Labor Statistics
CMB_TOT	Workers that have a combination of both OASDI covered wages and self-employed income.
COV	Economic Sub-Process: Covered Employment and Earnings
CPI	The Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W) is an official measure of inflation in consumer prices, published by the BLS.
CPS	Current Population Survey, conducted monthly by the Bureau of Census for the Bureau of Labor Statistics. It is the source of historical monthly economic data (such as labor force, civilian noninstitutional population, and unemployment) used to project US employment.
CSE_TOT	Total OASDI covered self-employed income
CSRS	Civil Service Retirement System
CSW	Self-employed only workers (SEO) plus combination workers (CMB_TOT)
E	Total employment, CPS concept
EA	Total agricultural employment
EDMIL	Total number serving in the US Armed Forces estimated by the Department of Defense and published by the Census Bureau
EO	Total employment in the other immigrant population
EO_ESF	Total employment in the other immigrant population whose reported earnings are posted to the Earnings Suspense File
EO_MEF	Total employment in the other immigrant population whose earnings are reported and posted to the Master Earnings File
EO_MEFC	Total employment in the other immigrant population whose earnings are reported and posted to the Master Earnings File and are OASDI-covered
EO_UND	Total employment in the other immigrant population that is strictly in the underground economy (i.e., with no earnings reported)
ES	Self-employed workers
ETP	Effective annual taxable payroll, equal to total employer taxable OASDI wages plus total self-employed taxable income minus one half of the multi-employer refund wages
EU	Unpaid family workers

EW	Wage and salaried workers
FERS	Federal Employee Retirement System
GDP	Gross domestic product
HI	Hospital insurance
LC	US labor force, equal to the sum of number of persons employed and number of persons seeking employment
LFPR	Labor force participation rate, defined as the ratio of the number of persons in the US labor force to the number of persons in the US noninstitutional population.
MER	Multi-employer refund wages
N	Civilian noninstitutional population
NCE	Total noncovered employment
NIPA	The National Income and Product Accounts, published by the BEA, providing historical estimates of quarterly earnings and output measures
NRA	Normal retirement age
OASDI	Old-Age, Survivors, and Disability Insurance
OASDHI	Old-Age, Survivors, Disability, and Health Insurance
OLI	Employer contributions for employee pension and insurance funds
PIA	Primary insurance amount
PIARR	PIA replacement rate, defined as the ratio of a hypothetical medium scale worker's PIA to his/her career average indexed earnings.
PGDP	GDP price deflator
PBNFXGE	Private nonfarm business excluding government enterprises
RCMB	Proportion of wage workers who are also self-employed (CMB_TOT/WSW)
RCSE	Covered self-employed ratio, defined as the ratio of total covered self-employment income to total proprietor income (CSE_TOT/Y).
RD	Disability prevalence ratio, defined as the ratio of disabled worker beneficiaries to the Social Security area population.
RELMAX	Ratio of the TAXMAX to averaged covered earnings
RET	Earnings test ratio, defined the ratio of the maximum amount of earnings before an OASDI benefit is reduced to the average wage index.
RFS	Family size ratio, defined as the ratio of the number of children under 6 to mothers of a certain age.
RM	Military ratio, the ratio of the US armed forces to the non-institutionalized population.
RMER	Multi-employer refund wage ratio, defined as the ratio of multi-employer refund wages to total OASDI wages.
RSET	Self-employed net income taxable ratio, defined as the ratio of total self-employed taxable income to total OASDI wages.
RTE	Ratio of total employment to the sum of wage & salary, self-employed workers, and the military (TE/(EW + ES + military))
RTP	A summary measure of the business cycle equal to the ratio of real GDP to potential GDP.

RU	Civilian unemployment rate defined as the ratio of the unemployed US labor force to the total US labor force.
RWSC	Covered wage ratio, defined as the ratio of OASDI covered wages to total wage and salary disbursements (WSC/WSD)
RWSD	Earnings ratio, defined as the ratio of total wage and salary disbursements to total wage and worker compensation(WSD/WSS)
RWSSY	Total compensation ratio, defined as the ratio of total compensation to gross domestic product (WSSY/GDP)
RWTEE	Employee taxable ratio, defined as the ratio total employee OASDI taxable wages to total OASDI covered wages.
RY	Income ratio, defined as the percentage of total compensation attributable to proprietor income (Y/WSSY).
S&L	State and Local government
SEL	Self-employment tax liabilities
SELQC	Quarterly self-employed net income tax collections
SEO	Workers that report only OASDI covered self-employed earnings
SEPR	The self employed participation rate is the proportion of employed persons that are self employed
SET	Total self-employed taxable income
SOC	Employer contributions for government social insurance
SSA	Social Security Administration
TAXMAX	OASDI contribution and benefit base
TAXPAY	Economic Sub-Process: Taxable Payroll
TCE	Total OASDI covered employment
TE	Total at-any-time employment
TEO	Total “at any time” employment in the other immigrant population
TEO_ESF	Total “at any time” employment in the other immigrant population whose reported earnings are posted to the Earnings Suspense File
TEO_MEF	Total “at any time” employment in the other immigrant population whose earnings are reported and posted to the Master Earnings File
TEO_MEF C	Total “at any time” employment in the other immigrant population whose earnings are reported and posted to the Master Earnings File and are OASDI-covered
TEO_UND	Total “at any time” employment in the other immigrant population that is strictly in the underground economy (i.e., with no earnings reported)
TRSE	OASDI self-employed tax rate
TRW	Combined OASDI employee-employer tax rate
U	The number of persons in the labor force who are unemployed
USEAR	Economic Sub-Process: U.S. Earnings
USEMP	Economic Sub-Process: U.S. Employment
WSC	Total OASDI covered wages
WSD	Total wage and salary disbursements
WSS	Total wage worker compensation
WSSY	Total compensation for wage and salary workers and proprietors

WSW	Wage and salary workers that report some OASDI covered earnings
WTEE	Total employee OASDI taxable wages
WTER	Total employer OASDI taxable wages
WTL	Annual OASDI wage tax liabilities
WTLQ	Quarterly OASDI wage tax liabilities
WTLQC	Quarterly OASDI wage tax collections
Y	Total proprietor income

Process 3:

Beneficiaries

3. Beneficiaries

OCACT uses the Beneficiaries process to project the fully insured and disability insured population, the number of disabled worker and their dependent beneficiaries, the number of retired worker and their dependent beneficiaries, and the number of dependent beneficiaries of deceased workers. The Beneficiaries process receives input data from the Demography and Economics sections along with data received from the Social Security Administration and other government agencies. Output data is provided to the Economics and Trust Fund Operations and Actuarial Status processes.

The Beneficiaries Process is composed of three subprocesses: INSURED, DISABILITY, and OLD-AGE AND SURVIVORS. As a rough overview, INSURED projects the number of people in the Social Security area population that have sufficient work histories for disability and retirement benefit eligibility. DISABILITY projects the number of disabled worker and their dependent beneficiaries. OLD-AGE AND SURVIVORS projects the number of retired workers, their dependent beneficiaries, and the dependent beneficiaries of deceased workers.

All programs output data on an annual basis.

3.1. INSURED

3.1.a. Overview

Insured status is a critical requirement for a worker, who has participated in the covered economy, to receive Social Security benefits upon retirement or disability. The requirement for insured status depends on the age of a worker and his (or her) accumulation of quarters of coverage (QC).

INSURED is a simulation model that estimates the percentage of the population that is fully insured (FPRO) and disability insured (DPRO) throughout the projection period. These estimates are used in conjunction with estimates of the Social Security area population to estimate the number of people that are fully insured (FINPOP) and disability insured (DINPOP). FINPOP is then used by the OLD-AGE AND SURVIVORS INSURANCE subprocess, and both FINPOP and DINPOP are used by the DISABILITY subprocess. FINPOP and DINPOP are projected by age, sex, and cohort.

For each sex and birth cohort, INSURED simulates 30,000 work histories which represent the Social Security area population (SSAPOP). These histories are constructed from past and projected cover worker rates, median earnings, and amounts required for crediting QC.

The equations for this subprocess are given below:

$$\text{FPRO} = \text{FPRO}(\cdot) \quad (3.1.1)$$

$$\text{DPRO} = \text{DPRO}(\cdot) \quad (3.1.2)$$

$$\text{FINPOP} = \text{FPRO} * \text{SSAPOP} \quad (3.1.3)$$

$$\text{DINPOP} = \text{DPRO} * \text{SSAPOP} \quad (3.1.4)$$

3.1.b. Input Data

All data are updated annually, except those that are noted.

Long-Range OCACT Data

Demography

- Social Security area population as of year end (1940 – 2085) by age (0 -100, age 100 including age 100 and older), marital status (single, married, widowed, divorced) and sex (M, F).
- Number of new “net legal immigrants” (legal immigrants – estimated legal emigrants) entering the Social Security area each year (1940 – 2085) by age (14 – 69)

- and sex (M, F).
- “Other immigrant” population as of year end (2007 – 2085) by age (0 -100, age 100 including age 100 and older), marital status (single, married, widowed, divorced) and sex (M, F).

Economics

- Historical annual estimates of covered workers by sex (M, F), age (15 – 74), and marital status (single, married, widowed, divorced) for years (1937 – 2007).
- Annual projections (2008 – 2085) of covered workers that are based on the assumption that the portion of “other immigrants” in the Social Security Area stays constant at the level of the latest historical estimates by cohort and sex. The data format is the same as those in historical period.
- Annual projection (2005 – 2085) of average wage index and median covered earnings.
- Total “other immigrant” workers by sex (M, F), age (16 – 95), and for years (2007 – 2085).
- “Other immigrant” workers with earnings posted to the Master Earnings File (MEF) by sex (M, F), age (16 – 95), and for years (2007 – 2085).

Beneficiaries

- Disabled-worker beneficiaries at year end (2007 – 2085) by age (15 – 66), sex (M, F) and duration (0 – 10, duration 10 including duration 10 and above) from the previous year’s Trustees Report. These data are read in from files that are generated annually from the Beneficiaries/DISABILITY (#3.2) area.

Short-Range OCACT data

- FINPOP by age (14 – 95, age 95 including age 95 and older) and sex (M, F) from the end of year 1969 to the end of Short-Range projection period (2018).
- DINPOP by age (15 – 66) and sex (M, F) from the end of year 1969 to the end of Short-Range projection period (2018).

Other input data

- Historical series of annual median earnings of covered workers by age group (<20, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-61, 62-64, 65-69) and sex (M, F) for years 1937-2005. Data are updated using the data in the most recent Social Security Annual Statistical Supplement Table 4.B6.
- Number of disabled workers by age (20-69) and sex (M, F) for years 1958 to the most recent data year. Ages 66-69 are zeros. Data are updated using the data from the

- historical disability file “wkrben”.
- The amount required for crediting one quarter of coverage for years 1937-2005 from SSA publication.
 - Historical series of annual median earnings of all covered workers for years 1937-2005. Data are updated using the data in the most recent Social Security Annual Statistical Supplement Table 4.B6.
 - The number of all covered workers (wage/salary workers, self-employed workers) by sex and amount of earnings for 2005 in the most recent Social Security Annual Statistical Supplement Table 4.B7 & 4.B9. These are used to produce the input data for the distribution of earnings (FRAC).
 - ANNUAL factor (comparability factor between quarterly and annual reporting of earnings) by age (13-69) and sex (M, F) for years prior to 1978. These data are not updated.

3.1.c. Development of Output

Equation 3.1.1 & 3.1.2 -

Determining the QC distribution

There are three variables playing important roles in the simulation process starting from age 13 through 69 of a birth cohort by sex. They include historical and projected covered worker rates, the amounts required for crediting QC, and a cumulative worker distribution by earnings level.

Covered worker rates by age and sex are defined as the ratio of covered workers to the Social Security area population. Historical and projected numbers of covered workers and the Social Security area population, which are provided by the Economics and Demography sections respectively, are used to calculate the rates for ages 15 through 69. For ages 13 and 14, INSURED estimates covered worker rates based on the covered worker rates at age 15.

The amount of earnings needed to earn one QC is specified by law for each year of the historical period. Its projection assumes the same growth rate as the Social Security average wage index.

The cumulative worker distribution by earnings level is called ‘FRAC’. It is a function of covered earnings relative to median earnings. For a given ratio of covered earnings relative to median earnings, FRAC returns the percentage of covered workers whose earnings relative to median earnings are less than the given ratio. It is constructed based on the latest historical data. It is used for each age and sex and is assumed to remain constant throughout the projection period. The program uses FRAC to estimate the percentage of covered workers that earn 0, 1, 2, 3 or 4 QC in a given year. Thus, for a particular age and sex, the percentage of covered workers earning at least n QC is defined as:

$$\text{QCDist} = 1 - \text{FRAC} \left[\frac{n * \text{QC amount}}{\text{median earnings}} \right], \quad \text{for } n = 1, 2, 3, 4$$

where *median earnings* is for that age and sex.

Simulation process – assigning QC to records

Once the QCDist is known, the simulation process begins with 30,000 records for each sex and birth cohort. Starting with the QC distribution at age 13, INSURED randomly assigns a number of QC (1, 2, 3 or 4) to these records based on QCDist.

For ages 14 to 69, INSURED begins the simulation process by randomly selecting records to represent new net legal immigrants from the covered worker portion of 30,000 records. For each record, a number of QC (1, 2, 3 or 4) is assigned on a uniform basis. Once a record is assigned a number of QC, INSURED nullifies the previous earnings of the record.

After the records for new immigrants are selected, the rest of the records for ages 14 to 69 are either non-covered workers or covered workers. The total number of records assigned as non-covered workers is set equal to (1-covered worker rate) * 30,000. These records receive no QC. To identify records as non-covered workers, INSURED uses two parameters (SRCH, SLCT), which vary by age and sex.

SRCH sets a limit on the number of consecutive records to be searched for a non-covered worker. In general, the younger age groups have lower SRCH values. SLCT is the number of consecutive prior years in which no QC were earned that is required in order for a simulated record to be assigned as a non-covered worker. Lower SLCT values are set for the very young and older age groups. Sensitivity analyses show that insured percentages are negatively correlated with these two parameters. When the female covered worker rates approach the male rates, the female SRCH and SLCT values are graded toward the male values³¹.

For each sex and birth cohort, the simulation process of assigning records as non-covered workers uses the following approach. This approach is repeated until the targeted number of non-covered workers is achieved.

1. One of the records, which is designated as one for legal immigrants, is randomly selected as the starting record.
2. Beginning with the starting record, each record is examined until a record that matches the SLCT criterion is found.
3. However, if the number of records examined equals the value of SRCH and no

³¹ This occurs when the female covered worker rate is greater than 80 percent of the male rate. When the female rate is equal to or greater than the male rate, the female SLCT and SRCH parameters are set equal to the male parameters. The parameters are linearly interpolated when the female covered worker rate is between 80 and 100 percent of the male rate.

record matches the SLCT criterion, then the record closest to the SLCT criterion is assigned no QC as a non-covered worker.

Initially, values for SRCH and SLCT are the same as those used in the prior Trustees Report. Adjustments to these values are only made when the results are not consistent with historical data.

The final step of the simulation process is to use QCDis to randomly assign QC of 0, 1, 2, 3 or 4 to the remaining covered worker records, which are not new net legal immigrants, for ages 14-69.

Determining Insured Status

Once the simulation process is complete, the insured status for each record at any age can be determined based on the total QC assigned up to that age. The simulated fully insured percentage (FSIM) is calculated as the percentage of the 30,000 simulated records meeting the QC requirements for insured status. The same calculation is applied to the disability-insured percentage (DSIM).

For each sex and cohort, FSIM and DSIM are determined at ages 13 to 69. FSIM is assumed to remain the same beyond age 69.

A multiplicative adjustment is applied to both FSIM and DSIM to reflect future changes in the proportion of ‘other immigrants’ in the Social Security area population (OIMPOP) and the proportion of ‘other immigrants’ whose earnings are posted to the MEF (ADJLPR) from that in the base year for each cohort. Thus the adjustment factor is

$$\frac{1 - [OIMPOP(\textit{projection year}) * (1 - ADJLPR(\textit{projection year}))]}{1 - [OIMPOP(\textit{base year}) * (1 - ADJLPR(\textit{base year}))]} \quad \text{by cohort and sex.}$$

A second additive adjustment, DINADD, is made to DSIM. It accounts for workers who fail to meet the requirement for disability-insured status solely because of having no earnings while receiving disability benefits. INSURED assumes that workers who have been on the disability rolls for more than 3 years would be in this situation³². Thus, DINADD is

$$\frac{\textit{\#of workers on the disability rolls more than 3 years}}{\textit{Social Security Area population}} \quad \text{by age, sex, and cohort.}$$

³² Those who are on the rolls for less than 4 years are assumed to meet the requirement for disability-insured status based on their earnings histories.

If the adjusted results for DSIM are not consistent with historical data, an additional age-sex-specific additive adjustment (DIADJ) is used to bring the simulated results in line with the historical estimates.

Finally, incorporation of Short-Range projections produces FPRO and DPRO. For the first 10 years, FPRO and DPRO are set equal to Short-Range estimates. The difference in terms of the percentage between the Long-Range and Short-Range projections at the end of 10th year is linearly phased out during the next ten years by cohort and sex. The Long-Range projections are assumed thereafter.

Number of Fully Insured and Disability Insured Workers

The numbers of Fully Insured and Disability Insured workers are obtained by applying FPRO and DPRO, respectively, to the Social Security area population. The result is an estimate of the number of people that are fully insured (FINPOP) and disability insured (DINPOP) by single year of age and sex, respectively. For a given age and sex, the proportion of the Social Security area population that is insured (FPRO) is assumed to be the same for each marital status.

3.2. DISABILITY

3.2.a. Overview

The Social Security Administration pays monthly disability benefits to workers who are insured for disability benefits and meet the definition of “disabled”. Provided that they meet certain requirements, spouses and children of disabled-worker beneficiaries may also receive monthly benefits.

DISABILITY projects the number of disabled-worker beneficiaries in current-payment status (DIB) at the end of each year by age at entitlement, sex, and duration from entitlement. The number of DIB at the end of each year is based on the number of disabled-worker beneficiaries who are currently entitled to benefits (CE). The number of CE at the end of year is obtained by adding the number of newly entitled CE (New Entitlements) during the year and subtracting the number of CE who leave the disability rolls (Exits) during the year to the number of CE at the end of the prior year. Disabled-worker beneficiaries who leave the disability rolls (Exits) do so by recovering from disabilities (Recoveries), by dying (Deaths), or by converting to retired worker status (Conversions). A disabled-worker beneficiary converts to retired worker status upon reaching Normal Retirement Age (NRA), the age at which a person first becomes entitled to unreduced retirement benefit.

DISABILITY also projects the number of future dependent beneficiaries of DIB by category, age, and sex. The six categories are minor child, student child, disabled adult child, young spouse, married aged spouse and divorced aged spouse. The numbers of dependent beneficiaries of DIB are obtained by multiplying the relevant subset of the SSA area population (Exposures) by a series of probabilities that relate to the regulations and requirements for obtaining benefits (Linkages).

$$\text{New Entitlements}(\text{year}) = \text{Exposure}_{\text{BOY}} \times \text{Incidence Rate}(\text{year}) \quad (3.2.1)$$

where BOY is beginning of year.

$$\text{Exits}(\text{year}) = \text{Recoveries}(\text{year}) + \text{Deaths}(\text{year}) + \text{Conversions}(\text{year}) \quad (3.2.2)$$

where Recoveries(year) = $\text{CE}_{\text{BOY}} \times \text{Recovery Rate}(\text{year})$
where Deaths(year) = $\text{CE}_{\text{BOY}} \times \text{Death Rate}(\text{year})$.

$$\text{CE}_{\text{EOY}} = \text{CE}_{\text{EOY-1}} + \text{New Entitlements}(\text{year}) - \text{Exits}(\text{year}), \quad (3.2.3)$$

where EOY is end of year, EOY-1 is end of prior year.

$$\text{Dependent Beneficiaries of DIB}_{\text{EOY}} = \text{Exposures}_{\text{EOY}} \times \text{Linkages}_{\text{EOY}} \quad (3.2.4)$$

3.2.b. Input Data

Trustees Assumptions

Each year, the Trustees set the assumption for the ultimate age-sex-adjusted incidence rate and the ultimate age-sex-adjusted recovery rate. This is achieved by setting the age-adjusted incidence rate and the age-adjusted recovery rate for each sex. The ultimate level for the age-sex-adjusted incidence rate and for the age-sex-adjusted recovery rate is set for the twentieth year of the projection period. Then for each age group and sex, the ultimate incidence rate is adjusted to match a “target” incidence rate. The following chart shows these “target” incidence rates by age group and sex.

Target Incidence Rates by Age Group

	Age Group										
	<19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69
Male	2.43	2.15	1.71	2.36	3.26	4.38	5.79	9.14	15.70	20.29	9.00
Female	1.65	1.50	1.72	2.46	3.39	4.56	6.03	9.41	14.09	14.84	8.34

Using a standard population of disability insured who are not in current pay as of December 1999, the age-sex-adjusted incidence rate for the 2009 Trustees Report was 5.6 per 1,000. Using a standard population of DIBs as of December 1999, the age-sex-adjusted recovery rate for the 2009 Trustees Report was 9.5 per 1,000.

Long-Range OCACT Data

All data is updated annually except those noted otherwise. Population data are as of December 31. Data as of December 31 of year $z-1$ are assumed equal to data as of January 1 of year z .

Demography

- January Social Security area population by age, sex, and marital status³³ (dimensioned (0:100,1:2,1:4)) for years 1971-2086.
- Total children by sex of parent, age of parent and age of child (dimensioned (1:2,19:71,0:18)) for years 1971-2086.
- Total married lives by age of husband crossed with age of wife (dimensioned (14:100,14:100)) for years 1971-2086.
- Average number of children under 16 per couple with children by age group (<25, 25-29, ..., 60-64) of head of household (dimensioned (1:9)) for years 1971-2086.
- Probabilities of death by sex, age and year (1:2,15:148,2009:2085).

Economics

- Unemployment rates by age group (16-19, 20-24, ..., 60-64), sex and year (1990:2019).

Beneficiaries (from INSURED subprocess #3.1)

³³ Single, married, widowed, divorced.

- Disability-insured population by age, sex and year (15:69,1:2,1969:2085).
- Fully insured population by age, sex and marital status (14:95,1:2,1:4) for years 1970-2085.

Other input data

- The December 2008 data from the Master Beneficiary Record (MBR) containing the number of DIB by duration of entitlement, age of entitlement, sex and time of year (BOY or EOY) (0:50,15:66,1:2,1:2).
- December data from the MBR containing the number of DIB by age, sex and year (15:66,1:2,1969:2008).
- December data from the MBR containing the number of dependent beneficiaries by age, sex of the account holder, and year for the following beneficiary categories.
 - 1) Minor child (0:17,1:2,1970:2008)
 - 2) Student child (18:21,1:2,1970:2008)
 - 3) Disabled adult child (age group 1:9³⁴,1:2,1970:2008)
 - 4) Young spouse (19:64,1:2,1970:2008)
 - 5) Married aged spouse (62:100,1:2,1970:2008)
 - 6) Divorced aged spouse (62:100,1:2,1970:2008)

Totals for each category are also read.

- December data from the MBR containing the number of DIB awards by age, sex and year (15:67,1:2,1970:2008).
- December data from the MBR containing (1) the number of DIB total terminations (recoveries and deaths) and (2) the number of conversions³⁵. These data are by sex and year (1:2,1970:2008).
- December data from the MBR containing the number of DIB deaths by age, sex and year (15:67,1:2,1975:2008).
- December data from the MBR containing the number of estimated DIB recoveries by age, sex and year (15:67,1:2,1975:2008).
- December data from the MBR containing (1) the number of DIB and (2) the number of old-age beneficiaries who at some point in time were converted to retired worker status. These data are by age, sex and year: (62:66,1:2,1970:2008) and (65:95+,1:2,1970:2008), respectively.
- December data from the MBR containing the number of DIB entitled to the Hospital Insurance portion of Medicare by age group (<25,25-29,...,60-64,65+), sex and year (2:11,1:2,1973:2008).
- Average incidence rates by age and sex (15:66,1:2) for the period 1993-2002 based on awards data from 1993-2007 (also known as the base incidence rates). These values are updated when time and data are available.
- Probability of death for DIB's – in a multiple-decrement environment by duration, age and sex (0:10,15:64,1:2) for the period 1996-2000 from Actuarial Study No. 118 (also

³⁴ Age groups 1 through 9 are 18-19, 20-24, 25-29, ..., 55-59.

³⁵ Conversions are DIB beneficiaries who become eligible for old-age benefits due to reaching the normal retirement age.

known as the base probability of death). These values are updated when time and data are available.

- Probability of recovery for DIB's – in a multiple-decrement environment by duration, age and sex (0:10,15:64,1:2) for the period 1996-2000 from Actuarial Study No. 118 (also known as the base probability of recovery). These values are updated when time and data are available.
- Death rate projection factors from the 2008 Trustees Report by age group (15-19,20-24,...,60-64), sex and year (1:2,1:10,2008:2085).
- Recovery rate projection factors and recovery rates from the 2008 Trustees Report by age group (15-19,20-24,...,60-64), sex and year (1:2,1:10,2008:2085).
- Initial Incurred but not reported (IBNR)³⁶ factors by duration, age and sex (0:10, 15: 69, 1:2) based on 2002 entitlements (awards data from 2002-2007).
- Ultimate IBNR factors by duration, age and sex (0:10, 15: 69, 1:2) based on 1993-2002 entitlements (awards data from 1993-2007). These values are updated when time and data are available. The IBNR calculation linearly grades between the initial and ultimate IBNR factors during the ten years from 2008-2017. The ultimate IBNR factor table applies for all years after 2017.
- For each year 2000-2085, (1) the Normal Retirement Age (NRA), (2) the proportion of DIBs who stay on the DI roll for that age, and (3) the proportion of DIBs who convert to an old-age benefit during that year for that age. These values are updated only when there is a change in the NRA or in the present law.
- For the years 2016-2026, weights for ages 61-66 for linear interpolating between one set of incidence rates at a particular NRA and another set of incidence rates at the next higher NRA. These values are updated only when there is a change in the NRA or in the present law.
- Special disability workload (SDW) of SSI beneficiaries by age, sex and year (29:54,1:2,2009:2010). The total values are from Short-Range and broken down by age and sex in a spreadsheet. These values will be updated every year until the SDW is done.
- The following linkages for the calculations of auxiliary beneficiaries; the probability that student is in an eligible school, the probability that adult child is disabled, the probability that beneficiary is not subject to the earnings test, and the probability that beneficiary was married 10 or more years are estimated and are updated when time and data are available.
- Retroactive factors³⁷ by year (1969:2007). These values are estimated using OCACT beneficiary data.
- APROJ factors by auxiliary beneficiary category (1:7) for years 2009-2028. These seven categories are; minor child, student child, disabled adult child, young wife, young husband, age wife and aged husband. These values are calculated using Short-Range numbers for auxiliary beneficiaries and pure Long- Range numbers for auxiliary beneficiaries.

³⁶ IBNR factors reflect the proportion of DIBs entitled to benefits who have been awarded since the year of their entitlement.

³⁷ Retroactive factors for each calendar year are the ratio of the total monthly payments to DIBs to the monthly DIBs in current payment status times the average DIB monthly benefit.

3.2.c. Development of Output

Equation 3.2.1 – New Entitlements

New entitlements are calculated by multiplying age-sex-specific incidence rates to the exposed population at the beginning of the year. The exposed population is the disability-insured population less the currently entitled population. Future age-sex-specific incidence rates are calculated by multiplying the base incidence rates by the incidence rate projection factors (IPROJGs). For the first ten years of the projection (short-range period), IPROJGs by 5-year age group and sex are obtained by using regression equations and independent variables of unemployment rates and previous years' incidence rates. The regression equations are described in detail in Appendix 3.2-1. Then the IPROJGs are run through the main model and the resulting incidence rates by age group and sex are analyzed. If necessary, an adjustment for each year is made to the female IPROJG for any age group in order to limit the ratio of (1) female incidence rate to (2) male incidence rate to 1.10. Again, the IPROJGs are run through the main model with the IPROJGs at the end of the short-range period (2018) held constant throughout the rest of the long-range period and the resulting ultimate age-adjusted incidence rates (years 2028 and later) for men and women are analyzed. If the ultimate age-adjusted incidence rates³⁸ for men (5.76) and women (5.55) are not achieved then adjustments are made to the IPROJGs in the twentieth year of the projection period (2028). The IPROJGs are further adjusted by age and sex to reach “target” incidence rates. These “target” incidence rates were determined based on a recent review of historical and projected award rates by age group and sex. Ultimate age-sex-specific incidence rates are reached in the twentieth year of the projection period. For projection periods between the tenth and twentieth years, the IPROJGs are linearly interpolated between the ultimate IPROJGs values and the IPROJGs values at the end of short-range period. Additional adjustments to the IPROJGs during the short-range period may be made for reconciliation between the long-range model and the short-range model on an aggregate level by sex. For the 2009 Trustees Report, IPROJG adjustments were made in the short-range period to anticipate a spike in projected disability claims resulting from the current economic recession.

Equation 3.2.2 – Exits

The long-range model projects three types of exits from the disability rolls; death, recovery and conversion to an old-age beneficiary upon reaching normal retirement age (NRA). Deaths and recoveries are projected by multiplying the beginning currently entitled population by the probabilities of death only and recovery only, $(q_x^{(d)})$ and $(q_x^{(r)})$, respectively. Projected $(q_x^{(d)})$ and $(q_x^{(r)})$ by age, sex, and duration are calculated by multiplying the base probabilities by the respective projection factors by age group and sex for that year.

³⁸ The ultimate age-adjusted incidence rates assumed in the 2008 TR were 5.61 for men and 5.30 for women. These rates were increased for the 2009 TR to incorporate a decrease in the number of disability-insureds due to a correction in the methodology for determining disability-insured status.

For the first ten years, the recovery projection factors (RPROJGs) by age group and sex are based on linear interpolation between an estimated starting level for the RPROJGs and an estimated tenth year projection target level for the RPROJGs. For each age group and sex, the starting RPROJGs are calculated the following way:

$$\text{RPROJG}^{\text{TR09}}(2008) = \text{RPROJG}^{\text{TR08}}(2008) \times \text{actual recovery rate}(2008) / \text{estimated recovery rate}^{\text{TR08}}(2008)$$

Because there is no apparent upward or downward trend, the average recovery rates for the last ten historical years are used as the target values for the 10th year (2018). Then, for each age group and sex, the tenth year level of the RPROJGs is calculated the following way:

$$\text{RPROJG}^{\text{TR09}}(2018) = \text{RPROJG}^{\text{TR09}}(2008) \times \text{target value recovery rate}(2018) / \text{actual recovery rate}(2008)$$

For the second 10 years of the projection period, the RPROJGs are linearly interpolated between the ultimate RPROJG value and the RPROJG value at the end of short-range period (2018). Ultimate recovery rates by sex are assumed to be reached in the twentieth year of the projection period. Ultimate recovery rates by age group and sex are determined by analyzing historical recovery rates. Additional adjustments may be made to the RPROJGs to reconcile with the short-range model.

For the first year of the projection period, the death projection factors (DPROJGs) by age group and sex are determined so that they achieve a targeted death rate. The targeted death rate is determined by fitting an exponential curve to historical death rates for DIBs by age group and sex (see Appendix 3.2-1). For the rest of the projection period, the DPROJGs are assumed to improve at the same rate as the general population for that age group and sex. The DPROJGs for each year by 5-year age group and sex are calculated the following way:

$$\text{DPROJG}(\text{year}) = \text{DPROJG}(\text{year}-1) \times \left(\frac{\sum_{\text{age}=x}^{x+4} q_x \times \frac{1999}{\text{age}}}{\sum_{\text{age}=x}^{x+4} q_x \times \frac{\text{year}-1}{\text{age}}} \right) / \left(\frac{\sum_{\text{age}=x}^{x+4} q_x \times \frac{1999}{\text{age}}}{\sum_{\text{age}=x}^{x+4} q_x \times \frac{\text{year}-1}{\text{age}}} \right)$$

Equation 3.2.3 – Disabled-Worker Beneficiaries

The projection begins with the latest data available from the mainframe of disabled-worker beneficiaries in current-payment status (DIBs who have been awarded benefits). This data is based on a 100 percent sample of the Master Beneficiary Record (MBR) at the end of the year. Disabled-worker beneficiaries are disaggregated by age at entitlement, sex and duration of entitlement. This population is converted to a currently entitled population by dividing each age, sex and duration cell by the appropriate duration-age-sex-year-specific IBNR factor. An iterative process begins with new entitlements added to and exits subtracted from the previous year's

currently entitled population to get the following year's currently entitled population with advancement of duration within the age of entitlement. This currently entitled population is then reduced by multiplying by the appropriate duration-age-sex-year-specific IBNR factor. The result is the following year's disabled-worker beneficiaries in current-payment status. The process is performed over each sex, age of entitlement and duration of entitlement throughout the projection period.

Equation 3.2.4 – Dependent Beneficiary of Disabled Workers

There are six dependent-beneficiary categories; minor child, student child, disabled adult child, young spouse, married aged spouse and divorced aged spouse. Projections are disaggregated by age of the beneficiary and sex of the account holder. Presented below are the linkages and the exposure used in each of the projections of dependent beneficiaries.

Minor Child

Exposure: Single SSA population by single ages 0-17
 Linkages: pMCAGA = Probability that parent is under NRA
 pMCDIA = Probability that parent is disability insured given
 that the parent is under NRA
 pMCDPA = Probability that disability insured parent under
 NRA is disabled
 MCRES = Residual Factor

Student Child

Exposure: Single SSA population by single ages 18-19
 Linkages: pSCAGA = Probability that parent is under NRA
 pSCDIA = Probability that parent is disability insured given
 that the parent is under NRA
 pSCDPA = Probability that disability insured parent under
 NRA is disabled
 pSCDPC = Probability that student is in an eligible school
 SCRES = Residual Factor

Disabled Adult Child

Exposure: Single SSA population by age groups 18-19, 20-24, 25-29,
 30-34, 35-39, 40-44, 45-49, 50-54, 55-59
 Linkages: pDCAGA = Probability that parent is under NRA
 pDCDIA = Probability that parent is disability insured given
 that the parent is under NRA
 pDCDPA = Probability that disability insured parent under
 NRA is disabled
 pDCDPC = Probability that adult child is disabled
 DCRES = Residual Factor

Young Spouse

Exposure: Married SSA population by sex and by single ages 20-64

Linkages: pYSAGA	= Probability that account holder is under NRA
pYSDIA	= Probability that account holder is disability insured given that the account holder is under NRA
pYSDPA under NRA is disabled	= Probability that disability insured account holder under NRA is disabled
pYSETB	= Probability that young spouse is not subject to earnings test
pYSMCB	= Probability that young spouse has a minor child beneficiary in his/her care
pYSDCB	= Probability that young spouse has a disabled child beneficiary in his/her care
YSRES	= Residual Factor

Married Aged Spouse

Exposure: Married SSA population by sex and by single ages 62-100

Linkages: pMSAGA	= Probability that account holder is under NRA
pMSDIA	= Probability that account holder is disability insured given that the account holder is under NRA
pMSDPA under NRA is disabled	= Probability that disability insured account holder under NRA is disabled
pMSFIB	= Probability that beneficiary is not insured
MSRES	= Residual Factor

Divorced Aged Spouse

Exposure: Divorced SSA population by sex and by single ages 62-100

Linkages: pDSDEA	= Probability that account holder is living
pDSAGA	= Probability that account holder is under NRA
pDSDIA	= Probability that account holder is disability insured given that the account holder is under NRA
pDSDPA under NRA is disabled	= Probability that disability insured account holder under NRA is disabled
pDSFIB	= Probability that beneficiary is not insured
pDSDMB	= Probability that beneficiary was married 10 or more years

The residual factors for each of the dependent categories are estimated using a 10-year Least Squares regression formula. These residual factor values are then held constant for the duration of the long range period. If the 10-year Least Squares method results in a negative residual factor, the last historical residual factor is held instead.

APROJ factors may be developed for a dependent beneficiary category to match short range results during the first 10 years of the projection period. These factors are phased out linearly over the second ten years of the projection period.

Appendix: 3.2-1

The following information provides details about the regression equations used in determining incidence rates and IPROJG values by age group and sex for the first ten years of the projection period.

Male

Independent Variable: 16-19 unemployment rates
Independent Variable: 15-19 incidence rates
Dependent Variable: 15-19 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.82125303
Standard Deviation: 0.24993771
Coefficient Intercept: 0.46216177
Coefficient Slope1: 0.05733928
Coefficient Slope2: 0.81395138

Independent Variable: 20-24 unemployment rates
Independent Variable: 20-24 incidence rates
Dependent Variable: 20-24 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.79677195
Standard Deviation: 0.21299341
Coefficient Intercept: 0.54574101
Coefficient Slope1: 0.12757822
Coefficient Slope2: 0.76723809

Independent Variable: 25-29 unemployment rates
Independent Variable: 25-29 incidence rates
Dependent Variable: 25-29 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.79909623
Standard Deviation: 0.10170368
Coefficient Intercept: 0.64624710
Coefficient Slope1: 0.18280553
Coefficient Slope2: 0.65486127

Independent Variable: 30-34 unemployment rates
Independent Variable: 30-34 incidence rates
Dependent Variable: 30-34 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.80188920
Standard Deviation: 0.12148817
Coefficient Intercept: 0.49270586
Coefficient Slope1: 0.15624312
Coefficient Slope2: 0.76982298

Independent Variable: 35-39 unemployment rates
Independent Variable: 35-39 incidence rates
Dependent Variable: 35-39 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.87019651
Standard Deviation: 0.12429186
Coefficient Intercept: 0.62926597
Coefficient Slope1: 0.36638844
Coefficient Slope2: 0.79045442

Independent Variable: 40-44 unemployment rates
Independent Variable: 40-44 incidence rates
Dependent Variable: 40-44 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.65680782
Standard Deviation: 0.21927462
Coefficient Intercept: 1.03009505
Coefficient Slope1: 0.32039690
Coefficient Slope2: 0.74803127

Independent Variable: 45-49 unemployment rates
Independent Variable: 45-49 incidence rates
Dependent Variable: 45-49 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.60690394
Standard Deviation: 0.26444653
Coefficient Intercept: 1.01527841
Coefficient Slope1: 0.41547723
Coefficient Slope2: 0.81381808

Independent Variable: 50-54 unemployment rates
Independent Variable: 50-54 incidence rates
Dependent Variable: 50-54 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.63347340
Standard Deviation: 0.39736325
Coefficient Intercept: 0.87000696
Coefficient Slope1: 0.67626900
Coefficient Slope2: 0.90039631

Independent Variable: 55-59 unemployment rates
Independent Variable: 55-59 incidence rates
Dependent Variable: 55-59 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.65459313
Standard Deviation: 0.54850119
Coefficient Intercept: 0.99321845

Coefficient Slope1: 0.73810236
Coefficient Slope2: 0.92521959

Independent Variable: 60-64 unemployment rates
Independent Variable: 60-64 incidence rates
Dependent Variable: 60-64 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.57064512
Standard Deviation: 0.52832774
Coefficient Intercept: 1.82287711
Coefficient Slope1: 0.40480236
Coefficient Slope2: 0.88212560

Female

Independent Variable: 16-19 unemployment rates
Independent Variable: 15-19 incidence rates
Dependent Variable: 15-19 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.88766357
Standard Deviation: 0.14346593
Coefficient Intercept: 0.35569255
Coefficient Slope1: 0.09303545
Coefficient Slope2: 0.80442814

Independent Variable: 20-24 unemployment rates
Independent Variable: 20-24 incidence rates
Dependent Variable: 20-24 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.78577726
Standard Deviation: 0.18370216
Coefficient Intercept: 0.36757482
Coefficient Slope1: 0.08747636
Coefficient Slope2: 0.80143272

Independent Variable: 25-29 unemployment rates
Independent Variable: 25-29 incidence rates
Dependent Variable: 25-29 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.82217911
Standard Deviation: 0.12972459
Coefficient Intercept: 0.42630887
Coefficient Slope1: 0.16999173
Coefficient Slope2: 0.77351844

Independent Variable: 30-34 unemployment rates
Independent Variable: 30-34 incidence rates
Dependent Variable: 30-34 incidence rates
Observation Period: 1994-2005

Adjusted R square: 0.73043907
Standard Deviation: 0.15207654
Coefficient Intercept: 0.51564908
Coefficient Slope1: 0.23663369
Coefficient Slope2: 0.79957410

Independent Variable: 35-39 unemployment rates
Independent Variable: 35-39 incidence rates
Dependent Variable: 35-39 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.71545135
Standard Deviation: 0.21164063
Coefficient Intercept: 0.84949642
Coefficient Slope1: 0.31870758
Coefficient Slope2: 0.76670238

Independent Variable: 40-44 unemployment rates
Independent Variable: 40-44 incidence rates
Dependent Variable: 40-44 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.78234758
Standard Deviation: 0.22725305
Coefficient Intercept: 1.32811371
Coefficient Slope1: 0.61065772
Coefficient Slope2: 0.73050540

Independent Variable: 45-49 unemployment rates
Independent Variable: 45-49 incidence rates
Dependent Variable: 45-49 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.65575649
Standard Deviation: 0.28168565
Coefficient Intercept: 1.42478421
Coefficient Slope1: 0.52178474
Coefficient Slope2: 0.77422799

Independent Variable: 50-54 unemployment rates
Independent Variable: 50-54 incidence rates
Dependent Variable: 50-54 incidence rates
Observation Period: 1994-2005
Adjusted R square: 0.37189356
Standard Deviation: 0.35934957
Coefficient Intercept: 2.22383408
Coefficient Slope1: 0.23273734
Coefficient Slope2: 0.76748013

Independent Variable: 55-59 unemployment rates
Independent Variable: 55-59 incidence rates

Dependent Variable: 55-59 incidence rates
 Observation Period: 1994-2005
 Adjusted R square: 0.57197346
 Standard Deviation: 0.41413357
 Coefficient Intercept: 1.35275350
 Coefficient Slope1: 0.82362260
 Coefficient Slope2: 0.90064335

Independent Variable: 60-64 unemployment rates
 Independent Variable: 60-64 incidence rates
 Dependent Variable: 60-64 incidence rates
 Observation Period: 1994-2005
 Adjusted R square: 0.57828474
 Standard Deviation: 0.36094908
 Coefficient Intercept: 3.09089172
 Coefficient Slope1: 0.39350220
 Coefficient Slope2: 0.76673100

The following information provides details about the exponentially fitted equations used in determining death rates by age group and sex for the first year of the projection period.

Male

Independent Variable: Year
 Independent Variable: 15-19 death rates
 Observation Period: 1999-2008
 Adjusted R square: 0.37799
 Standard Deviation: 0.22913
 Coefficient Intercept: 131.55250
 Coefficient Slope1: -0.06416

Independent Variable: Year
 Independent Variable: 20-24 death rates
 Observation Period: 1999-2008
 Adjusted R square: 0.60554
 Standard Deviation: 0.06372
 Coefficient Intercept: 56.80948
 Coefficient Slope1: -0.02700

Independent Variable: Year
 Independent Variable: 25-29 death rates
 Observation Period: 1999-2008
 Adjusted R square: 0.88445
 Standard Deviation: 0.04173
 Coefficient Intercept: 79.56979
 Coefficient Slope1: -0.03841

Independent Variable: Year

Independent Variable: 30-34 death rates
Observation Period: 1999-2008
Adjusted R square: 0.93613
Standard Deviation: 0.03160
Coefficient Intercept: 83.14833
Coefficient Slope1: -0.04011

Independent Variable: Year
Independent Variable: 35-39 death rates
Observation Period: 1999-2008
Adjusted R square: 0.90974
Standard Deviation: 0.02674
Coefficient Intercept: 59.45369
Coefficient Slope1: -0.02819

Independent Variable: Year
Independent Variable: 40-44 death rates
Observation Period: 1999-2008
Adjusted R square: 0.75268
Standard Deviation: 0.03092
Coefficient Intercept: 39.51385
Coefficient Slope1: -0.01814

Independent Variable: Year
Independent Variable: 45-49 death rates
Observation Period: 1999-2008
Adjusted R square: 0.79112
Standard Deviation: 0.02410
Coefficient Intercept: 34.88004
Coefficient Slope1: -0.01572

Independent Variable: Year
Independent Variable: 50-54 death rates
Observation Period: 1999-2008
Adjusted R square: 0.43620
Standard Deviation: 0.02315
Coefficient Intercept: 17.98176
Coefficient Slope1: -0.00719

Independent Variable: Year
Independent Variable: 55-59 death rates
Observation Period: 1999-2008
Adjusted R square: 0.81970
Standard Deviation: 0.02271
Coefficient Intercept: 36.13212
Coefficient Slope1: -0.01619

Independent Variable: Year

Independent Variable: 60-64 death rates
Observation Period: 1999-2008
Adjusted R square: 0.92794
Standard Deviation: 0.01793
Coefficient Intercept: 46.61078
Coefficient Slope1: -0.02135

Female

Independent Variable: Year
Independent Variable: 15-19 death rates
Observation Period: 1999-2008
Adjusted R square: 0.20648
Standard Deviation: 0.43257
Coefficient Intercept: 177.21757
Coefficient Slope1: -0.08706

Independent Variable: Year
Independent Variable: 20-24 death rates
Observation Period: 1999-2008
Adjusted R square: 0.65879
Standard Deviation: 0.10052
Coefficient Intercept: 97.69050
Coefficient Slope1: -0.04744

Independent Variable: Year
Independent Variable: 25-29 death rates
Observation Period: 1999-2008
Adjusted R square: 0.93097
Standard Deviation: 0.03731
Coefficient Intercept: 93.51550
Coefficient Slope1: -0.04544

Independent Variable: Year
Independent Variable: 30-34 death rates
Observation Period: 1999-2008
Adjusted R square: 0.80146
Standard Deviation: 0.04338
Coefficient Intercept: 61.04544
Coefficient Slope1: -0.02918

Independent Variable: Year
Independent Variable: 35-39 death rates
Observation Period: 1999-2008
Adjusted R square: 0.85658
Standard Deviation: 0.02815
Coefficient Intercept: 48.66417
Coefficient Slope1: -0.02293

Independent Variable: Year
Independent Variable: 40-44 death rates
Observation Period: 1999-2008
Adjusted R square: 0.75450
Standard Deviation: 0.03274
Coefficient Intercept: 41.57478
Coefficient Slope1: -0.01929

Independent Variable: Year
Independent Variable: 45-49 death rates
Observation Period: 1999-2008
Adjusted R square: 0.86409
Standard Deviation: 0.02027
Coefficient Intercept: 37.21146
Coefficient Slope1: -0.01702

Independent Variable: Year
Independent Variable: 50-54 death rates
Observation Period: 1999-2008
Adjusted R square: 0.93244
Standard Deviation: 0.01738
Coefficient Intercept: 46.15317
Coefficient Slope1: -0.02141

Independent Variable: Year
Independent Variable: 55-59 death rates
Observation Period: 1999-2008
Adjusted R square: 0.96585
Standard Deviation: 0.01248
Coefficient Intercept: 47.39754
Coefficient Slope1: -0.02197

Independent Variable: Year
Independent Variable: 60-64 death rates
Observation Period: 1999-2008
Adjusted R square: 0.94662
Standard Deviation: 0.01638
Coefficient Intercept: 49.29775
Coefficient Slope1: -0.02285

3.3. Old-Age and Survivors Insurance

3.3.a. Overview

Every month, the Social Security program pays benefits to retired workers and their dependents. It also provides benefits to eligible dependents of deceased workers. The OLD-AGE AND SURVIVORS subprocess projects the number of people expected to receive benefits over the next 75 years. The projection method is very similar to the method used for dependent beneficiaries of disabled workers in the DISABILITY subprocess. The projection of beneficiaries is computed by multiplying a subset of the Social Security area population by a series of probabilities of the conditions that a person must meet to receive benefits. The main program receives all necessary input data and performs all preliminary calculations. It then calls each individual beneficiary type subroutine where all beneficiary calculations are made.

Retired workers and their dependent beneficiaries are categorized as follows:

- retired workers (*RWN*) by age (62-95+), sex, and marital status (single, married, widowed, divorced)
- aged spouses of retired workers (*ASRWN*), by age (62-95+), sex of the account holder, and marital status of the beneficiary (married, divorced)
- young spouses of retired workers (*YSRWN*) by age-group (under 25, 25-29, ..., 65-69) and sex of the account holder
- minor, student, and disabled adult children of retired workers (*MCRWN*, *SCRWN*, and *DCRWN*, respectively) by age of the child (0-17 for minor, 18-19 for student, age groups 18-19, 20-24, ..., 55-59, 60+ for disabled adult) and sex of the account holder

Dependent beneficiaries of deceased workers include:

- aged spouses of deceased workers, *ASDWN*, by age (60-95+), sex of the account holder, marital status (widowed, divorced) and insured status (insured, uninsured)
- disabled spouses of deceased workers (*DSDWN*) by age (50-69), sex of the account holder and marital status (widowed, divorced)
- young spouses of deceased workers (*YSDWN*) by age-group (under 25, 25-29, ..., 65-69), sex of the account holder and marital status of the beneficiary (widowed, divorced)
- minor, student, and disabled adult children of deceased workers (*MCDWN*, *SCDWN*, and *DCDWN*, respectively) by age of the child (0-17 for minor, 18-19 for student, age groups 18-19, 20-24, ..., 55-59, 60+ for disabled adult) and sex of the account holder

Lastly, the number of deaths of insured workers (*LUMSUM*) is estimated by 5-year age group (20-24, 25-29, ..., 80-84, 85+) and sex.

Equations 3.3.1-13 indicate the flow of calculations of beneficiaries.

$$ASDWN = ASDWN(\cdot) \quad (3.3.1)$$

$$RWN = RWN(\cdot) \quad (3.3.2)$$

$$ASRWN = ASRWN(\cdot) \quad (3.3.3)$$

$$DSDWN = DSDWN(\cdot) \quad (3.3.4)$$

$$MCRWN = MCRWN(\cdot) \quad (3.3.5)$$

$$MCDWN = MCDWN(\cdot) \quad (3.3.6)$$

$$SCRWN = SCRWN(\cdot) \quad (3.3.7)$$

$$SCDWN = SCDWN(\cdot) \quad (3.3.8)$$

$$DCRWN = DCRWN(\cdot) \quad (3.3.9)$$

$$DCDWN = DCDWN(\cdot) \quad (3.3.10)$$

$$YSRWN = YSRWN(\cdot) \quad (3.3.11)$$

$$YSDWN = YSDWN(\cdot) \quad (3.3.12)$$

$$LUMSUM = LUMSUM(\cdot) \quad (3.3.13)$$

The appendix 3.3-1 at the end of this section provides a listing with explanation of the acronyms used in this documentation.

3.3.b. Input Data

All data are updated annually unless otherwise noted. Timing of data received is denoted ‘BOY’ (beginning of year) or ‘EOY’ (end of year).

Long-Range OCACT Data

Demography

- Social Security area population by year (EOY 1970-2085), single year of age (0-100+), sex, and marital status (single, married, widowed, divorced)
- Deaths by year (during years 2008-2085), age group (20-24, ..., 80-84, 85+) and sex
- Average number of children per family by year (EOY 1970-2085), and age group of the householder (20-24, ..., 60-64)
- Children by year (EOY 1970-2085), single year of age (0-17), sex of primary account holder (parent), status of primary (retired or deceased), and age of the other parent (15-24, 25-29, ..., 65-69)
- Married couples by year (EOY 1970-2085), age of husband (62-95+) and age of wife (62-95+)

- Persons with an aged spouse by year (EOY 1970-2085), age group (15-24, 25-29,....,65-69) and sex

Economics

- Covered wages and employment in the Federal Civilian and State and Local Sectors (during years 1998-2085)
- Labor force participation rates for age 62 by year (during years 1970-2085) and sex

Beneficiaries

- Fully insured persons by year (EOY 1970-2085), age (14-95+), sex, and marital status (single, married, widowed, divorced)
- Disabled-worker beneficiaries in current pay by year (EOY 1970-2085), age (62-66) and sex
- Converted DI to OAI beneficiaries by year (EOY 1970-2085), age(65-95+) and sex
- Disability prevalence rates by year (EOY 1970-2085), age (50-66) and sex

Short-Range OCACT Data

- Insured aged spouses of deceased workers by year (EOY 1998-2008), age (60-95+) and sex
- The following is received for EOY 2007 and 2008:
 - a. Aged spouses of deceased workers by age (60-95+), sex and marital status (widowed, divorced)
 - b. Retired workers by age (62-95+) and sex
 - c. Aged spouses of retired workers by age (62-95+), sex and marital status (married, divorced)
 - d. Disabled widow(er)s by age (50-64), sex and marital status (widowed, divorced)
 - e. Minor children by age (0-17), sex of parent and status of parent (retired, deceased)
 - f. Student children by age (18-19), sex of parent and status of parent (retired, deceased)
 - g. Disabled adult children by age (20-95+), sex of parent and status of parent (retired, deceased)
 - h. Young spouses of retired workers by age group (under 25, 25-29,....,60-64) and sex
 - i. Young spouses of deceased workers by age group (under 25, 25-29,....,65-69), sex and marital status (widowed, divorced)
 - j. Total parent beneficiaries

Note: Each year, data for the most recent historical year is appended.

- The following is received for EOY 2018:
 - a. Retired workers by age group (62-64, 65-69) and sex
 - b. Insured widows by age group (60-64,....,80-84,85+)
 - c. Uninsured widows by age group (60-64, 65+)
 - d. Total disabled widows

- e. Female young spouses of deceased workers
 - f. Female aged spouses of retired workers by age group (62-64, 65+)
 - g. Female young spouses of retired workers
 - h. Minor children by status of parent
 - i. Student children by status of parent
 - j. Disabled adult children by status of parent
- Total amount of lump-sum death payments during 2007

Other Input Data

- For EOY 1970-2004, obtained from the MBR10PER dataset on the mainframe:
 - a. Aged spouses of deceased workers by age (60-95+), sex and marital status (widowed, divorced)
 - b. Retired workers by age (62-95+) and sex
 - c. Aged spouses of retired workers by age (62-95+), sex and marital status (married, divorced)
 - d. Disabled widow(er)s by age (50-64), sex and marital status (widowed, divorced)
 - e. Minor children by age (0-17), sex of parent and status of parent (retired, deceased)
 - f. Student children by age (18-19), sex of parent and status of parent (retired, deceased)
 - g. Disabled adult children by age (20-95+), sex of parent and status of parent (retired, deceased)
 - h. Young spouses of retired workers by age group (under 25, 25-29,....,60-64) and sex
 - i. Young spouses of deceased workers by age group (under 25, 25-29,....,65-69), sex and marital status (widowed, divorced)
 - j. Total parent beneficiaries

Note: this data will not be updated
- Number of beneficiaries with benefits withheld due to receipt of a significant government pension by sex and marital status (married, widowed) for EOY 2007 from the 2008 Annual Statistical Supplement
- Age distribution of beneficiaries with benefits withheld due to receipt of a significant government pension by age (60-95+) and sex, computed as an average from the 2001 through 2005 WEP 100-percent sample (Note: these values are only updated about every five years and will be based on averages from five years of data.)
- Proportions of disabled adult children of retired and deceased workers (proportioned by age and sex of the child) from the 2003 MBR ten-percent sample. (Note: The RSB program calculates disabled adult children by sex of the primary account holder, not by sex of the child. The RSB program outputs a file which is used for Annual Update #9, which calculates beneficiaries by sex. Therefore the 2003 proportions are applied to

estimate the breakdown of disabled adult children by sex of the child. This input file will not be updated.). Not used for SOSI.

- Schedule of normal retirement age (*NRA*), delayed retirement credit, and actuarial reduction factors for ages more than 3 years below *NRA* and less than 3 years below *NRA* for years 1970-2085 from the Social Security website (Note: these values are only updated when there is a Social Security law change regarding the *NRA*)
- Elasticity factors by age (65-69) and sex for the elimination of the earnings test after *NRA*
- Prevalence rate regression coefficients (slope and y-intercept value by sex)
- Regressed prevalence rate by sex for the most recent historical year
- Adjustment factors which account for the difference between estimated and actual historical retired worker prevalence rates by year (EOY 1970-2085), age (63-69) and sex
- Adjustment factors which account for the difference between projected beneficiary values for the tenth year of the projection period made by the Long-Range and Short-Range offices. Factors are computed for:
 - a. Retired workers by age group (62-64, 65-69) and sex
 - b. Insured widows by age group (60-64, ..., 80-84, 85+)
 - c. Uninsured widows by age group (60-64, 65+)
 - d. Total disabled widows
 - e. Female young spouses of deceased workers
 - f. Female aged spouses of retired workers by age group (62-64, 65+)
 - g. Female young spouses of retired workers
 - h. Minor children by status of parent
 - i. Student children by status of parent
 - j. Disabled adult children by status of parent

3.3.c. *Development of Output*

Several acronyms are used to describe the equations presented below. Acronyms that are not preceded with a subscript generally refer to the number of beneficiaries. For example, *RWN* refers to the number of retired workers. Acronyms that are preceded with a '*p*' refer to probabilities. For example, pRW_{FLA} refers to the probability that a person is fully insured.

Equation 3.3.1 – Aged Spouses of Deceased Workers (ASDWN)

The number of aged spouses of deceased workers (widow(er)s), along with all linkage factors, are projected by age, sex of the account holder (sa=1 for male, sa=2 for female), marital status and insured status. Age ranges from 60 to 95+, marital status includes widowed (mb=1) and divorced (mb=2), and insured status includes insured (in=1) and uninsured (in=2). Note that all variables preceded by the letter *p* are calculated probabilities. The projected number of aged spouses of deceased workers is calculated as follows:

$$ASDWN = ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FIA} \times pASDW_{MBB} \times pASDW_{FIB} \times pASDW_{GPB} \times pASDW_{RES} \quad (3.3.1)$$

$ASDW_{POP}$ represents the subset of the population from which these beneficiaries are drawn and is set equal to the Social Security area population ($SSAPOP_{mb}$) for each possible marital status.

$$ASDW_{POP} = SSAPOP_{mb}$$

$pASDW_{DEA}$ represents the probability that the primary account holder (PAH) is deceased. For the widowed population, this factor is set equal to one. For the divorced population, this factor is set equal to the portion of the total widowed ($SSAPOP_{wid}$) and married ($SSAPOP_{mar}$) population who are widowed.

$$pASDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pASDW_{FIA}$ represents the probability that the PAH was fully insured at death. For a given age of widow, AW , it is assumed that the age of her deceased husband, AH , ranges from $AW-6$ to $AW+12$ with a lower and upper bound of 60 and 95+. Further, it is assumed that the more likely age of the husband is $AW+3$. For each age, $pASDW_{FIA}$ is calculated as a weighted average of the portion of the Social Security area population who are fully insured at each possible age of the husband ($FINS_{AH}$). For example, for a widow age 70, it is assumed that the age of her husband is between 64 and 82, therefore the weighted average of the portion of the population who are fully insured males is calculated, applying the highest weight of 10 to age 73, and a linearly reduced weight to zero for each age above and below 73. The same concept is used for widow(er)s with the assumption that the age of his deceased wife ranges from $AH-12$ to $AH+6$, with a greater likelihood of her age being $AH-3$. Let $WEIGHT$ represent the specific weight applied to each potential age of the spouse.

$$WEIGHT_{AH} = 10 - |AW + 3 - AH|$$

$$WEIGHT_{AW} = 10 - |AH - 3 - AW|$$

$$pASDW_{FLA} = \left\{ \begin{array}{l} \frac{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH} \times FINS_{AH}}{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH}}, \quad sa = 1 \\ \frac{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW} \times FINS_{AW}}{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW}}, \quad sa = 2 \end{array} \right.$$

$pASDW_{MBB}$ represents the probability that the widow(er) is not receiving a young-spouse benefit for the care of a child. A young-spouse benefit can be received up to age 69 if all other eligibility requirements are met. Since the minimum age requirement to receive a widow(er) benefit is 60, it is necessary to remove those receiving a young-spouse benefit ($YSDWN^{ab}$), where ab represents the 5-year age bracket³⁹. A uniform breakdown is assumed to divide the age groups into single-age estimates.

$$FACTOR_{age} = \begin{cases} 1, & 65 \leq age < NRA \text{ and } in = 2 \\ 0, & \text{elsewhere} \end{cases}$$

$$pASDW_{MBB} = \left\{ \begin{array}{l} 1 - \frac{0.2 \times YSDWN^{60-64}}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FLA}}, \quad \begin{array}{l} age = 60-64 \\ \text{and } in = 2 \end{array} \\ 1 - \frac{YSDWN^{65-69}}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FLA}} * \frac{FACTOR_{age}}{\sum_{65}^{69} FACTOR_{age}}, \quad \begin{array}{l} 65 \leq age < NRA \\ \text{and } \underline{in} = 2 \end{array} \\ 1, \quad \begin{array}{l} age = 60-69 \\ \text{and } in = 1 \end{array} \end{array} \right.$$

³⁹ There are no young spouses at NRA or above.

$pASDW_{FIB}$ represents the probability that the aged widow(er) is fully insured. For insured widow(er)s, $pASDW_{FIB}$ is the portion of the Social Security area population that is fully insured (*FINS*) at each age, sex, and marital status. For uninsured widow(er)s, $pASDW_{FIB}$ is simply one minus the probability for insured widow(er)s.

$$pASDW_{FIB} = \begin{cases} \frac{FINS}{SSAPOP_{mb}}, & in = 1 \\ 1 - \frac{FINS}{SSAPOP_{mb}}, & in = 2 \end{cases}$$

$pASDW_{GPB}$ represents the probability that the aged-widow(er)'s benefits are not withheld or not offset totally because of receipt of a significant government pension based on earnings in noncovered employment. According to the 1977 amendments, Social Security benefits are subject to reduction by up to two-thirds of non-covered government pension. *GPWHL* represents the total number of widow(er) beneficiaries (for all ages) expected to receive a significant government pension. *rGPOAGE* represents the ratio of the total for each given age. If a person is insured, this implies that he/she is eligible to receive Social Security benefits based on his/her own earnings regardless of a government pension. Therefore no factor is applied.

$$pASDW_{GPB} = \begin{cases} 1 & in = 1 \text{ OR } year \leq 1978 \\ 1 - \frac{rGPOAGE \times GPWHL}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FLA} \times pASDW_{MBB} \times pASDW_{FIB}}, & in = 2 \end{cases}$$

$pASDW_{RES}$ represents the probability that a widow(er), who is eligible to receive widow(er)'s benefits, will actually receive benefits. In particular, for $in = 1$, this factor is equivalent to the probability that a widow(er) eligible to receive his/her own retired-worker benefits would instead apply for and receive widow(er) benefits. For all historical years, $pASDW_{RES}^{year}$ is calculated as the ratio of *ASDWN*, the actual number of widow(er)s, to the number of persons meeting all previously mentioned requirements by age, sex, insured status, and marital status.

$$pASDW_{RES}^{year} = \frac{ASDWN}{ASDW_{POP} \times pASDW_{DEA} \times pASDW_{FLA} \times pASDW_{MBB} \times pASDW_{FIB} \times pASDW_{GPB}}, \quad year < TRYR,$$

where TRYR is the Trustees Report year.

For each age, sex, insured status, and marital status, a least squares regression is used over the last ten years of historical data to determine a starting value in TRYR-1 for $pASDW_{RES}^{year}$ from which future values are projected. In addition, for each sex, insured status, and marital status, the regressed values of $pASDW_{RES}^{TRYR-1}$ are graduated over age using a

weighted minimized third-difference formula to produce $ESTRES^{ASDW}$. $ESTRES^{ASDW}$ are the preliminary estimates of $pASDW_{RES}^{TRYR+9}$, the values in the tenth year of the projection period. In addition, adjustments by age group (60-64, 65-69, 70-74, 75-79, 80-84, 85+ for insured; 60-64, 65+ for uninsured), $SRADJ^{ASDW}$, are applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pASDW_{RES}^{year}$ for intermediate years are linearly interpolated between $pASDW_{RES}^{TRYR-1}$ and $pRW_{RES}^{N,TRYR+9}$ (equal to $ESTRES^{ASDW} * SRADJ^{ASDW}$). After the 10th year, these adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to long-range projections.

Equation 3.3.2 – Retired Workers (RWN)

The numbers of retired-worker beneficiaries, along with all linkage factors, are projected by age, sex, and marital status. Age ranges from 62 to 95+, and marital status includes single, married, widowed, and divorced ($ms = 1$ to 4). The projected number of retired-worker beneficiaries is calculated as follows:

$$RWN = RW_{POP} \times pRW_{FIA} \times pRW_{DBB} \times pRW_{WBB} \times pRW_{RES} \quad (3.3.2)$$

RW_{POP} represents the subset of the population from which these beneficiaries are drawn and is set equal to the Social Security area population ($SSAPOP_{ms}$) for $ms = 1$ to 4.

$$RW_{POP} = SSAPOP_{ms}$$

pRW_{FIA} represents the probability that the primary account holder (PAH) is insured, and is set equal to the portion of the Social Security area population that is fully insured ($FINS$) for $ms=1$ to 4.

$$pRW_{FIA} = \frac{FINS}{RW_{POP}}$$

pRW_{DBB} represents the probability that the PAH is not receiving a disabled-worker or disability-conversion benefit. This factor is set equal to the portion of fully insured workers who are neither disabled-worker beneficiaries nor converted from disabled-worker beneficiaries ($DIBCON$). $ASDWN$ represents the number of aged spouses of deceased workers.

$$pRW_{DBB} = \begin{cases} 1 - \frac{DIBCON}{RW_{POP} \times pRW_{FIA}}, & ms = 1-2 \\ \left(1 - \frac{DIBCON + ASDWN}{RW_{POP} \times pRW_{FIA}}\right)^{\left(\frac{DIBCON}{DIBCON + ASDWN}\right)}, & ms = 3-4 \end{cases}$$

pRW_{WBB} represents the probability that the PAH is not receiving a widow(er) benefit, and is set equal to the portion of fully insured workers that is not aged spouses of deceased workers.

$$pRW_{WBB} = \begin{cases} 1, & ms = 1-2 \\ \left(1 - \frac{DIBCON + ASDWN}{RW_{POP} \times pRW_{FIA}}\right)^{\left(\frac{ASDWN}{DIBCON + ASDWN}\right)}, & ms = 3-4 \end{cases}$$

$pRW_{RES}^{N,year}$ represents the retirement prevalence rate, which is the probability that a fully insured worker not receiving disability or widow(er)'s benefits would receive retired-worker benefits as of the given age, N , for the given year. In order to estimate the future prevalence rate, the program first calculates the historical values of $pRW_{RES}^{N,year}$.

For each historical year and sex, $pRW_{RES}^{N,year}$ is calculated as the ratio of RWN , the actual number of retired workers, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status.

$$pRW_{RES}^{N,year} = \frac{RWN}{RW_{POP} \times pRW_{FIA} \times pRW_{DBB} \times pRW_{WBB}}, N = 62-95+ \text{ and } year < \text{TRYR}$$

Historical prevalence rates at age 62 and labor force participation rates ($LFPR^{year}$) at age 62, by sex, follow an inverse linear relationship over the historical period. This relationship is assumed to hold in the projection period, and therefore used to calculate $REGPR^{year}$, the regressed prevalence rate based on the projected $LFPR^{year}$ at age 62 for each year and sex. Note that prevalence rates are calculated on a cohort basis⁴⁰. The regression equation used to estimate the prevalence rates is:

⁴⁰ For example, to calculate the projected number of 65 year olds in a given year, the prevalence rate at age 62 is needed. This is actually the prevalence rate that occurred three years ago at age 62.

$REGPR^{year} = -0.97353 \times LFPR^{year} + 0.91037$ for male with an R^2 value of 0.90363, and

$REGPR^{year} = -0.82426 \times LFPR^{year} + 0.77462$ for female with an R^2 value of 0.81395.

The future prevalence rate at age 62, $pRW_{RES}^{62,year}$, is then set equal to the sum of the regressed prevalence rate ($REGPR^{year}$) and $ERROR$, the difference between the actual prevalence rate and the regressed prevalence rate in the most recent historical year, which is phased out linearly.

$$pRW_{RES}^{62,year} = REGPR^{year} + (ERROR) \times \max\left(0, \frac{TRYR + 9 - year}{10}\right), N = 62 \text{ and } year \geq TRYR - 1$$

To compute $pRW_{RES}^{N,year}$ for ages 63 to 69 in the projection period, several preliminary variables must be calculated. These include:

- $MBAPIA_N$, for $N = 62, 70$ (same for both sexes),
- $ESTPR_N^{year}$, for $N = 63, 69$ and by sex,
- $DIFFADJ_N$, for $N = 63, 69$ and by sex,
- $ESTPR2_N^{year}$ for $N = 63, 69$ and by sex, and
- $EARN_N$, for $N = 65, 69$ and by sex.

$MBAPIA_N$ is the ratio of the monthly benefit amount (MBA) to the primary insurance amount (PIA) at age N and is calculated on a cohort basis for $N = 62, 70$. The calculation of $MBAPIA_N$ is based on the normal retirement age (NRA), delayed retirement credits (DRC), and actuarial reduction factors; $ARFLE3$ when the difference between NRA and age at retirement is less than 3, and $ARFGT3$ when the difference is greater than 3 within each cohort. If a person retires after NRA , his/her benefits are increased by DRC for each year the age exceeds NRA . If a person retires before NRA , his/her benefits are decreased by $ARFLE3$ for each of the first three years that NRA exceeds the age, and further decreased by $ARFGT3$ for any remaining years.

$$MBAPIA_N = \begin{cases} 1 + (N - NRA) \times DRC, & N \geq NRA \\ 1 - (NRA - N) \times ARFLE3, & NRA - 3 \leq N < NRA \\ 1 - 3 \times ARFLE3 - (NRA - 3 - N) \times ARFGT3, & N < NRA - 3 \end{cases}$$

$ESTPR_N^{year}$, the estimated prevalence rate at age N , is then calculated as the prevalence rate at age 62 ($pRW_{RES}^{62,year-(N-62)}$) plus an estimate on the expected portion of the remaining probability ($1 - pRW_{RES}^{62,year-(N-62)}$), that a potential retired worker will actually retire by that given age. This estimate is based on $MBAPIA_N$, assuming that the retirement decision by a worker is totally and completely influenced by the expected change in the portion of PIA that is payable at each age relative to the potential change after the initial eligibility.

$$ESTPR_N^{year} = pRW_{RES}^{62, year-(N-62)} + (1 - pRW_{RES}^{62, year-(N-62)}) \times \frac{MBAPIA_N - MBAPIA_{62}}{MBAPIA_{70} - MBAPIA_{62}}, \quad N = 63-69$$

In the first year of the projection period, an adjustment ($DIFFADJ_N$) is made which accounts for the difference between the actual and estimated prevalence rate at each age in the most recent historical years; For ages 63 and 64, the value used beginning in TRYR is the average of the last 5 years' differences between the actual and estimated PR. This value is held constant throughout the projection period. For ages 65 to 69 the difference between the actual and estimated prevalence rate is computed for 1999, the last year prior to the elimination of the earnings test at NRA. The ultimate value of $DIFFADJ_N$, reached in 2010, is calculated as the average difference of the five years; 1995 through 1999. The values for $DIFFADJ_N$ for years 1999 through 2009 are linearly interpolated.

$$ESTPR2_N = ESTPR_N + DIFFADJ_N$$

Another adjustment ($EARN_N$) is needed for ages 65 to 69 which take into account the scheduled increases in NRA and the elimination of the earnings test at NRA. This adjustment measures what portion of the remaining workers, who would not have applied for benefits if the earnings test legislation in 2000 had not have been enacted, decide to receive benefits and continue to work.

$$pRW_{RES}^{N, year} = ESTPR2_N + (1 - ESTPR2_N) \times EARN_N, \quad N = 65-69 \text{ and } year \geq \text{TRYR}$$

For age 70, it is assumed that the values of the latest actual $pRW_{RES}^{N, year}$ by sex would change linearly to the ultimate level of 0.995 for male and 0.99 for female over the first 20 years of the projection period.

$$pRW_{RES}^{70, year} = \begin{cases} 0.995 - [0.995 - pRW_{RES}^{70, \text{TRYR}-1}] \times \max\left(0, \frac{\text{TRYR} + 19 - year}{20}\right), & sa = 1 \text{ and } year \geq \text{TRYR} \\ 0.99 - [0.99 - pRW_{RES}^{70, \text{TRYR}-1}] \times \max\left(0, \frac{\text{TRYR} + 19 - year}{20}\right), & sa = 2 \text{ and } year \geq \text{TRYR} \end{cases}$$

For ages 71 and older, $pRW_{RES}^{N, year}$ is assumed to stay constant at the level when the age was 70 because there is no incentive to delay applying for benefits beyond age 70.

$$pRW_{RES}^{N, year} = pRW_{RES}^{70, year-(N-70)}, \quad \text{for } N = 71-95+ \text{ and } year \geq \text{TRYR}$$

In addition, adjustments by age group (62-64, 65-69) and sex, $SRADJ^{RW}$, are applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. These adjustments are also applied to $pRW_{RES}^{N,year}$ for all years after TRYR+9. The values of $pRW_{RES}^{N,year}$ for intermediate years are linearly interpolated between $pRW_{RES}^{N,TRYR-1}$ and $pRW_{RES}^{N,TRYR+9}$.

For each age, retired workers are further broken down by age at entitlement, AE , by multiplying the number of retired workers at age N by the ratio of the incidence rate at AE ($N - AE$ years prior) to the prevalence rate at age N .

$${}_{AE}^N RWN^{year} = {}^N RWN^{year} \times \frac{{}_{AE} INCRATE^{year-(N-AE)}}{pRW_{RES}^{N,year}}, AE \leq N$$

where the incidence rate for a given $AE = N$ and year is calculated as the change in the prevalence rate at age N to the prevalence rate at age $N-1$ in the previous year.

$${}_{AE} INCRATE^{year} = \begin{cases} pRW_{RES}^{N,year}, & \text{for } N = AE = 62 \\ pRW_{RES}^{N,year} - pRW_{RES}^{N-1,year-1}, & \text{for } 63 \leq N = AE \leq 69 \\ 1 - pRW_{RES}^{N-1,year-1}, & \text{for } N = AE = 70 \end{cases}$$

Equation 3.3.3 – Aged Spouses of Retired Workers (ASRWN)

The number of aged spouses of retired workers, along with all linkage factors, are projected by age, sex of the account holder ($sa = 1,2$), and marital status of the beneficiary. Age ranges from 62 to 95+, and marital status includes married ($mb = 1$) and divorced ($mb = 2$). The projected number of aged spouses of retired workers is calculated as follows:

$$ASRWN = ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA} \times pASRW_{MBB} \times pASRW_{FIB} \times pASRW_{GPB} \times pASRW_{RES} \quad (3.3.3)$$

$ASRW_{POP}$ represents the subset of the population from which these beneficiaries are drawn and is set equal to the Social Security area population ($SSAPOP_{mb}$) for $mb = 1,2$.

$$ASRW_{POP} = SSAPOP_{mb}$$

$pASRW_{DEA}$ represents the probability that the PAH is not deceased. For the married population, no factor is applied. For the divorced population, the factor is set equal to the portion of the total married and widowed population who are married.

$$pASRW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (married)} \\ \frac{SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pASRW_{AGA}$ represents the probability that the PAH is of the required age, and is set equal to the portion of the married population with a spouse (PAH) at least age 62 ($MAR62PLUS$).

$$pASRW_{AGA} = \frac{MAR62PLUS}{SSAPOP_{mar}}$$

$pASRW_{FIA}$ represents the probability that the PAH is fully insured, and is set equal to the portion of married couples of the required age where the PAH is fully insured (FI_PAH). For example, when the program is estimating the number of female aged spouse of retired workers, this factor will find the portion where their spouse, the male PAH, is fully insured.

$$pASRW_{FIA} = \frac{\sum[MAR62PLUS \times FI_PAH]}{\sum MAR62PLUS}$$

$pASRW_{CPA}$ represents the probability that the PAH is receiving benefits. This factor is set equal to the portion of eligible married couples where the PAH is receiving benefits ($RETIRED$). If the beneficiary is divorced, no factor is applied, since it is not required for the retired worker to be receiving benefits for the divorced aged spouse to receive benefits.

$$pASRW_{CPA} = \begin{cases} 1, & year \geq 1985 \text{ and } mb = 2 \\ \frac{\sum[MAR62PLUS \times FI_PAH \times RETIRED]}{\sum[MAR62PLUS \times FI_PAH]}, & \text{elsewhere} \end{cases}$$

$pASRW_{MBS}$ represents the probability that the beneficiary is not receiving a young-spouse benefit. If the beneficiary is age 70 or older or if the beneficiary is divorced, no factor is applied. Otherwise, this factor is set equal to the portion of potentially eligible widow(er)s where the spouse of the PAH is not receiving a young-spouse benefit ($YSRWN^{ab}$), where ab represents the 5-year age group.⁴¹

⁴¹ There are no young spouses at NRA or above.

$$FACTOR_{age} = \begin{cases} 1, & 65 \leq age < NRA \\ 0, & elsewhere \end{cases}$$

$$pASRW_{MBB} = \begin{cases} 1 - \frac{0.2 \times YSRWN^{60-64}}{ASRW_{POP} \times pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FLA} \times pASRW_{CPA}}, & age = 62-64 \\ & \text{and } mb = 1 \\ 1 - \frac{YSRWN^{65-69}}{ASRW_{POP} \times ASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FLA} \times pASRW_{CPA}} \\ \times \frac{FACTOR_{ag}}{\sum_{65}^{69} FACTOR_{ag}}, & 65 \leq age < NRA \\ & \text{and } mb = 1 \\ 1, & elsewhere \end{cases}$$

$pASRW_{FIB}$ represents the probability that the aged spouse is not fully insured, and is therefore not receiving a retired-worker benefit based on his/her own earnings. This factor is set equal to the portion of the married and divorced population that is not fully insured. For example, when the program is estimating the number of female aged spouse of retired workers, this factor will find the portion of female beneficiaries that is fully insured.

$$pASRW_{FIB} = 1 - \frac{FINS}{SSAPOP}, \quad mb = 1-2$$

$pASRW_{GPB}$ represents the probability that the aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in noncovered employment. $GPWHL D$ represents the total number of aged spouse of retired-worker beneficiaries (for all ages) expected to receive a significant government pension. $rGPOAGE$ represents the ratio of the total for each given age.

$$pASRW_{GPB} = \begin{cases} 1, & year \leq 1978 \\ 1 - \frac{rGPOAGE \times GPWHL D}{ASRW_{POP} * pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FLA} \times pASRW_{CPA} \times pASRW_{FIB}}, & elsewhere \end{cases}$$

$pASRW_{RES}$ represents the probability that a person who is eligible to receive aged-spouse benefits actually receive the benefits. For all historical years, $pASRW_{RES}^{year}$ is calculated as the

ratio of $ASRWN$, the actual number of aged spouses receiving benefits, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status.

$$pASRW_{RES}^{year} = \frac{ASRWN}{ASRW_{POP} * pASRW_{DEA} \times pASRW_{AGA} \times pASRW_{FIA} \times pASRW_{CPA} \times pASRW_{FIB}}, \quad year < TRYR$$

For each age, sex, and marital status, a least squares regression is used over the last ten years of historical data to determine a starting value in TR-1 for $pASRW_{RES}^{year}$ from which future values are projected. In addition, for each sex and marital status, the regressed values of $pASRW_{RES}^{TRYR-1}$ are graduated over age using a weighted minimized third-difference formula to compute $ESTRES^{ASRW}$. $ESTRES^{ASRW}$ are the preliminary estimates of $pASRW_{RES}^{TRYR+9}$, the values in the tenth year of the projection period. For female spouses, additional adjustments by age group (62-64, 65+), $SRADJ^{ASRW}$, are applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pASRW_{RES}^{year}$ for intermediate years are linearly interpolated between $pASRW_{RES}^{TRYR-1}$ and $pASRW_{RES}^{TRYR+9}$ (equal to $ESTRES^{ASRW} \times SRADJ^{ASRW}$). After the 10th year of the projection period, these adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

Equation 3.3.4 – Disabled Spouses of Deceased Workers (DSDWN)

The number of disabled spouses of deceased workers, along with all linkage factors, are projected by age, sex of the account holder ($sa = 1$ for male, $sa = 2$ for female) and marital status. Age ranges from 50 to 69, and marital status includes widowed ($mb = 1$) and divorced ($mb = 2$). The projected number of disabled spouses of deceased workers is calculated as follows:

$$DSDWN = DSDW_{POP} \times pDSDW_{DEA} \times pDSDW_{FIA} \times pDSDW_{SSB} \times pDSDW_{DEB} \times pDSDW_{RES} \quad (3.3.4)$$

$DSDW_{POP}$ represents the subset of the population from which these beneficiaries are drawn and is set equal to the Social Security area population ($SSAPOP_{mb}$) for $mb = 1, 2$.

$$DSDW_{POP} = SSAPOP_{mb}$$

$pDSDW_{DEA}$ represents the probability that the primary account holder is deceased. For the widowed population, no factor is applied. For the divorced population, this factor is set equal to the portion of the total widowed and married population that is widowed.

$$pDSDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{wid} + SSAPOP_{mar}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pDSDW_{FLA}$ represents the probability that the PAH was fully insured at death. Given the age of the widow, AW , it is assumed that the age of her deceased husband, AH , ranges from $AW-6$ to $AW+12$ with a lower and upper bound of 50 and 95+. Further, it is assumed that the more likely age of the husband is $AW+3$. For each age, $pDSDW_{FLA}$ is calculated as a weighted average of the portion of the Social Security area population that is fully insured ($FINS$), at each possible age of the husband. For example, if the widow is age 65, it is assumed that the age of the husband is between 59 and 77, therefore the weighted average of the portion of the population who are fully insured males is calculated, applying the highest weight of ten to age 68 and a linearly reduced weight to zero for each age above and below 68. The same concept is used for widow(er)s with the assumption that the age of his deceased wife ranges from $AH-12$ to $AH+6$, with a greater likelihood of her age being $AH-3$. Let $WEIGHT$ represent the specific weight applied to each age.

$$WEIGHT_{AH} = 10 - |AW + 3 - AH|$$

$$WEIGHT_{AW} = 10 - |AH - 3 - AW|$$

$$pDSDW_{FLA} = \begin{cases} \frac{\sum_{AH=AW-6}^{AW+12} WEIGHT_{AH} \times FINS_{AH}}{\sum_{AH=AW-6}^{AW+12} WEIGHT}, & sa = 1 \\ \frac{\sum_{AW=AH-12}^{AH+6} WEIGHT_{AW} \times FINS_{AW}}{\sum_{AW=AH-12}^{AH+6} WEIGHT}, & sa = 2 \end{cases}$$

$pDSDW_{SSB}$ represents the probability that the spouse is indeed disabled and is set equal to the disability prevalence rates ($DISPREV$) by age and sex received from the DISABILITY subprocess.

$$pDSDW_{SSB} = DISPREV$$

$pDSDW_{DEB}$ represents the probability that the disabled spouse is not dually eligible for another type of benefit. This factor is assumed to remain at a constant level by sex.

$$pDSDW_{DEB} = \begin{cases} 0.85, & sa = 1 \\ 0.06, & sa = 2 \end{cases}$$

$pDSDW_{RES}$ represents the probability that a person who is eligible to receive disabled-spouse benefits actually receive the benefits. For all historical years, $pDSDW_{RES}^{year}$ is calculated as the ratio of $DSDWN$, the actual number of disabled spouses of deceased workers receiving benefits, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status.

$$pDSDW_{RES}^{year} = \frac{DSDWN}{DSDW_{POP} \times pDSDW_{DEA} \times pDSDW_{FIA} \times pDSDW_{DEB}}, \quad year < TRYR$$

For ages 50 to 64, and each sex, and marital status, a least squares regression is used over the last ten years of historical data to determine a starting value in TR-1 for $pDSDW_{RES}^{year}$ from which future values are projected. In addition, for each sex and marital status, the regressed values of $pDSDW_{RES}^{TRYR-1}$ are graduated over age using a weighted minimized third-difference formula to compute $ESTRES^{DSDW}$. $ESTRES^{DSDW}$ are the preliminary estimates of $pDSDW_{RES}^{TRYR+9}$, the values in the tenth year of the projection period. For female disabled spouses, an adjustment, $SRADJ^{DSDW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pDSDW_{RES}^{year}$ for intermediate years are exponentially interpolated between $pDSDW_{RES}^{TRYR-1}$ and $pDSDW_{RES}^{TRYR+9}$ (equal to $ESTRES^{DSDW} \times SRADJ^{DSDW}$). After the 10th year of the projection period, the adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

For the projection period, for ages 65 to 69 where age is less than NRA, $pDSDW_{RES}^{year}$ is equal to $pDSDW_{RES}^{year}$ at age 64 times an adjustment which accounts for the additional ages as NRA changes.

$$FACTOR_{age} = \begin{cases} 1, & NRA \geq age + 1 \\ NRA - age, & age < NRA < age + 1 \end{cases}$$

$$pDSDW_{RES}^{year} = pDSDW_{RES}^{year,64} \times \left(\frac{pDSDW_{RES}^{year,64}}{pDSDW_{RES}^{year,63}} \right)^{(age-64)} \times FACTOR_{age}, \quad \begin{matrix} age = 65-69 \text{ and} \\ age < NRA \end{matrix}$$

Equation 3.3.5-6 – Minor Children of Retired and Deceased Workers (MCRWN and MCDWN)

The number of minor children of retired workers, $MCRWN$, is projected by age of the minor ($am = 0$ to 17) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female), and is calculated by multiplying the number of minor children in the most recent historical year, $BASE_MCRWN$, by two factors which reflect changes in (1) the number of children in the population, and (2) the proportion of retired workers to the population, at ages where a worker is most likely to have a minor child.

$$MCRWN_{sa,am}^{year} = BASE_MCRWN_{sa,am}^{TRYR-1} \times POPRATIO_{sa,am}^{year} \times \frac{RW_RATIO_{sa,62-71}^{year}}{POP_RATIO_{sa,20-71}^{year}} \quad (3.3.5)$$

An adjustment, $SRADJ^{MCRW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. These adjustments are also applied to $MCRWN_{sa,am}^{year}$ for all years after $TRYR+9$. The values of $MCRWN_{sa,am}^{year}$ for intermediate years are linearly interpolated between $MCRWN_{sa,am}^{TRYR-1}$ and $MCRWN_{sa,am}^{TRYR+9} \times SRADJ^{MCRW}$.

The number of minor children of deceased workers, along with all linkage factors, is projected by age of the minor ($am = 0$ to 17) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female), and is calculated as follows:

$$MCDWN = MCDW_{POP} \times pMCDW_{DEA} \times pMCDW_{FIA} \times pMCDW_{RES} \quad (3.3.6)$$

$MCDW_{POP}$ represents the subset of the population from which these beneficiaries are drawn and is set equal to the Social Security area population ($SSAPOP$).

$$MCDW_{POP} = SSAPOP$$

$pMCDW_{DEA}$ represents the status of the parent (PAH). This is set equal to the portion of the minor population where at least one parent is deceased. CHI_DEA represents the number of children having at least one deceased parent.

$$pMCDW_{DEA} = \frac{CHI_DEA}{MCDW_{POP}}$$

$pMCDW_{FIA}$ represents the probability that the parent (PAH) is fully insured. This is set equal to the portion of the population aged $25 + am$ to $35 + am$ where the PAH is fully insured (FI_PAH).

$$pMCDW_{FIA} = \frac{\sum_{25+am}^{35+am} [SSAPOP \times FI_PAH]}{\sum_{25+am}^{35+am} SSAPOP}$$

$pMCDW_{RES}$ represents the probability that a child who is eligible to receive minor-child benefits actually receive the benefits. For all historical years, $pMCDW_{RES}^{year}$ is calculated as the ratio of $MCDWN$, the actual number of minor children of deceased workers receiving benefits, to the number of number of persons meeting all previously mentioned requirements by age and sex of the parent.

$$pMCDW_{RES}^{year} = \frac{MCDWN}{MCDW_{POP} \times pMCDW_{DEA} \times pMCDW_{FIA}}, \quad year < TRYR$$

For each age and sex of parent, a least squares regression is used over the last ten years of historical data to determine a starting value in TR-1 for $pMCDW_{RES}^{year}$ from which future values are projected. An adjustment, $SRADJ^{MCDW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pMCDW_{RES}^{year}$ for intermediate years are linearly interpolated between the regressed values for $pMCDW_{RES}^{TRYR-1}$ and $pMCDW_{RES}^{TRYR+9} \times SRADJ^{MCDW}$. After the 10th year of the projection period, the adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections.

Equation 3.3.7-8 – Student Children of Retired and Deceased Workers (SCRWN and SCDWN)

The number of student children of retired and deceased workers, along with all linkage factors, are projected by age of the student ($as = 18$ to 19) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). The projected number of student children of retired and deceased workers is calculated as follows:

$$SCRWN = SCRW_{POP} \times pSCRW_{DEA} \times pSCRW_{AGA} \times pSCRW_{FIA} \times pSCRW_{CPA} \times pSCRW_{SSB} \times pSCRW_{RES} \quad (3.3.7)$$

$$SCDWN = SCDW_{POP} \times pSCDW_{DEA} \times pSCDW_{AGA} \times pSCDW_{FIA} \times pSCDW_{CPA} \times pSCDW_{SSB} \times pSCDW_{RES} \quad (3.3.8)$$

$SCRW_{POP}$ and $SCDW_{POP}$ represent the subset of the population from which these beneficiaries are drawn and are set equal to the Social Security area population ($SSAPOP$).

$$SCRW_{POP} = SCDW_{POP} = SSAPOP$$

$pSCRW_{DEA}$ and $pSCDW_{DEA}$ represent the status of the parent (PAH). For student children of retired workers, this is set equal to the proportion of the subset of the population where neither parents are deceased. For student children of deceased workers, this is set equal to the proportion of the subset of the population where at least one parent is deceased. CHI_DEA represents the number of student children having at least one deceased parent.

$$pSCRW_{DEA} = 1 - \frac{CHI_DEA}{SCRW_{POP}}$$

$$pSCDW_{DEA} = \frac{CHI_DEA}{SCRW_{POP}}$$

$pSCRW_{AGA}$ and $pSCDW_{AGA}$ represent the probability that the PAH is age 62 or older. For student children of retired workers, this is set equal to the proportion of the student population that has one parent age 62 or older, CHI_62+ . For student children of deceased workers, the factor is set equal to one.

$$pSCRW_{AGA} = \frac{CHI_62+}{SCRW_{POP}}$$

$$pSCDW_{AGA} = 1$$

$pSCRW_{FIA}$ and $pSCDW_{FIA}$ represent the probability that the PAH is fully insured. For student children of retired workers, this is set equal to the portion of the population aged 62 to 64+as where the PAH is fully insured (FI_PAH). For student children of deceased workers, the factor is calculated similarly with the population being aged 25+as to 35+as.

$$pSCRW_{FIA} = \frac{\sum_{62}^{64+as} [SSAPOP \times FI_PAH]}{\sum_{62}^{64+as} SSAPOP}$$

$$pSCDW_{FIA} = \frac{\sum_{25+as}^{35+as} [SSAPOP \times FI_PAH]}{\sum_{25+as}^{35+as} SSAPOP}$$

$pSCRW_{CPA}$ and $pSCDW_{CPA}$ represent the probability that the PAH is receiving benefits. For student children of retired workers, this factor is set equal to the portion of the population aged 62 to 64+as where the PAH is receiving benefits (*RETIRED*). For student children of deceased workers, this factor is set equal to one.

$$pSCRW_{CPA} = \frac{\sum_{62}^{64+as} [SSAPOP \times FI_PAH \times RETIRED]}{\sum_{62}^{64+as} [SSAPOP \times FI_PAH]}$$

$$pSCDW_{CPA} = 1$$

$pSCRW_{SSB}$ and $pSCDW_{SSB}$ represent the probability that the child is indeed attending school (full-time elementary or secondary school). This factor is dependent upon the age of the child, and is calculated as follows.

$$pSCRW_{SSB} = pSCDW_{SSB} = \begin{cases} \frac{1}{as-16}, & year \leq 1981 \\ \frac{0.5}{as-16}, & year > 1981 \end{cases}$$

$pSCRW_{RES}$ and $pSCDW_{RES}$ represent the probability that a child who is eligible to receive student-child benefits actually receive the benefits. For all historical years, $pSCRW_{RES}^{year}$ and $pSCDW_{RES}^{year}$ are calculated as the ratio of $SCRWN$ and $SCDWN$, the actual number of student children receiving benefits, to the number of number of persons meeting all previously mentioned requirements by age and sex of the parent.

$$pSCRW_{RES}^{year} = \frac{SCRWN}{SCRW_{POP} \times pSCRW_{DEA} \times pSCRW_{AGA} \times pSCRW_{FLA} \times pSCRW_{CPA} \times pSCRW_{SSB}}, \quad year < TRYR$$

$$pSCDW_{RES}^{year} = \frac{SCDWN}{SCDW_{POP} \times pSCDW_{DEA} \times pSCDW_{AGA} \times pSCDW_{FLA} \times pSCDW_{CPA} \times pSCDW_{SSB}} \quad year < TRYR$$

For each age and sex of parent, a least squares regression is used over the last ten years of historical data to determine a starting value in TR-1 for $pSCRW_{RES}^{year}$ from which future values are projected. An adjustment, $SRADJ^{SCRW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pSCRW_{RES}^{year}$ for intermediate years are linearly interpolated between the regressed values for $pSCRW_{RES}^{TRYR-1}$ and $pSCRW_{RES}^{TRYR+9} \times SRADJ^{SCRW}$. After the 10th year of the projection period, the adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. Values of $pSCDW_{RES}^{year}$ are calculated similarly.

Equation 3.3.9-10 – Disabled Adult Children of Retired and Deceased Workers (DCRWN and DCDWN)

The number of disabled adult children of retired and deceased workers, along with all linkage factors, are projected by age-group of the disabled adult child ($ad = 1-10$) and sex of the account holder ($sa = 1$ for male, $sa = 2$ for female). The age groups are 18-19, 20-24, ..., 55-59, 60+. The projected number of disabled adult children of retired and deceased workers is calculated as follows:

$$DCRWN = DCRW_{POP} \times pDCRW_{AGA} \times pDCRW_{DEA} \times pDCRW_{FIA} \times pDCRW_{CPA} \times pDCRW_{SSB} \times pDCRW_{RES} \quad (3.3.9)$$

$$DCDWN = DCDW_{POP} \times pDCDW_{AGA} \times pDCDW_{DEA} \times pDCDW_{FIA} \times pDCDW_{CPA} \times pDCDW_{SSB} \times pDCDW_{RES} \quad (3.3.10)$$

All factors are calculated similar to those for student children with the exception of the following.

$pDCRW_{DEA}$ is set equal to the proportion of the married and widowed population who are married (for ages of the parent that are reasonable based on the given age range of the disabled child). $pDCDW_{DEA}$ is calculated similarly for disabled children of deceased workers.

$$pDCRW_{DEA} = \begin{cases} \frac{SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 1-9 \\ \frac{0.25 \times SSAPOP_{mar}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 10 \end{cases}$$

$$pDCDW_{DEA} = \begin{cases} \frac{SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}}, & ad = 1-9 \\ \frac{0.25 \times SSAPOP_{wid}}{SSAPOP_{mar} + SSAPOP_{wid}} + 0.75, & ad = 10 \end{cases}$$

$pDCRW_{SSB}$ and $pDCDW_{SSB}$ represent the probability that the adult child is indeed disabled. DCPREM is the preliminary calculation of this factor and is assumed to remain constant. For

the projection period, for $ad=6-10$, $pDCRW_{SSB}$ and $pDCDW_{SSB}$ are set equal to the preliminary factor, plus an adjustment which accounts for the year.

$$DCPREM = \begin{cases} 0.012, & ad = 1-2 \\ 0.009, & ad = 3 \\ 0.007, & ad = 4 \\ 0.006, & ad = 5 \\ 0.005, & ad = 6 \\ 0.004, & ad = 7-10 \end{cases}$$

$$pDCRW_{SSB} = pDCDW_{SSB} = \begin{cases} \min[0.005, DCPREM + 0.0001 \times (year - TRYR)], & ad = 7-10 \text{ and } year > TRYR+1 \\ DCPREM, & \text{elsewhere} \end{cases}$$

$pDCRW_{RES}$ and $pDCDW_{RES}$ represent the probability that a child who is eligible to receive disabled-child benefits actually receive the benefits. For all historical years, $pDCRW_{RES}^{year}$ and $pDCDW_{RES}^{year}$ are calculated as the ratio of $DCRWN$ and $DCDWN$, the actual number of disabled children receiving benefits, to the number of number of persons meeting all previously mentioned requirements by age and sex of the parent.

$$pDCRW_{RES}^{year} = \frac{DCRWN}{DCRW_{POP} * pDCRW_{DEA} * pDCRW_{AGA} * pDCRW_{FLA} * pDCRW_{CPA} * pDCRW_{SSB}},$$

$$pDCDW_{RES}^{year} = \frac{DCDWN}{DCDW_{POP} * pDCDW_{DEA} * pDCDW_{AGA} * pDCDW_{FLA} * pDCDW_{CPA} * pDCDW_{SSB}} \begin{matrix} year < TRYR \\ year < TRYR \end{matrix}$$

An adjustment, $SRADJ^{DCRW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pDCRW_{RES}^{year}$ for intermediate years are linearly interpolated between $pDCRW_{RES}^{TRYR-1}$ and $pDCRW_{RES}^{TRYR+9} \times SRADJ^{DCRW}$. After the 10th year of the projection period, the adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. Values of $pDCDW_{RES}^{year}$ are calculated similarly.

Equation 3.3.11-12 – Young Spouses of Retired and Deceased Workers (YSRWN and YSDWN)

The number of young spouses of retired and deceased-workers, along with all linkage factors, are projected by age-group ($ab = 1-10$) of the young spouse and sex of the account

holder ($sa = 1$ for male, $sa = 2$ for female). Young spouses of deceased workers are also projected by marital status of the young spouse ($mb = 1$ for widowed and $mb = 2$ for divorced). The age groups are under 25, 25-29 ..., 65-69. The projected number of young spouses of retired and deceased-workers is calculated as follows:

$$YSRWN = YSRW_{POP} \times pYSRW_{AGA} \times pYSRW_{ECB} \times pYSRW_{FSB} \times pYSRW_{RES} \quad (3.3.11)$$

$$YSDWN = YSDW_{POP} \times pYSDW_{DEA} \times pYSDW_{ECB} \times pYSDW_{FSB} \times pYSDW_{RMB} \times pYSDW_{RES} \quad (3.3.12)$$

$YSRW_{POP}$ and $YSDW_{POP}$ represent the subset of the population from which these beneficiaries are drawn. $YSRW_{POP}$ is set equal to the married Social Security area population ($SSAPOP_{mar}$) and $YSDW_{POP}$ is set equal to $SSAPOP_{mb}$ for $mb = 1-2$.

$$YSRW_{POP} = SSAPOP_{mar}$$

$$YSDW_{POP} = SSAPOP_{mb}$$

$pYSDW_{DEA}$ represent the probability that the PAH is deceased. For $mb=1$, no factor is applied. For $mb=2$, this factor is set equal to the portion of young spouses that is widowed.

$$pYSDW_{DEA} = \begin{cases} 1, & mb = 1 \text{ (widowed)} \\ \frac{SSAPOP_{wid}}{SSAPOP_{wid} + SSAPOP_{mar}}, & mb = 2 \text{ (divorced)} \end{cases}$$

$pYSRW_{AGA}$ and represent the probability that the PAH is of the required age. $pYSRW_{AGA}$ is set equal to the portion of the married population who has an aged spouse ($AGSP$).

$$pYSRW_{AGA} = \frac{AGSP}{YSRW_{POP}}$$

$pYSRW_{ECB}$ and $pYSDW_{ECB}$ represent the probability that the young spouse has an entitled child in their care. $pYSRW_{ECB}$ is set equal to the portion of persons meeting the previously mentioned requirements who have a minor or disabled adult child in their care. $pYSDW_{ECB}$, by marital status, is set equal to the portion of persons meeting the previously mentioned requirements who have a minor or disabled adult child in their care. $MCRWN^{ab}$ and

$DCRWN^{ab}$ represent the total number of minor and disabled adult children of retired workers where the other parent (young spouse) is in the age bracket ab .

$$pYSRW_{ECB}^{ab} = \frac{MCRWN^{ab} + DCRWN^{ab}}{YSRW_{POP} \times pYSRW_{AGA}}$$

$$pYSDW_{ECB}^{mb} = \frac{(MCDWN^{ab} + DCDWN^{ab}) \times \left[\frac{YSDW_{POP}^{mb} \times pYSDW_{DEA}^{mb} \times pYSDW_{AGA}^{mb}}{YSDW_{POP}^{total} \times pYSDW_{DEA}^{total} \times pYSDW_{AGA}^{total}} \right]}{YSDW_{POP} \times pYSDW_{DEA}}$$

$pYSRW_{FSB}$ and $pYSDW_{FSB}$ represent the probability that the young spouse is not already receiving benefits based on another child in their care. This factor is set equal to one divided by the number of children in the average family ($ASOF_{ab}$) for the given age bracket of the spouse. For young spouses of retired workers, no factor is applied for $sa=2$.

$$pYSRW_{FSB} = \begin{cases} \frac{1}{ASOF_{ab}}, & sa = 1 \\ 1, & sa = 2 \end{cases}$$

$$pYSDW_{FSB} = \frac{1}{ASOF_{ab}}$$

$pYSDW_{RMB}$ represents the probability that the spouse is not remarried. This factor is assumed to remain constant at 0.600.

$$pYSDW_{RMB} = 0.600$$

$pYSRW_{RES}$ and $pYSDW_{RES}$ represent the probability that a person who is eligible to receive young-spouse benefits actually receive the benefits. For all historical years, $pYSRW_{RES}^{year}$ is calculated as the ratio of $YSRWN$, the actual number of young spouses of retired workers receiving benefits, to the number of persons meeting all previously mentioned requirements by age, sex, and marital status. $pYSDW_{RES}^{year}$ is calculated similarly, using the number of young spouses of deceased workers.

$$pYSRW_{RES} = \frac{YSRWN}{YSRW_{POP} \times pYSRW_{AGA} \times pYSRW_{ECB} \times pYSRW_{FSB}}, \text{ year} < \text{TRYR}$$

$$pYSDW_{RES} = \frac{YSDWN}{YSDW_{POP} \times pYSDW_{DEA} \times pYSDW_{ECB} \times pYSDW_{FSB} \times pYSDW_{RMB}}, \text{ year} < \text{TRYR}$$

For each age, sex, and marital status, a least squares regression is used over the last ten years of historical data to determine a starting value in TR-1 for $pYSRW_{RES}^{year}$. In addition, for each sex and marital status the regressed values of $pYSRW_{RES}^{TRYR-1}$ are graduated over age using a weighted minimized third-difference formula to compute $ESTRES^{YSRW}$. $ESTRES^{YSRW}$ are the preliminary estimates of $pYSRW_{RES}^{TRYR+9}$, the values in the tenth year of the projection period. For female young spouses, an adjustment, $SRADJ^{YSRW}$, is applied to the tenth year of the projection period in order to match the projections made by the Short-Range office. The values of $pYSRW_{RES}^{year}$ for intermediate years are exponentially interpolated between $pYSRW_{RES}^{TRYR-1}$ and $pYSRW_{RES}^{TRYR+9}$ (equal to $ESTRES^{YSRW} \times SRADJ^{YSRW}$). After the 10th year of the projection period, the adjustment factors are linearly graded to one over the 10 years beyond the end of the short-range period, thus gradually eliminating the effect of the short-range adjustment factors, so that we ultimately return to the long-range projections. Values of $pYSDW_{RES}^{year}$ are calculated similarly.

Equation 3.3.13 – Number of Deaths of Insured Workers ($LUMSUM_{ab}$)

The number of deaths of insured workers is projected by sex and 5-year age group ($ab = 1-14$). Age groups include 20-24, 25-29, ..., 80-84, 85+. $EXPOSURE_{ab}$, the estimated number of lump-sum payments paid during the year for age group ab , is calculated as the number of total deaths during the year times the probability that the deceased was fully insured and has a surviving spouse or child. $BASE$ is calculated as the ratio of the actual total amount of lump-sum death payments paid in TRYR-1 to the estimated total amount of lump-sum payments paid in TRYR-1. $LUMSUM_{ab}$ is then calculated for each year in the projection period.

$$LUMSUM_{ab} = EXPOSURE_{ab} \times BASE \quad (3.3.13)$$

Appendix 3.3-1: Glossary

AB: age group of the beneficiary

AD: age of the disabled child

AGSP: married population where at least one spouse is age 62 or older

AM: age of the minor child

ARFGT3: actuarial reduction factor for ages more than 3 years below normal retirement age

ARFLE3: actuarial reduction factor for ages less than 3 years below normal retirement age

AS: age of the student child

ASDW: aged spouse of deceased worker by linkage factor, age (60-95+), sex of the account holder, marital status (widowed, divorced) and insured status (insured, uninsured). Linkage factors are:

$ASDW_{POP}$: population of potential aged spouse of retired workers

$pASDW_{DEA}$: probability that the primary account holder (PAH) is deceased

$pASDW_{FLA}$: probability that the PAH was fully insured at death

$pASDW_{MBA}$: probability that the widow(er) is not receiving a young-spouse benefit for the care of a child

$pASDW_{FIB}$: probability that the aged widow(er) is fully insured

$pASDW_{GPB}$: probability that the aged-widow(er)'s benefits are not withheld or offset totally because of receipt of a significant government pension based on earnings in noncovered employment

$pASDW_{RES}$: probability that a widow(er) eligible to receive his/her own retired-worker benefits would instead apply for and receive widow(er) benefits

ASDWN: final number of aged spouse of deceased workers (product of all linkage factors)

ASOF: average number of children in a family, by age group (under 25, 25-29 ..., 65-69)

ASRW: aged spouse of retired worker by linkage factor, age (62-95+), sex of the account holder, and marital status of the beneficiary (married, divorced). Linkage factors are:

$ASRW_{POP}$: population of potential aged spouse of retired worker beneficiaries

$pASRW_{DEA}$: probability that the primary account holder (PAH) is not deceased

$pASRW_{AGA}$: probability that the PAH is of the required age

$pASRW_{FLA}$: probability that the PAH is fully insured

$pASRW_{CPA}$: probability that the PAH is receiving benefits

$pASRW_{MBA}$: probability that the beneficiary is not receiving a young-spouse benefit

$pASRW_{FIB}$: probability that the aged spouse is not fully insured

$pASRW_{GPB}$: probability that the aged-spouse's benefits are not withheld because of receipt of a significant government pension based on earnings in noncovered employment

$pASRW_{RES}$: probability that a person who is eligible to receive aged-spouse benefits actually receive the benefits

ASRWN: final number of aged spouse of retired workers (product of all linkage factors)

AH: age of husband

AW: age of wife

BASE: ratio of actual to estimated total amount of lump-sum death payments paid in TRYR-1

CHI_62+: number of children having at least one parent aged 62 or older

CHI_DEA: number of children having at least one deceased parent

CON: number of persons converted from disabled-worker beneficiaries

DCDW: disabled child of deceased workers by linkage factor, age group of the child (18-19, 20-24,..., 55-59, 60+) and sex of the account holder. Linkage factors are same as SCDW.

$DCDW_{POP}$: population of potential disabled children

$pDCDW_{AGA}$: probability that the PAH is age 62 or older

$pDCDW_{DEA}$: probability that the parent is either retired or deceased

$pDCDW_{FIA}$: probability that the PAH is fully insured

$pDCDW_{CPA}$: probability that the PAH is receiving benefits

$pDCDW_{SSB}$: probability that the child is indeed disabled

$pDCDW_{RES}$: probability that a child who is eligible to receive disabled-child benefits actually receive the benefits

DCDWN: final number of disabled children of deceased workers (product of all linkage factors)

DCPREM: preliminary calculation of the probability that a child is disabled, by age

DCRW: disabled child of retired workers by linkage factor, age group of the child (18-19, 20-24,..., 55-59, 60+) and sex of the account holder. Linkage factors are same as those for DCDW.

DCRWN: final number of disabled children of retired workers (product of all linkage factors)

DIB: number of disabled-worker beneficiaries

DIFFADJ: adjustment which accounts for the difference between the actual and estimated prevalence rate at each age in the most recent historical years

DISPREV: disability prevalence rate by age and sex

DRC: delayed retirement credit

DSDW: disabled spouse of deceased worker by linkage factor, age (50-69), sex of the account holder, and marital status (widowed, divorced). Linkage factors are:

$DSDW_{POP}$: population of potential beneficiaries

$pDSDW_{DEA}$: probability that the primary account holder (PAH) is deceased

$pDSDW_{FIA}$: probability that the PAH was fully insured at death

$pDSDW_{SSB}$: probability that the spouse is indeed disabled

$pDSDW_{DEB}$: probability that the disabled spouse is not receiving another type of benefit

$pDSDW_{RES}$: probability that a person who is eligible to receive disabled-spouse benefits actually receive the benefits

DSDWN: final number of disabled spouse of deceased workers (product of all linkage factors)

ERROR: actual prevalence rate minus the regressed prevalence rate in the most recent historical year

ESTPR: preliminary estimate of the prevalence rate for retired workers

ESTPR2: secondary estimate of the prevalence rate for retired workers

ESTRES: preliminary estimate of the RES factor for the tenth year of the projection period

EXPOSURE: estimated number of lump-sum payments by age group (20-24, 25-29, ..., 80-84, 85+)

FACTOR: adjustment for calculation of MBB factor of aged spouse of deceased worker

FINS: portion of the SSA population that is fully insured

FI_PAH: portion of married population where one spouse is fully insured

FP: status of the parent (retired, deceased)

GPOAGE: portion, by age, of the total beneficiaries expected to receive a significant government pension

GPWHLD: total number of beneficiaries (for all ages) expected to receive a significant government pension

IN: insured status of the beneficiary

LFPR: labor force participation rates for age 62, by sex

LUMSUM: number of deaths of insured workers by sex and age group (20-24, ..., 80-84, 85+)

MAR62PLUS: number of couples where both husband and wife are age 62 and over

MS: marital status of the primary account holder

MB: marital status of the beneficiary

MBAPIA: ratio of the monthly benefit amount (MBA) to the primary insurance amount (PIA) by age (62-70) and sex

MCDW: minor children of deceased workers by linkage factor, age of the child (0-17) and sex of the account holder. Linkage factors are:

$MCDW_{POP}$: population of potential minor children

$pMCDW_{DEA}$: probability that the parent is either retired or deceased

$pMCDW_{FIA}$: probability that the PAH is fully insured

$pMCDW_{RES}$: probability that a child who is eligible to receive minor-child benefits actually receive the benefits

MCDWN: final number of minor children of deceased workers (product of all linkage factors)

MCRW: minor children of retired workers by linkage factor, age of the child (0-17) and sex of the account holder.

MCRWN: final number of minor children of retired workers (product of all linkage factors)

NRA: normal retirement age

PAH: primary account holder

REGPR: regressed prevalence rate for retired workers

RETIRED: number of retired workers receiving benefits

RW: retired workers by linkage factor, age (62-95+), sex, and marital status (single, married, widowed, divorced). Linkage factors are:

RW_{POP} : population of potential retired-worker beneficiaries

pRW_{FIA} : probability that the primary account holder (PAH) is insured

pRW_{DBB} : probability that the PAH is not receiving a disabled-worker benefit
 pRW_{WBB} : probability that the PAH is not receiving a widow(er) benefit
 pRW_{RES} : retirement prevalence rate; probability that a fully insured worker (not receiving disability or widow(er)'s benefits) would receive a retired-worker benefit

RWN: final number of retired workers (product of all linkage factors)

SA: sex of the account holder

SCDW: student children of deceased workers by linkage factor, age of the student (18-21) and sex of the account holder. Linkage factors are:

$SCDW_{POP}$: population of potential student children
 $pSCDW_{DEA}$: probability that the parent is either retired or deceased
 $pSCDW_{AGA}$: probability that the PAH is age 62 or older
 $pSCDW_{FLA}$: probability that the PAH is fully insured
 $pSCDW_{CPA}$: probability that the PAH is receiving benefits
 $pSCDW_{SSB}$: probability that the child is indeed attending school
 $pSCDW_{RES}$: probability that a child who is eligible to receive student-child benefits actually receive the benefits

SCDWN: final number of student children of deceased workers (product of all linkage factors)

SCRW: student children of retired workers by linkage factor, age of the student (18-21) and sex of the account holder. Linkage factors are same as SCDW.

SCRWN: final number of student children of retired workers (product of all linkage factors)

SRADJ: adjustment to match short-range projections in 10th year of projection period

SSAPOP: Social Security area population by age (0:100), sex, and marital status (single, married, widowed, divorced)

SX: sex of the beneficiary

TRYR: first year of the projection period

WEIGHT: estimated probability applied to each possible age of the spouse, given the age of the primary account holder

YSDW: young spouse of deceased worker by linkage factor, age group (under 25, 25-29,...,65-69), sex of the account holder and marital status (widowed, divorced). Linkage factors are:

$YSDW_{POP}$: population of potential young spouse of deceased workers
 $pYSDW_{DEA}$: probability that the primary account holder (PAH) is of the required age
 $pYSDW_{ECB}$: probability that the young spouse has an entitled child in their care
 $pYSDW_{FSB}$: probability that the young spouse is not already receiving benefits based on another child in their care
 $pYSDW_{RMB}$: probability that the young spouse is not remarried
 $pYSDW_{RES}$: probability that a person who is eligible to receive young-spouse benefits actually receive the benefits

YSDWN: final number of young spouse of deceased workers (product of all linkage factors)

YSRW: young spouse of retired worker by linkage factor, age group (under 25, 25-29,...,65-69) and sex of the account holder. Linkage factors are:

$YSRW_{POP}$: population of potential young spouse of retired workers
 $pYSRW_{AGA}$: probability that the primary account holder (PAH) is of the required age
 $pYSRW_{ECB}$: probability that the young spouse has an entitled child in their care
 $pYSRW_{FSB}$: probability that the young spouse is not already receiving benefits based on another child in their care
 $pYSRW_{RES}$: probability that a person who is eligible to receive young-spouse benefits actually receive the benefits

YSRWN: final number of young spouse of retired workers (product of all linkage factors)

Process 4:

**Trust Fund
Operations &
Actuarial Status**

4. Trust Fund Operations and Actuarial Status

OCACT uses the Trust Fund Operations and Actuarial Status Process to project (1) the annual flow of income from payroll taxes, taxation of benefits, and interest on assets in the trust fund and (2) the annual flow of cost from benefit payments, administration of the program, and railroad interchange. The annual flows are projected for each year of the 75-year projection period. In addition, this subprocess produces annual and summarized values to help access the financial status of the Social Security program.

The Trust Fund Operations and Actuarial Status Process is composed of three subprocesses: TAXATION OF BENEFITS, AWARDS, and COST. As a rough overview, TAXATION OF BENEFITS projects, for each year during the 75-year projection period, the amount of income from taxation of benefits as a percent of benefits paid. AWARDS projects information needed to determine the benefit levels of newly awarded retired workers and disabled workers by age and sex. COST uses information from the AWARDS and TAXATION OF BENEFITS subprocesses, as well as information from other processes, to project the annual flow of income and cost to the trust funds. In addition, COST produces annual and summarized measures of the financial status of the Social Security program.

4.1. TAXATION OF BENEFITS

4.1.a. Overview

The 1983 Social Security Act specifies including up to 50 percent of the Social Security benefits to tax return filer's adjustable gross income for tax liability if tax return filer's adjusted gross income plus half of his (or her) Social Security benefits is above the specified income threshold amount of \$25,000 as a single filer (or \$32,000 as a joint filer). Moreover, the 1993 OBRA (Omnibus Budget Reconciliation Act) increased taxable benefits up to 85 percent if tax return filer's adjusted gross income plus half of his (or her) Social Security benefits is above the specified income threshold amount of \$34,000 as a single filer (or \$44,000 as a joint filer).

The proceeds from taxing up to 50 percent of the OASDI benefits, as a result of the 1983 Act, are credited to the OASI and DI Trust Funds, while additional taxes on the OASDI benefits, as a result of the 1993 Act, are credited to the HI Trust Fund.

Income to the trust funds from such taxation is done by estimating ratios of taxes on benefits to benefits for the OASI and DI programs separately. These ratios, called "RTBs," are applied to the projected OASI and DI benefit amounts to estimate tax revenues to the OASI, DI, and HI Trust Funds.

For the short range period (first 10 years of the projection), OCACT (Office of the Chief Actuary) uses OTA (Office of Tax Analysis)'s projected estimates: (1) percent of benefits taxable, and (2) average marginal tax rates applicable to those OASI and DI benefits. The multiplication of the (1) and (2) produces the projected RTBs under the 1983 Act and 1993 Act, respectively.

For each year (yr) after the short range period (11th through 75th year of the projection period), the RTB ratios for the OASI benefits and for the DI benefits under the 1983 Act and the combined 1983 and 1993 Act are computed with the following formula for each projection year.

$$RTB(yr) = RTB(tryr+9) * \{AWI(tryr+9)/AWI(yr)\}^P + RTB(ultimate)*\{1- AWI(tryr+9)/AWI(yr)\}^P, \quad (4.1.1)$$

where

tryr = first year of the projection period (year of the Trustees Report)

RTB(ultimate) = ratio of taxes on benefits to benefits assuming income threshold amounts equal zero.

AWI = SSA average wage index series

P = exponential parameter for a trend curve line.

Finally, OCACT applies the projected RTB ratios to its own estimates of the projected OASI and DI benefit payments to produce taxation of benefit revenues to the OASI, DI, and HI Trust Funds, respectively.

4.1.b. *Input Data*

OCACT Data

Economics

- Projected SSA wage index series by year, updated yearly

Other input Data

- OTA's projected percent of benefits taxable and average marginal tax rates by type of benefit (OASI, DI) and by OASDI Trust Fund and HI Trust Fund for the short range period. Updated yearly.
- OTA's ultimate ratios of taxes on benefits to benefits (i.e., with income thresholds, assumed equal to 0). Such ultimate ratios are provided by OASDI Trust Fund and HI Trust Fund on a combined OASDI benefit type basis, and are expected to be updated periodically based on OTA's update.

4.1.c. *Development of Output*

For the short range period, OCACT uses OTA's estimates of benefits taxable and average marginal tax rate projections for OASI and DI benefits, under the 1983 Act and 1993 Act, respectively. OCACT multiplies these two estimated ratios to compute the projected RTB ratios.

After the short range period, formula 4.1.1 computes projected ratios of taxes on OASI benefits to OASI benefits and projected ratios of taxes on DI benefits to DI benefits under the 1983 Act (up to 50 percent of benefits taxable) and under the combined 1983 and 1993 Acts (up to 85 percent of benefits taxable). This formula essentially provides more weight to the ultimate RTB ratios as time progresses, using the ratio of AWI (10th year) to AWI (projection year) as the "weight." Additionally, an exponential parameter P value to the AWI "weights" is set judgmentally such that the estimate continues the short range trend into the transitional 11th through 20th projection years before it approaches the ultimate RTB ratio. For the RTB ratios for up to 50 percent of benefits taxable, the P values were set at 0.99 and 0.99 to project smooth transitional RTB ratios for OASI and DI benefits, respectively.⁴²

⁴² For RTB ratios for up to 85 percent of benefits taxable, P values were set at .99 and .99 for OASI and DI benefits, respectively.

The ultimate RTB ratios used in the projection are based on OTA's ultimate ratios, reduced by about 5 percent. The 5 percent reduction reflects estimates of the effect of the higher proportion of "old elderly" beneficiaries in the 2085 OASDI beneficiary distribution relative to the current OASDI beneficiary distribution, due to improved mortality.

For the 2009 Trustees Report, the ultimate RTB ratios for up to 50 percent of OASI and DI benefits taxable were set at 0.058 and 0.029, respectively.⁴³

Lastly, OCACT applies these projected RTB ratios to its own estimates of OASI and DI benefit payments to develop estimated taxation of benefit revenues to the OASI, DI, and HI Trust Funds, respectively.

⁴³ Ultimate ratios for up to 85 percent of OASI and DI benefits taxable, with the higher \$34,000/\$44,000 threshold amounts, were set at 0.105 and 0.0525, respectively.

4.2. AWARDS

Each year over 2 million workers begin receiving either retired-worker or disabled-worker benefits. The monthly benefits for these new awards are based on their primary insurance amount or simply PIA. The PIA is computed using the PIA benefit formula as specified in the 1977 amendments and the average indexed monthly earnings (AIME). The AIME depends on the number of computation years, Y , and the amount earned (earnings level) by a worker in each year. For retired-worker beneficiaries who have attained or will attain age 62 in 1991 or later, $Y = 35$.

The AWARDS subprocess selects records from a 10% sample of newly entitled worker beneficiaries obtained from the Master Beneficiary Record (MBR).⁴⁴ The selected sample, referred to as “sample”, contains 221,100 beneficiary records, and each record, r , includes a worker’s history of taxable earnings under the OASDI program as well as additional information such as sex, birth date, month of initial entitlement, and type of benefit awarded. To estimate the benefit levels of future newly entitled worker beneficiaries, the earnings records in the sample are modified to reflect the expected work histories and earnings levels of future beneficiaries (equation 4.2.1). After the modifications, an AIME is computed for each record in the future sample of beneficiaries (equation 4.2.2). Each AIME value is then subdivided into *bend point subintervals*⁴⁵ (equation 4.2.3). As input to the Cost subprocess, the AIME values are used to calculate aggregate percentages of AIME in each *bend point subinterval* for each age at entitlement, sex and trust fund (equation 4.2.4). Equations 4.2.1 through 4.2.4 outline the overall structure and solution sequence. The subscript n refers to the *bend point subinterval* and r refers to the sample record.

$$\text{Projected Earnings} = \text{Projected Earnings}(\cdot) \quad (4.2.1)$$

$$\text{AIME}(r) = \frac{\sum \text{Highest } Y \text{ Indexed Earnings}(r)}{Y * 12} \quad (4.2.2)$$

$$\text{AIME}_n(r) = \text{AIME}_n(\cdot) \quad (4.2.3)$$

$$\text{PAP}_n = \frac{\sum_r \text{AIME}_n(r)}{\sum_r \text{bp}_n} \quad (4.2.4)$$

where bp_n is the length of the n th bend point subinterval,

⁴⁴ A record is selected if the year of initial entitlement equals 2004 and the beneficiary is in current pay status as of Dec. 2004, 2005 or 2006. Retired beneficiaries over age 70 and disability beneficiaries under age 20 or over age 64 were excluded.

⁴⁵ The current law PIA formula has two bend points. For the purposes of PAP, the Awards subprocess instead uses 30 subintervals.

Y is the number of computation years, and
AIME_n(r) is the AIME amount contained within the *n*th interval for record r.

4.2.b. *Input Data*

Long-Range OCACT Projection Data

Demography–

- Social Security area population by sex and age.
 - From 1951 to 2085
 - Updated annually

Economics –

- Covered workers by sex and age--all.
 - From 1951 to 2085
 - Updated annually
- Covered workers by sex and age—with earnings posted to the Master Earnings File (MEF) only. - used with CWHS data to project future earnings levels
 - From 1951 to 2085
 - Updated annually
- Average Wage Index (AWI), projected values.
 - From 2008 to 2085
 - Updated annually
- Average Taxable Earnings (ATE) —with earnings posted to the Master Earnings File (MEF) only. – used with CWHS data to project future earnings levels
 - From 1951 to 2006
 - Updated annually
- Projected Covered Worker Rate (for validation)
 - From 2008 to 2085
 - Updated annually
- COLA (Cost Of Living Adjustment)
 - From 2008 to 2085
 - Updated annually
- CPI (Consumer Price Index)
 - From 2008 to 2085
 - Updated annually

Other input data

- 10% Awards Sample from the MBR and Master Earnings File
 - Newly awarded OASI / DI beneficiaries, whose initial entitlement year was 2004, are in current pay status as of Dec. 2004, 2005 or 2006.
 - SSN
 - Type of benefit

- Type of claim (retirement or disability)
 - Sex
 - Date of birth
 - Date of initial entitlement
 - PIA amount
 - Type of dual entitlement
 - Dual entitlement status code
 - PIFC
 - LAF
 - Trust fund
 - Earnings histories for each worker from 1951 to 2003
 - Generally updated annually, pending validation of the sample
- Total taxable earnings and number of workers with taxable earnings by age, sex, and year from the 2006 Continuous Work History Sample (CWHS).
 - From 1951 to 2006
 - Updated annually
- AWI, Average Wage Index
 - From 1951 to 2007
 - Data obtained from OCACT internet site.
 - Updated annually
- Wage base
 - From 1951 to 2009
 - Data obtained from OCACT internet site.
 - Updated annually
- COLA, cost of living adjustment
 - From 1975 to 2008
 - Data obtained from OCACT internet site.
 - Updated annually
- Amount of earnings needed to earn one quarter of coverage
 - From 1951 to 20089
 - 1978-2009 data obtained from OCACT internet site. 1951-1977 values estimated by applying projection methodology backwards from 1978.
 - Updated annually
- Windfall Elimination Provision (WEP) factors, the % of sample cases affected by the WEP which will no longer be affected by the WEP, by sex and projection year
 - From 2008 to 2085
 - Data obtained from OCACT internal calculations
 - Updated once every 2 years
- PIA bend points
 - From 1979 to 2009
 - Data obtained from OCACT internet site.
 - Updated annually

4.2.c. Development of Output

All equations described below are projected separately for the OASI and DI program.

Equation 4.2.1 – Projected Earnings

In order to estimate future benefit levels, the work histories and earnings levels in the current sample must be modified to represent those for a sample of worker beneficiaries who are newly entitled in future years. There are three distinct modifications that must be made to the earnings records. For each future year, changes are made to the earnings records in order to reflect:

- *Changes in Wage Bases.*
For some years, the projected wage base (contribution and benefit base), on an AWI discounted basis, is higher than the historical wage base. Therefore, the taxable earnings of future beneficiaries may need to include covered earnings above the reported historical wage base. Thus, for each record with reported taxable earnings at the wage base in a given year, AWARDS imputes his/her covered earnings.
- *Changes in Covered Worker Rates.*
Adjustments are made to work histories to be consistent with the projected changes in the economy-wide covered worker rates. Economy-wide covered worker rates are defined as the ratio of covered workers (from Economics subprocess) to the Social Security area population (from Demography subprocess).
- *Earnings Experience in the CWHS⁴⁶.*
Earning levels are modified to capture the changes to date that are reflected in the average taxable earnings reported in the CWHS by age and sex and the changes expected in the future.

Change in Wage Bases

The earnings posted in the sample are limited by the historical wage base (contribution and benefit base). Prior to 1975, the maximum annual amount of earnings on which OASDI taxes were paid was determined by ad hoc legislation. After 1974, however, the annual maximum level was legislated to be determined automatically, based on the increase in the Social Security Average Wage Index (AWI). Prior to these automatic wage base increases, a relatively large portion of workers earned amounts above the base. Additional legislation raising the annual maximum taxable amount occurred in 1979, 1980, and 1981 to improve the financial future of the OASDI Trust Funds. In addition, the AWI used in the automatic calculation of the annual taxable maximum was modified in the early 1990s to include deferred compensation amounts. For the 2009 Trustees Report, years 2010 and 2011 are projected to have no increase in the wage

⁴⁶ This file is a 1% sample of individuals who had covered earnings at some point in their work histories.

base (will remain at the 2009 level of \$106,800) because the cost of living adjustment (COLA) for benefit payments is projected to be zero for December 2009 and December 2010.⁴⁷ Therefore, for each record in the sample with earnings at the wage base, the AWARDS process imputes covered earnings above the historical wage base in order to reflect higher maximum taxable amounts imposed on future newly entitled beneficiaries. Please refer to appendix 4.2-2 at the end of this subprocess for details of this imputation.

Change in Covered Worker Rates

The *sample's covered worker rate* by age group and sex is defined as the ratio of (1) the number of those beneficiaries with covered earnings in the sample to (2) the total number of beneficiaries in the sample. For both males and females, the work histories are modified to reflect changes in the covered worker rates that would apply to a future sample of beneficiaries. These changes in the covered worker rates are based on changes in the *economy-wide covered worker rates*. The *economy-wide covered worker rate* is defined for an age-sex group in a particular period which represents a future sample cohort as the ratio of (1) the number of workers in the economy in this group that have some earnings in this period, to (2) the total midyear population in this group in this period. *Economy-wide covered worker rates* are calculated separately for each age-sex group and each historical and projected calendar year based on input data from the Economics and Demography subprocesses.

In projecting *sample covered worker rates*, examination is done of the change in *economy-wide covered worker rates*, by age group, between the “base period” (representing individuals retiring in the sample year) and the “projection period” (representing individuals retiring in a year later than the sample year). Details of how this change is used to estimate the change in a covered worker rate for retired workers from a current period in the sample to a future period are given below for male and female, respectively. This presentation presumes that economy-wide covered rates increase over time, which is very common for females but not always true for males, based on 2009TR data. The calculation of the change in covered worker rate changes slightly if there is a reduction in relevant economy-wide covered worker rates. Example 1.1 gives an example of the calculations done for males if economy-wide covered worker rates decline.

MALE

- The ratio of (1) the absolute difference in the *economy-wide male covered worker rate* between the two periods to (2) the potential difference in the *economy-wide male worker rate in the sample year* (i.e., $1 - \text{economy-wide covered worker rate}$), multiplied by

⁴⁷ Per Section 230(a) of the Social Security Act, if the COLA does not increase, then the wage base does not increase.

- The corresponding potential difference in the *sample's male covered worker rates* (i.e. $1 - \text{sample male covered worker rate}$)).

FEMALE

- The ratio of (1) the absolute difference in the *economy-wide female covered worker rate* between the two periods to (2) the difference between the *projected sample male covered worker rate* and the *economy-wide female worker rate in the sample year*, is multiplied by
- The corresponding difference between the *projected sample male covered worker rate* and the *sample female covered worker rate*.

For additional explanation of this calculation, refer to example 1.1 and 1.2 in appendix 4.2-1 of this subprocess.

Once the covered worker rates for the future sample of beneficiaries are determined, modifications to work histories of the sample to attain these rates are generally done by randomly removing or adding earnings. For males, the procedure is to select records randomly. However, for females, an additional selection criterion is included in order to achieve a distribution of the number of years of earnings for retired female beneficiaries. Female records with 10 or fewer years of earnings are not modified. A distribution limit is set for those female workers with 11 to 25 total years of career earnings within the projection year. This distributional limit changes each projected year. In the first year, the distribution limit for females is equal to the male distribution plus 97% of the difference between the initial male and female distributions within the sample. In each subsequent year, the percentage decreases by three percent until it reaches 0%. Thus, the females' years of earnings distribution for those with 11 to 25 years of earnings is adjusted to approach that of the males.

If a record is selected for adding earnings in a particular year, the amount of earnings added is based on the career earnings pattern of the selected record. When earnings are added to a record, AWARDS calculates the ratio of (1) the record's Average Indexed Earnings, AIE⁴⁸, to (2) the AIE of a hypothetical worker, w , whose year of birth and sex are the same as the record and whose annual earnings are set equal to *average taxable earnings*. For this purpose, *average taxable earnings* are determined by averaging the earnings over all records in the sample with the same sex and year of birth. Then, the amount of earnings⁴⁹ in year t that is added to the record is

$$\text{Earnings}(r, t) = \text{Pre ATE}_t(\text{sex}, t) * \{ \text{AIE}(r) / \text{AIE}(w) \},$$

⁴⁸ AIE is the average indexed annual earnings, average over the highest Y years of earnings (similar to AIME, but an annual amount).

⁴⁹ In this subprocess, earnings histories of projected beneficiaries are all reflected as wage-indexed earnings histories in the 2004 sample.

where ${}^{\text{Pre}}\text{ATE}_f(\text{sex}, t)$ is the average taxable earnings in year t , for those in the sample with the same sex as that of the record and calculated using projected earnings adjusted to reflect wage base changes for those retiring in year f .

For additional explanation of this calculation, refer to example 2 in appendix 4.2-1 of this subprocess.

Earnings Experience in the CWHS

For historical years beginning with 1951, the AWARDS subprocess uses average taxable earnings by age and sex (${}_{\text{CWHS}}\text{ATE}_{\text{as}}$) and numbers of covered workers by age and sex (${}_{\text{CWHS}}\text{CW}_{\text{as}}$) as tabulated from the most recent CWHS file⁵⁰. The AWARDS subprocess then projects these values from the base year (the last historical year in the CWHS file). Projections are made for each year after the base year through the end of the 75-year projection period using projected economy-wide number of covered workers by age and sex and annual average taxable earnings (ATE) from the Economics process⁵¹.

The numbers of covered workers by age and sex (${}_{\text{CWHS}}\text{CW}_{\text{as}}$) are projected by applying the annual growth rates by age and sex in the numbers of economy-wide covered workers⁷ produced by the Economics subprocess. In addition, ${}_{\text{CWHS}}\text{ATE}_{\text{as}}$ are projected. The first step is to determine preliminary ${}_{\text{CWHS}}\text{ATE}'_{\text{as}}$ by using the annual growth rate in the total economy-wide ATE⁷. A further multiplicative adjustment is made to each ${}_{\text{CWHS}}\text{ATE}'_{\text{as}}$ such that the resulting aggregate average taxable earnings, determined by combining the projected values of ${}_{\text{CWHS}}\text{CW}_{\text{as}}$ and ${}_{\text{CWHS}}\text{ATE}_{\text{as}}$ for the year, produces the same growth rate as the growth in total taxable earnings from the Economic process.

For additional explanation of this calculation, refer to example 3 in appendix 4.2-1 of this subprocess.

The historical and projected ${}_{\text{CWHS}}\text{ATE}_{\text{as}}$ are then used to change the earnings histories of the sample of newly entitled beneficiaries so that the earnings represent newly entitled beneficiaries in future years. For a given sex and age-group, the expected annual average taxable earnings of a future sample is denoted as ATE'_f . ATE'_f is computed by using the comparable changes⁵² in the ${}_{\text{CWHS}}\text{ATE}_{\text{as}}$. In addition, the annual average taxable earnings of the sample (after adjustments to the records' earnings levels for changes in wage bases and covered worker rates) are computed by sex and age-group and denoted as ATE_f . The difference between these values is the amount by which the average annual earnings levels are adjusted. Let

⁵⁰ These historical values are tabulated by the Economic subprocess.

⁵¹ These values are based on earnings posted to the Master Earnings File (MEF), excluding earnings posted to the suspense file.

⁵² The comparable changes reflect the wage-indexed changes in the ${}_{\text{CWHS}}\text{ATE}_{\text{as}}$ between the year of earnings in the sample of new beneficiaries and the year of earnings in the projected sample.

$$\delta(t) = ATE_f' - ATE_f,$$

for each year t . Then, $(\delta(t) * TotalWorkers(t))$ is the total amount of earnings which the model distributes for a given sex and age-group in a way so that the average taxable earnings after distribution is ATE_f' .

For additional explanation of the calculation $\delta(t)$, refer to example 4 in the appendix of this subprocess.

When $\delta(t)$ is negative, earnings for the year are decreased. To achieve ATE_f' for the given sex and age-group, AWARDS multiplies CoveredEarnings(r, t) by a ratio,

$$ratio(t) = 1 + \frac{\delta(t)}{ATE_f(t)} + \alpha.$$

The term, α , is an additional adjustment necessary because covered earnings above the wage base, have either no effect or only a partial effect on modifying ATE_f to ATE_f' . In the 2009 Trustees Report, α was set equal to -0.06 for both sexes and trust funds (OASI and DI).

When $\delta(t)$ is positive, earnings for the year are increased. However, the method of increasing earnings differs between disabled beneficiaries (DIBs) and retired worker beneficiaries (OABs).

For DIBs, this is done similar to the adjustment when $\delta(t)$ is negative, where the ratio is multiplied by covered earnings to raise each record's earnings for year t . If the resulting value exceeds the future wage base in year t , taxable earnings would equal the wage base. Hence, again, a constant α is added to account for the fact that increasing covered earnings which are greater or equal to the future wage base has no effect and covered earnings a little below the future wage base has only a partial effect on modifying ATE_f' to ATE_f . For the 2009 Trustees Report, $\alpha = 0.02$ for females and $\alpha = 0.03$ for males.

However, when earnings for the year are increased for OABs, the ratio takes into account that among OABs, there are many more workers with earnings near or above the future wage base. In order to account for this, AWARDS first computes z , such that

$$z(t) = \frac{\delta(t) * \left(\sum_{n=1}^{20} NumberWorkers(n, t) \right) + \frac{\delta(t)}{h(t)} * \left(\sum_{n=j}^{20} NumberWorkers(n, t) \right) * \frac{\delta(t)}{2}}{\sum_{n=1}^{20} NumberWorkers(n, t)},$$

where

(1) $\text{NumberWorkers}(n, t)$ equals the number of workers whose earnings in year t fall within the n th interval, that is the number of workers whose earnings are greater or equal to $\text{IntervalLength}(t) * (n-1)$ and less than $\text{IntervalLength}(t) * n$. For this calculation,

$$\text{IntervalLength}(t) = \frac{\text{WageBase}(t)}{20}. \text{ Also, } \text{TotalWorkers}(t) = \sum_{n=1}^{20} \text{NumberWorkers}(n, t).$$

(2) $j(t)$ is the interval, such that $j(t) = 20 - \left\lfloor \frac{\delta(t)}{\text{IntervalLength}(t)} \right\rfloor$.

(3) $h(t)$ is the dollar amount from interval $j(t)$ to the wage base,
 $h(t) = \text{IntervalLength}(t) * (21 - j(t))$.

Now the covered earnings of OAB records are multiplied by the ratio

$$\text{ratio}(t) = 1 + \frac{z(t)}{\text{ATE}_t(t)} + \alpha,$$

where $\alpha = 0.03$ for males and $\alpha = 0.02$ for females.

As AWARDS applies $\text{ratio}(t)$ to $\text{Earnings}(r, t)$ by each record, it makes sure total adjustment in a year does not exceed $\delta(t) * \text{TotalWorkers}(t)$.

For additional explanation of this calculation, refer to example 5 in appendix 4.2-1 of this subprocess.

Equation 4.2.2 – Average Indexed Monthly Earnings (AIME)

Step 1: Index Earnings

To compute an individual's AIME, all taxable earnings after 1950 are considered. First, the earnings are indexed up to the index year, i , which is defined as the year of attaining age 60 for retired-worker beneficiaries (eligible for benefits at age 62). For disabled-worker beneficiaries, i generally equals 2 years before the year of disability onset when disability insured. Thus,

$$\text{IndexedEarnings}(r, t) = \begin{cases} \text{Earnings}(r, t) * \frac{\text{AverageWage}(i)}{\text{AverageWage}(t)}, & \text{if } t < i \\ \text{Earnings}(r, t), & \text{if } t \geq i \end{cases}$$

Step 2: Determine Computation Years

For each record, the number of computation years, Y, is determined. For a retired-worker beneficiary in the sample, Y is 35.

For a disabled-worker beneficiary, Y is calculated as follows:

- Determine the number of elapsed years, which is equal to the year of disability onset (not later than the year the worker turned age 62) *minus* the greater of either the year the disabled worker turned age 22 or 1951.

$$\text{Elapsed Years} = \min\{\text{Year of disability onset, Year attained age 62}\} - \max\{1951, \text{Year attained age 22}\}$$

- Divide the elapsed years by five and truncate. Subtract this number (cannot exceed five) from the number of elapsed years.

$$Y = \text{Elapsed Years} - \min\left\{\left\lfloor \frac{\text{Elapsed Years}}{5} \right\rfloor, 5\right\}$$

- Y must be at least 2.

Step 3: Determine AIME

Finally, an individual's AIME is computed by summing the highest Y indexed earnings and dividing by the number of months in those years. Hence, for each record,

$$\text{AIME}(r) = \frac{\sum \text{Highest Y Indexed Earnings}(r)}{Y * 12}$$

Equation 4.2.3 – AIME_n(r)

The Possible AIME value is divided into 30 intervals (*bend point subintervals*). The length of each interval in 1979 dollars is given below:

$$\text{bp}_n = \begin{cases} \$45, & \text{if } 0 < n \leq 13 \\ \$100, & \text{if } 14 \leq n \leq 18 \\ \$200, & \text{if } 19 \leq n \leq 28 \\ \$1000, & \text{if } 29 \leq n \leq 30 \end{cases}$$

Thus, the interval points of AIME division given below in 1979 dollars, y_k , are equal to $\sum_{n=1}^k \text{bp}_n$

and

$$y_k = \begin{cases} \$180, & \text{if } k = 4 \\ \$1085, & \text{if } k = 18 \\ \$5085, & \text{if } k = 30. \end{cases}$$

For each record (r), the values for bp_n are indexed from 1977 to his/her indexing year i using the Social Security average wage index (AWI). So for $n = 1$ to 30,

$$bp_n(r) = bp_n * \frac{AWI(i)}{AWI(1977)}$$

Next the record's AIME amount, $AIME(r)$, is compared to the indexed intervals. If

$$\sum_{n=1}^{k-1} bp_n(r) < AIME(r) \leq \sum_{n=1}^k bp_n(r),$$

then $AIME(r)$ falls within the k th interval. And for $n = 1$ to 30,

$$AIME_n(r) = \begin{cases} bp_n(r), & \text{if } n < k \\ AIME(r) - \sum_{n=1}^k bp_n(r), & \text{if } n = k \\ 0, & \text{if } n > k \end{cases}$$

Equation 4.2.4 – Potential AIME Percentages (PAPS)

Finally, for $n = 1$ to 30, AWARDS sums the values of $AIME_n$ and bp_n across all the records for years 2008 to 2085 by sex, age (20-64 for disabled workers, and 62-70 for retired workers), and trust fund. The ratio of these values gives the average potential AIME percentages (PAPS)

$$PAP_n = \frac{\sum_r AIME_n(r)}{\sum_r bp_n(r)}.$$

Appendix 4.2-1

This appendix provides examples to help understand the calculations described in the model documentation of the AWARDS subprocess. **These examples do not reflect actual values.**

Example 1.1: (OASI-Male)

Task: In projecting the 2004 sample of newly entitled male beneficiaries to represent newly entitled male beneficiaries in 2050, an adjustment to the earnings histories for those males age 40-44 is needed to reflect lower covered worker rates expected for males in this age group.

This example illustrates the calculation of the projected covered worker rate for males who are age 40-44 in the projection period. We will be comparing the group of males age 40-44 in the base period with its counterpart group of males age 40-44 in the projection period.

Information given:

- Newly entitled retired male beneficiaries represented in the 2004 sample are age 40-44 in the base period, 1974-1986, and the counterpart group of males retiring in 2050 is age 40-44 in the projection period, 2020-2032.
- Based on the 2004 sample, covered worker rates for males age 40-44 in the base period = 88.0%.
- Economy-wide covered worker rate for males age 40-44 = 85.0% in the base period.
- Economy-wide covered worker rate for males age 40-44 = 75.0% in the projection period.

Calculations:

1. The potential difference in the economy-wide covered worker rate for males age 40-44 in the projection period is 100.0% - 75.0% or 25.0%.
2. The potential difference in the economy-wide covered worker rate for males age 40-44 in the base period is 100.0% - 85.0% or 15.0%.
3. The ratio from *steps 1 and 2* is 25/15 or 166 2/3 %.
4. The potential difference in the sample's covered worker rate for the males age 40-44 in the base period is 100.0% - 88.0% or 12.0%.
5. The ratio from *step 3* is multiplied by the potential difference in the sample's covered worker rate for males age 40-44 in the base period to yield 20.0% (166 2/3 % * 12.0% = 20.0%).
6. The amount in *step 5* (20.0%) would be subtracted from 100% to yield the sample covered worker rate for males who are age 40-44 in the projection period (80.0%).

Example 1.2 : (OASI - Female)

Task: In projecting the 2004 sample of newly entitled female beneficiaries to represent newly entitled female beneficiaries in 2050, an adjustment to the earnings histories for those females age 40-44 is needed to reflect higher covered worker rates expected for females in this age group.

This example illustrates the calculation of the projected covered worker rate for females who are age 40-44 in the projection period. We will be comparing the group of females age 40-44 in the base period with its counterpart group of females age 40-44 in the projection period.

Information given:

- Newly entitled retired female beneficiaries represented in the 2004 sample are age 40-44 in the base period, 1974-1986, and the counterpart group of females retiring in 2050 is age 40-44 in the projection period, 2020-2032.
- Based on the 2004 sample, covered worker rates for females age 40-44 in the base period = 75.0%.
- Economy-wide covered worker rate for females age 40-44 = 68.0% in the base period.
- Economy-wide covered worker rate for females age 40-44 = 72.0% in the projection period.
- Projected sample covered worker rate for males age 40-44 = 80.0% in the projection period. (as shown in example 1.1 above)

Calculations:

1. The absolute difference in the economy-wide covered worker rate for females age 40-44 from the base period to the projection period is $72.0\% - 68.0\%$ or 4.0% .
2. The difference between the projected covered worker rate for males age 40-44 and the economy-wide covered worker rate for females age 40-44 in the base period is $80.0\% - 68.0\%$ or 12.0% .
3. The ratio from *steps 1 and 2* is $4.0/12.0$ or $33 \frac{1}{3} \%$.
4. The difference between projected covered worker rate for males age 40-44 and sample's covered worker rate for the females age 40-44 in the base period is $80.0\% - 75.0\%$ or 5.0% .
5. The ratio from *step 3* is multiplied by the potential difference in the sample's covered worker rate for females age 40-44 in the base period to yield 1.667% ($33 \frac{1}{3} \% * 5.0\% = 1.667\%$).

6. The amount in *step 5* (1.667%) would be added to the sample covered worker rate for females age 40-44 in the base period (75.0%) to yield the projected covered worker rate for females who are age 40-44 in the projection period (76.667%).

Example 2:

Task: In projecting the 2004 sample of newly entitled male OASI beneficiaries to represent newly entitled male OASI beneficiaries in 2021, an adjustment to the earnings histories for those males age 40-44 is needed to reflect higher covered worker rates expected for males in this age group. The amount of increase in the covered worker rates is computed as described in example 1 and found to be 4 percent. To achieve this target, the desired number of records with zero reported earnings in this age group are randomly selected and assigned earnings.

This example illustrates the calculation of earnings to be assigned to a randomly chosen newly entitled retired male record with zero taxable earnings in the base year.

Information given:

- Newly entitled retired male beneficiaries represented in the 2004 sample are age 40-44 in the base period, 1974-1986, and the counterpart group of males retiring in 2021 will be age 40-44 in the projection period, 1991-2003.
- Based on the 2004 sample, a male record, $r = 60,000$ has been randomly selected to replace his zero taxable earnings reported in the base year 1981 with an amount based on his career earnings pattern. This record is age 63 in 2004, and his year of birth is 1941.
- A beneficiary retiring at age 63 in 2021 will have a year of birth of 1958. And the corresponding projection year to the base year of 1981 is the projection year 1998.
- The Average Indexed Earnings for this record, AIE (60,000), is computed to be \$43,465. Note: This value is calculated by (1) using the record's annual taxable earnings reported each year through 2003, (2) converting them to 2003 year dollars, and then (3) summing the highest 35 years of earnings and dividing by 35.
- The Average Indexed Earnings for a hypothetical worker, $AIE(w)$ whose year of birth is 1941 is \$41,995. This value is calculated as above given the hypothetical worker earned the *average taxable earnings* in each of the base years for males retiring at age 63 in the 2004 sample.

- The projected Average Taxable Earnings⁵³ of males retiring in 2021 for the projection year 1998 (converted to 1981 dollars⁵⁴) is $ATE_{2020}(\text{male}, 1998) = \$18,718$.

Calculations:

1. The ratio of the Average Indexed Earnings for record number 60,000, $AIE(60,000)$ to the Average Indexed Earnings of a hypothetical male worker born in 1941 and retiring at age 63, $AIE(w)$ is $\$43,465/\$41,995$ or 1.035.
2. The amount in *step 1* (1.035) would be multiplied to $ATE_{2021}(\text{male}, 1998)$ given to be $\$18,718$. This yields the amount of earnings assigned to record number 60,000 in the projection year 1998 representing a sample retiring in 2021, Thus, $Earnings(60,000, 1998) = 1.035 * \$18,718$ which equals $\$19,373$.

Example 3:

Task: The AWARDS subprocess estimates projected values of Average Taxable Earnings by age and sex using the values⁵⁵ in the 2006 CWHS file supplied by the Economic subprocess as the base year on which to build our projections.

This example illustrates the calculation of the projected Average Taxable Earnings of the CWHS in 2007 for 42 year old females, $CWHSATE_{42, \text{female}}(2007)$. We will be using the number of female covered workers age 42 and the total taxable earnings for females age 42 as given the in 2006 CWHS data.

Information given:

- The total number of female covered workers age 42 in 2006 reported in the CWHS, $CWHS CW_{42, \text{female}}(2006)$ is 17,440. .
- The total taxable earnings for females age 42 in 2006 reported in the CWHS is $\$544,410,075$.
- The economy-wide number of covered workers for females age 42 in 2006 is 1,764,582.

⁵³ The average taxable earnings have been computed using projected earnings adjusted for changes in the wage base. Adjustments to earnings for changes in covered worker rates and the earnings experience in the CWHS have not been applied at this stage in the process.

⁵⁴ All projection year dollars are converted back to ‘sample year’ dollar amounts.

⁵⁵ Because not all earnings are posted for the most recent years for a given CWHS file, adjustment factors, based on historical trends, are applied by the Economic subprocess to complete these earnings. For the 2006 CWHS, adjustment factors were applied to data in years 2003 through 2006.

- The economy-wide number of covered workers for females age 42 in 2007 is 1,689,023.
- The economy-wide Average Taxable Earnings (ATE) in 2006 is \$31,030.
- The economy-wide ATE in 2007 is \$32,089.

Calculations:

1. The growth in the economy-wide covered workers for females age 42 from 2006 to 2007 is $1,689,023/1,764,582$ or 0.9572.
2. The growth rate in the economy-wide covered workers for females age 42 from *step 1* is applied to the CWHS female covered workers age 42 in 2006, $CWHS_{42,female}(2006)$ to yield the projected covered workers for females who are age 42 in 2007, $CWHS_{42,female}(2007)$, $0.9572 * 17,440$ or 16,694
3. The CWHS Average Taxable Earnings for females age 42 in 2006, $CWHS_{ATE_{42,female}}(2006)$ is computed using the given CWHS total taxable earnings and total covered workers for females age 42 in 2006 ($\$544,410,075 / 17,440 = \$31,216$).
4. The economy-wide growth rate in the average taxable earnings from 2006 to 2007 is $\$32,089 / \$31,216$ or 1.028
5. The growth rate in *step 4* (1.028) is applied to the amount in *step 3* ($\$31,216$) to estimate the preliminary CWHS Average Taxable Earnings for females age 42 in 2007, $CWHS_{ATE'_{42,female}}(2007) = \$32,090$ ($1.028 * \$31,216$)
6. The value from *step 5* ($\$32,090$) is multiplied by the value in *step 2* (16,694) to yield the preliminary CWHS projected total taxable earnings for females age 42 in 2007 ($\$535,710,460$).
7. Note a final multiplicative adjustment is made to $CWHS_{ATE'_{42,female}}(2007)$ to obtain $CWHS_{ATE_{42,female}}(2007)$. This adjustment is made once the preliminary taxable earnings have been computed for each age and sex in 2007 in order to ensure the growth rate in the aggregate average taxable earnings between 2006 and 2007 matches the growth rate in the ATE from the Economic process.

Example 4:

Task: In projecting the 2004 sample of newly entitled female beneficiaries to represent newly entitled female beneficiaries in 2050, the projected Average Taxable Earnings of females in the sample (ATE_f)⁵⁶ for year $t = 2020$ must be adjusted by an amount, $\delta(2020)$, to meet a targeted Average Taxable Earnings(ATE_t') for 2020.

⁵⁶The average taxable earnings have been computed using projected earnings adjusted for changes in the wage base and for changes in covered worker rates.

This example illustrates the calculation of $\delta(2020)$ for the female cohort retiring at ages 62 to 70 in the projection year 2050. The cohort is age 32 to 40 in the projection year 2020. The value $\delta(2020)$ is the dollar amount in which the average annual earnings levels are adjusted for females in this age-group in the year 2020 and retiring in 2050. We will be comparing this group of females age 32-40 in the projection 2020 year with its counterpart group of females age 32-40 in the base year 1974.

Information given:

- A cohort of newly entitled retired female beneficiaries retiring at ages 62 to 70 represented in the 2004 sample are ages 32-40 in the base year, 1974, and the counterpart group of females retiring in 2050 are ages 32-40 in the projection year, 2020.
- Based on the 2004 sample, the average taxable earnings for females in the base year 1974 is \$4,536.
- For a sample projected to be retiring in 2050, the average taxable earnings (ATE_f) for females in the projection year 2020 is \$4,564⁵⁷, after applying adjustments to the records' earning levels for changes in the wage base and in covered worker rates.
- Based on the 2006 CWHS data, the average taxable earnings for females ages 32-40 in the base year 1974 is \$4,218.
- The projected average taxable earnings in the CWHS for females ages 32-40 in the projection year 2020 (in 1974 dollar amounts) is \$4,640.

Calculations:

1. The increase in the average taxable earnings for females ages 32-40 in the CWHS from the base year 1974 (\$4,218) to the projection year 2020 (\$4,640)¹² is 10 percent.
2. Based on the value from *step 1* the expected annual average taxable earnings (ATE_f') of females ages 32-40 in 2020 and retiring in 2050 is $\$4,536 * 1.10$ or \$4,990
3. The difference in ATE_f' (\$4,990) and ATE_f (\$4,564) yields the $\delta(2020)$ value \$426 ($\$4,990 - \$4,564 = \426).

Example 5:

Task: In projecting the 2004 sample of newly entitled female beneficiaries to represent newly entitled female beneficiaries in 2033, for year $t = 2023$, $\delta(2023)$ is positive indicating an adjustment to earnings histories is needed to reflect higher average taxable earnings by this cohort for the projection year 2023.

⁵⁷ Amount is in 1974 dollars, 'sample year dollars'.

This example illustrates the calculation of the ratio(2023) in projection year 2023 for the female cohort retiring at ages 62 to 70 in the projection year 2033. The value, ratio(2023), is the adjustment ratio that will be applied to this cohort projected covered earnings in 2023 in order to achieve the targeted Average Taxable Earnings of this cohort for 2023.

Information given:

- Earnings in the projection year 2023 for a group of newly entitled female beneficiaries retiring in 2033 is the counterpart corresponding to earnings in the base year 1994 for the group of newly entitled female beneficiaries in the 2004 sample.
- For newly entitled females retiring in 2033, the average taxable earnings (ATE₂₀₂₃) for the projection year 2023 is \$19,970⁵⁸.
- $\delta(2023)$, the difference in the targeted average taxable earnings ATE₂₀₂₃' and ATE₂₀₂₃, is given to be \$1,590.
- The wage base for the projection year 2023 (in 1994 dollar amounts) is \$57,600
- The number of females in the projected 2033 sample with earnings below the wage base in 2023 is 52,800.
- The dollar amounts below the wage base in 2023 are divided into 20 equal intervals, IntervalLength(2023), of \$2,880 each.
- The number of females in the 2033 sample who in 2023 have earnings falling in the 20th interval (their earnings are between \$54,720 and \$57,600) is 465
- For female OABs the constant α when $\delta(t)$ is positive is 0.02.

Calculations:

- The interval in which \$56,010 (the wage base in 2023 minus $\delta(2023)$ or \$57,600-\$1,590) falls within is interval $j(2023)$ and is found by rounding $\delta(2023)/$ IntervalLength(2023) or (\$1590/\$2880) to the lowest integer and subtracting this value from 20. This yields interval 20.
- The dollar amount from interval 20 to the wage base is $h(2023)$, and is \$2,880*(21-20) or \$2,880
- Using the equation given on page 8 yields,

$$z(2023) = \frac{\$1,590 * (52,800) + \frac{\$1,590}{\$2,880} * (465) * \frac{\$1,590}{2}}{52,800} = \$1,594$$

Note, the value for $z(2023)$ is higher than $\delta(2023)$ in order to adjust for those records above or close to the wage base.

⁵⁸ Amount is in 1994 dollars, 'sample year dollars'.

- The ratio multiplied to the covered earnings in 2023 for females retiring in 2033 is $1 + (\$1,594/\$19,970) + 0.02$, or 1.10.

Appendix 4.2-2

This appendix provides additional details on how the AWARDS process imputes covered earnings above the historical wage base.

To do this, AWARDS first computes the cumulative distribution, F , of the workers in the sample by their earnings level. Each historical wage base is divided into 20 equal intervals, n , and each interval length in year t is

$$\text{IntervalLength}(t) = \frac{\text{WageBase}(t)}{20}.$$

The cumulative distribution $F(n, t)$ is the proportion of workers whose earnings are less than $\text{IntervalLength}(t)*n$, for $n = 1$ to 20. Let $\text{NumberWorkers}(n, t)$ be the number of workers whose earnings in year t fall within the n th interval, that is the earnings are greater or equal to $\text{IntervalLength}(t)*(n-1)$ and less than $\text{IntervalLength}(t)*n$. Also, let $\text{TotalWorkers}(t)$ be the total number of workers in the sample with earnings in year t . Then for any n , $1 \leq n \leq 20$,

$$F(n, t) = \frac{\sum_{m=1}^n \text{NumberWorkers}(m, t)}{\text{TotalWorkers}(t)}.$$

Once $F(n, t)$ is computed for $n = 1$ to 20, AWARDS extends the function for those who had earnings at the wage base. To extrapolate F past the historical base (define $F(n, t)$ for $n > 20$), AWARDS groups the maximum earners in each year in the sample based on the number of years they had earnings at the wage base during the next four years (0, 1, 2, 3, 4). Under the assumption of uniform distribution within each group, AWARDS assigns an $F(n_r, t)$ value to each record with earnings at the tax maximum beginning with the group that has no other earnings at the tax maximum during the next four years and ending with the group that has maximum earnings in each of the next four years. Note that for these beneficiaries $F(n_r, t) > F(20, t)$. Once $F(n_r, t)$ is computed for these beneficiaries, values for $F(n, t)$, where $n > 20$ are estimated.

To find $F(n, t)$, where $n > 20$, the log odds transformation is utilized. The odds ratio,

$$T(n, t) = \frac{F(n, t)}{1 - F(n, t)}, \text{ where } n \leq 20$$

is the ratio of (1) the proportion of beneficiaries with earnings levels below the n th interval to (2) the proportion of beneficiaries with earnings levels above the n th interval. Next, the natural logarithm of the odds ratio is computed, giving the log odds transformation,

$$Y(n,t) = \ln[T(n,t)] = \ln\left(\frac{F(n,t)}{1-F(n,t)}\right), \text{ where } n \leq 20$$

Utilizing the most linear portion of the function at the upper values of n , AWARDS regresses Y on those values. The regression line of Y has the form

$$\hat{Y}(n,t) = \beta_0 + \beta_1 * n$$

Finally, the amount of covered earnings of a record that has earnings at the taxable maximum is determined based on the $F(n_r, t)$ value assigned to the record. The $F(n_r, t)$ value for this record is used in the above equations to determine $T(n_r, t)$, $Y(n_r, t)$, and then n_r , the non-integer value for n in the regression equation of \hat{Y} above. Thus, if $\text{earnings}(r, t) = \text{wage base in year } t$ then

$$\text{CoveredEarnings}(r, t) = n_r * \text{IntervalLength}(t) + \text{error}^{59},$$

where n_r is the record's non-integer value for n in the regression equation of \hat{Y} above.

If $\text{earnings}(r, t) < \text{wage base in year } t$, then $\text{CoveredEarnings}(r, t) = \text{earnings}(r, t)$

At this point, AWARDS defines the expected taxable earnings of a future sample as,

$$\text{Earnings}(r, t) = \begin{cases} \text{CoveredEarnings}(r, t), & \text{CoveredEarnings}(r, t) < \text{future wage base (t)} \\ \text{future wage base (t)}, & \text{CoveredEarnings}(r, t) \geq \text{future wage base (t)} \end{cases}$$

⁵⁹ The difference between $Y(20,t)$ and $\hat{Y}(20,t)$.

4.3. Cost

4.3.a. Overview

The COST subprocess projects the trust fund operations for each year of the long-range 75 year period. The COST subprocess projects the income and cost for each trust fund (OASI and DI). The two components of non-interest income are payroll contributions and taxation of benefits. The other component of income is interest earned on the trust fund assets. The three components of cost are scheduled benefits, administrative expenses, and the railroad interchange. Each of these components is projected for each trust fund (OASI and DI). The end-of-year assets is computed by taking the beginning-of-year assets (*ASSETS*), adding payroll contributions (*CONTRIB*), taxation of benefits (*TAXBEN*), and interest income (*INT*), and subtracting scheduled benefits (*BEN*), administrative expenses (*ADM*), and the railroad interchange (*RR*).

Equations 4.3.1 through 4.3.6 outline this overall structure and sequence.

$$CONTRIB = CONTRIB(\cdot) \quad (4.3.1)$$

$$BEN = BEN(\cdot) \quad (4.3.2)$$

$$TAXBEN = TAXBEN(\cdot) \quad (4.3.3)$$

$$ADM = ADM(\cdot) \quad (4.3.4)$$

$$RR = RR(\cdot) \quad (4.3.5)$$

$$INT = INT(\cdot) \quad (4.3.6)$$

$$ASSETS_{EOY} = ASSETS_{BOY} + CONTRIB + TAXBEN + INT - BEN - ADM - RR$$

The COST subprocess produces annual values which help assess the financial status of the OASI, DI, and combined funds. These include the annual income rate (*ANN_INC_RT*), annual cost rate (*ANN_COST_RT*), and trust fund ratio (*TFR*) as outlined below.

$$ANN_INC_RT = ANN_INC_RT(\cdot) \quad (4.3.7)$$

$$ANN_COST_RT = ANN_COST_RT(\cdot) \quad (4.3.8)$$

$$TFR = TFR(\cdot) \quad (4.3.9)$$

The COST subprocess also produces summarized values. These values are computed for the entire 75-year projection periods, as well as 25- and 50-year periods. These include the actuarial balance (*ACT_BAL*), unfunded obligation (*UNF_OBL*), summarized income rate (*SUMM_INC_RT*), summarized cost rate (*SUMM_COST_RT*), and closed group unfunded obligation (*CLOSEDGRP_UNFOBL*).

$$ACT_BAL = ACT_BAL(\cdot) \quad (4.3.10)$$

$$UNF_OBL = UNF_OBL(\cdot) \quad (4.3.11)$$

$$SUMM_INC_RT = SUMM_INC_RT(\cdot) \quad (4.3.12)$$

$$SUMM_COST_RT = SUMM_COST_RT(\cdot) \quad (4.3.13)$$

$$CLOSEDGRP_UNFOBL = CLOSEDGRP_UNFOBL(\cdot) \quad (4.3.14)$$

The following notation is used throughout this documentation:

- ni represents the first year of the projection period-2009 for the 2009 TR
- $ni+74$ represents the final year of the projection period-2083 for the 2009 TR
- nf represents the last year the cost program will project-2085 for the 2009 TR
- $nim1$ is equal to $ni-1$
- $nim2$ is equal to $ni-2$
- ns is equal to $ni+9$
- $nbase$, the year of the sample, is equal to 2004

4.3.b. *Input Data*

Data received as input from the short-range office are presented first. Then data from long range and all other sources are identified separately for each equation.

Short-range OCACT Data

- Estimates for the first ten years of the projection period for the first six equations mentioned above.
- Assets at the beginning of year ni .

All of this information is updated annually.

Long-range OCACT and other Data

i. Equation 4.3.1 – Tax Contributions (CONTRIB)

Economics-Process 2

- Projected effective taxable payroll for years $nim1$ through nf , updated yearly

Other

- Projected employee/employer payroll tax rate, by trust fund and year, for years 1981 through nf , updated as needed (e.g., as required due to legislative changes)

ii. Equation 4.3.2 – Scheduled Benefits (BEN)

Demography-Process 1

- Projected number of married and divorced people in the Social Security area population age 65 and above for beginning of years *nim1* through 2101, updated yearly

Economics-Process 2

- Historical COLA for years 1975 through *nim2*, updated yearly
- Historical CPI for years 1990 through *nim1*, updated yearly
- Projected cost of living adjustment (COLA) for years *nim1* through *nf*, updated yearly
- Historical SSA average wage index for year *nim2*, updated yearly
- Projected percent increases in the average wage index for years *nim1* through *nf*, updated yearly
- Coefficients for five dual entitlement equations, updated as needed

Beneficiaries-Process 3

- Initial and ultimate incurred but not reported (IBNR) DI beneficiary distribution by age, duration of disability (0 through 9 years and 10+ years) and sex, updated every few years (subprocess #3.2). Factors are read in such that there is a ten year linear phase-in from initial factors to ultimate factors. The number of DI beneficiaries in current-payment status is equal to the number of currently entitled DI workers times the IBNR factor.
- Projected number of disabled workers in current-pay status by sex, age in current-pay, and duration of disability (0 through 9 and 10+) for years *nim1* through *nf*, updated yearly from subprocess 3.2
- Projected number of retired worker beneficiaries in current-pay status by sex, age in current-pay, and age at entitlement for years *nim1* through *nf*, updated yearly from subprocess 3.3
- Projected number of auxiliary beneficiaries (by benefit category) of retired-worker, deceased-worker, and disabled-worker beneficiaries for years *nim1* through *nf*, updated yearly from subprocess 3.3
- Projected number of disability insurance beneficiaries who convert to retirement insurance status upon the attainment of normal retirement age by age in current pay, for years *nim1* through *nf*, updated yearly from subprocess 3.2 and 3.3
- Retired Workers 65+ by sex, and marital status (single, married, widowed, and divorced)
- Retired Workers 62+ by sex, and marital status (single, married, widowed, and divorced)

Other

- Total (aggregate) PIA and MBA, not actuarially reduced, of DI male and female workers in current payment status, updated yearly from the 1-A Table Supplement
- Total (aggregate) PIA and MBA, actuarially reduced, of DI male and female workers in current payment status, updated yearly from the 1-A Table Supplement

- Cumulative distribution of AIME dollars for newly entitled retired-worker beneficiaries by age (62 through 70) and sex, for years *nim1* through *nf*, updated yearly from subprocess 4.2
- Cumulative distribution of AIME dollars for newly entitled disabled-worker beneficiaries by age (20 through 64) and sex, for years *nim1* through *nf*, updated yearly from subprocess 4.2. Ages 15 through 19 are assumed to have the same distribution of dollars as does age 20. Ages 65 and 66 are assumed to have the same distribution of dollars as does age 64
- Age distribution of newly entitled retired-worker beneficiaries in the AIME awards sample by sex, updated in years that the sample changes, from subprocess 4.2
- Starting average PIA matrix for retired-worker benefits for the year *nim1*, by age at entitlement, age in current-pay and sex, updated yearly
- Starting average PIA matrix for disabled-worker benefits, for the year *nim1*, by age in current-pay, duration and sex, updated yearly
- Starting average PIA matrix for beneficiaries who convert from disabled worker to retirement worker status for the year *nim1*, by age in current-pay and sex, updated yearly
- Starting average MBA matrix for retired-worker benefits for the year *nim1*, by age at entitlement, age in current-pay and sex, updated yearly
- Benefit relationships between worker and auxiliary benefits (linkages) for the year *nim1*, for all benefit categories and worker account holders of both sexes, updated yearly from qlink09.xls
- Retroactive payment loading factors for auxiliary beneficiary categories for all years, for each benefit category and both sexes, updated yearly
- Initial and ultimate post entitlement factors for retired workers by sex and duration updated yearly. Factors are read in such that there is a twenty year linear phase-in from initial factors to ultimate factors.
- Initial and ultimate post entitlement factors for disabled workers by sex and duration updated yearly. Factors are read in such that there is a twenty year linear phase-in from initial factors to ultimate factors.
- Initial and ultimate post entitlement factors for DI conversion workers by sex and duration updated yearly. Factors are read in such that there is a twenty year linear phase-in from initial factors to ultimate factors.
- Historical AIME deviation, by sex for years 1995 through *nim1*, updated yearly
- Average excess MBA amounts for dually entitled wives and widows, updated yearly from Statistical Supplement
- Initial Windfall Elimination Provision (WEP) factors by sex and age for attributed year, updated every other year
- Ultimate WEP factors by sex read in as a percentage of the way from initial factors to one, updated every other year
- Year in which ultimate WEP factor is reached by age at initial entitlement (62-70), updated every other year
- Trendline by which WEP factors are phased-in from initial value to ultimate value,

- updated every other year
- Workers Compensation cumulative factors by duration that adjust benefits to account for decreasing offsets (i.e – Workers Comp offsets decrease as duration increases), updated yearly
- Workers Compensation reduction factors (used in retroactive category) to reflect offsets starting and stopping in the year of DI entitlement
- Workers Compensation parameter to account for offsets that begin and end in the year of entitlement, updated yearly
- Dual Entitlement regression coefficients for wives, widows, widowers, excess wives and excess widows, updated yearly
- Target values for ratios relating to the five above dual entitlement categories, updated yearly
- Number of years in which the difference between the results from the regression coefficients and targeted values are phased in

iii. *Equation 4.3.3 – Taxation of Benefits*

Trust Fund Operations and Actuarial Status

- Taxation of benefits as a percentage of scheduled benefits by trust fund for years ni through nf , updated yearly from subprocess #4.1

iv. *Equation 4.3.4 – Administrative Expenses*

Economics-Process 2

- Average wage indexes for years ni through nf , updated yearly
- Ultimate value of productivity factor for the period ni through nf updated yearly

Beneficiaries-Process 3

- Total number of beneficiaries in current-pay status by trust fund for years ni through nf , updated yearly

v. *Equation 4.3.5 – Railroad Interchange*

Economics-Process 2

- Increase in the average wage index for years ni through nf , updated yearly
- Ultimate value of productivity factor for the period ni through nf updated yearly

Trust Fund Operations and Actuarial Status

- Taxation of benefits as a percent of the amount of benefits scheduled to be paid, by trust fund for years ni through nf , updated yearly (use same factors as in equation 4.3.3)

Other input data

- Nominal annual yield rate on the combined OASDI trust fund for years ni through nf , updated as the ultimate real interest rate and ultimate CPI are changed by the Trustees. Ratio of railroad retirement average benefit to OASI average benefit for the year $nim1$, updated yearly
- Number of railroad beneficiaries (retirement and disability) for December of year $nim3$, updated yearly
- Average taxable earnings in railroad employment for year $nim2$, updated yearly
- Average worker benefit by sex and trust fund, updated yearly from the December $nim1$ Table I-A Supplement
- Ratio of railroad worker benefit to worker benefit by trust fund, updated yearly using 10 years of historical financial interchange benefit data
- Auxiliary loading factor by trust fund, updated yearly using 10 years of historical financial interchange benefit data
- Railroad initial mortality rate by trust fund for year $nim2$, updated yearly using 10 years of historical financial interchange benefit data
- Railroad mortality improvement rate by trust fund, updated yearly using 10 years of historical financial interchange benefit data

vi. *Equation 4.3.6 – Interest Income*

Economics-Process 2

- Annual increase in the CPI for years ni through nf , updated yearly
- Factors for exposure to interest rate for benefits, payroll, taxation of benefits, railroad interchange and administrative expenses, updated periodically

Trustees assumptions

- Ultimate real interest rate, reviewed annually

vii. *Equations 4.3.7 through 4.3.13 – Annual Values and Summarized Values*

All inputs for equations 4.3.7 through 4.3.13 are estimated internally in the Cost program.

viii. *Equation 4.3.14 – Closed Group Unfunded Obligation*

Demographics-Process 1

- Single year population and mortality rate data for years 1941 through 2100, updated yearly
- Population projections by single year of age, 75-99, from $nim2$ to 2100, updated yearly

Economics-Process 2

- Historical and projected single-year COLA data and average wage indexing series (AWI) data for years 1975 through 2085 (for COLA) and 1951 through 2085 (AWI),

- updated yearly
- Projected number of covered workers by single year of age 15-74 from year ni through nf , updated yearly
- Aggregate number of covered workers aged less than 15, and aged 75 and over, from years ni through nf , updated yearly

Beneficiaries-Process 3

- Total projected disabled workers by age for years 2008 to 2085, updated yearly

Awards-Process 4

- Projected number of workers and total taxable earnings by single year of age (15-74) and sex from 2008 to 2085, updated yearly

Other

- Total count of beneficiaries in 20 of the 28 beneficiary categories (excluding retired workers, disabled workers, aged spouses (married, divorced, and dually entitled excess), and aged widow(er)s (married, divorced, and dually entitled excess) from the December 2008 Master Beneficiary Record (MBR)⁶⁰—updated yearly
- Total benefits paid in 20 of the 28 beneficiary categories (excluding retired workers, disabled workers, aged spouses (married, divorced, and dually entitled excess), and aged widow(er)s (married, divorced, and dually entitled excess) from the December 2008 MBR—updated yearly
- Consumer Price Index data from 1951-1974 from Bureau of Labor Statistics
- Number of covered workers and average taxable earnings by single year of age 1-14 for years 1991-2005 from 1 percent Continuous Work History Sample (CWHs), updated yearly to include year $ni-4$
- Number of covered workers and average taxable earnings by single year of age 75-99 for years 1991-2005 from 1 percent CWHs, updated yearly to include year $ni-4$
- Distribution of assumed age differentials of wives compared to age of husbands—separate distributions used for aged spouse benefits and aged widow(er) benefits, respectively

4.3.c. *Development of Output*

i. Equation 4.3.1 – Payroll Tax Contributions (CONTRIB)

It would be natural to estimate the payroll tax contributions by trust fund by multiplying the applicable employer/employee tax rate by effective taxable payroll. However, tax contributions

⁶⁰ For disabled adult children of deceased workers and lump-sum beneficiaries, data was extracted from a 1- percent sample of the December 2008 MBR, mainframe dataset ACT.TAPEL.CAN1208. For the other 18 auxiliary beneficiary categories, data was extracted from the 100 percent December 2008 MBR, mainframe dataset ACT.TAPEH.MBR100.D0812.CANSORT.

are reported on a cash basis. That is, tax contribution amounts are attributed to the year in which they are actually received by the trust funds, while taxable payroll is attributed to the year in which earnings are paid. In other words, the lag between the time the tax liability is incurred and when the taxes are actually collected must be reflected. If *lag* represents the proportion of incurred payroll taxes estimated to be received by the trust fund (*tf*) in year *yr*, then tax contributions (*CONTRIB*) are given by the formula

$$CONTRIB(tf, yr) = lag \times tax\ rate(tf, yr) \times payroll(yr) \\ + (1 - lag) \times tax\ rate(tf, yr - 1) \times payroll(yr - 1)$$

for $yr \geq ns$.

The value of *lag* is estimated from the combined OASI and DI tax contributions estimated to be collected in the final year of the short-range period, *ns*, and is given by

$$lag = \frac{\sum_{tf=1}^2 CONTRIB(tf, ns) - \sum_{tf=1}^2 (tax\ rate(tf, ns - 1) \times payroll(ns - 1))}{\sum_{tf=1}^2 (tax\ rate(tf, ns) \times payroll(ns) - tax\ rate(tf, ns - 1) \times payroll(ns - 1))}$$

For the first ten years of the long-range period, tax contributions are set equal to those provided by the short-range office. The same value of *lag* is used for all years, and both trust funds, thereafter.

ii. Equation 4.3.2 – Scheduled Benefits (BEN)

(1) Disabled-Worker Benefits

Disabled Worker Beneficiary Matrix

The number of disabled-worker beneficiaries for a given year and sex are provided from the subprocess 3.2. For each projection year, two matrices are provided – one for males and one for females. The structure of each matrix is as follows:

- 11 columns. The columns are indexed by duration of disability (0-9 and 10+).
- 52 rows. These rows correspond to the age in current pay, ages 15 through 66.

The COST subprocess, however, only uses 10 durations (0-8 and 9+), and 47 ages (ages 20 through 66). This requires a manipulation of the matrix of DI beneficiaries in current-pay status from subprocess 3.2. For ages in current pay greater than or equal to 30, the duration 9 and 10+ columns of this matrix are added to give the total number of duration 9+ beneficiaries. For ages (*ag*) between 20 and 30 inclusive, the number of beneficiaries in current-pay aged *ag* and duration *ag-20* is the value provided by the DISABILITY

subprocess added to the number of people in current pay aged $ag-j$ and duration $ag-20$ for $j=1,\dots,5$. (For example, the number of people aged 20 of duration 0 is combined with the number of people aged 15, 16, 17, 18 and 19 of duration 0; the number of people aged 21 of duration 1 is combined with the number of beneficiaries in current-payment status aged 16, 17, 18, 19, and 20 of duration 1, and so on. In other words, the five nonzero diagonal of the matrix provided by the DISABILITY subprocess is “combined with” the diagonal directly below it and then zeroed out.)

Building the Average Benefit Matrix for Disabled Workers

In each projection year, the COST subprocess produces an average benefit matrix for each sex. Each matrix is a 47 by 10 matrix whose entries are the average benefit amounts of disabled worker beneficiaries whose age in current pay is indexed by the rows (ages 20 through 66) and whose duration of disability is indexed by the columns (durations 0 through 8 and 9+).

The 100 percent Master Beneficiary Record (100% MBR) extract is processed by a side model. The final product of the side model is two matrices of benefits, one for males and one for females, for December *niml* (2008 for the 2009 TR).

For a given year of the projection period, a new average benefit matrix is obtained by moving the average benefit matrix from the previous year one year forward. The next few paragraphs describe this procedure.

In general, for each age in current-pay, the age and duration are incremented by 1 and the previous PIA amount is given a cost of living adjustment. In addition, the beneficiaries are given a workers compensation adjustment and a post-entitlement adjustment. For each duration $j = 0, 1, \dots, 7$, and 8+ and sex, let the workers compensation offset factor be denoted $wkcomp(yr, sx, dur)$. We have, for durations 0 through 8, that

$$avgmba(yr, sx, ag, dur) = avgmba(yr - 1, sx, ag - 1, dur - 1) \times (1 + COLA(yr)) \\ \times (1 + wkcomp(yr, sx, dur)).$$

A more careful explanation of the last factor, $wkcomp(yr, sx, dur)$, is given later in this document. See the section titled *Average PIAs and MBAs for Disabled-Worker Beneficiaries*, below. To move duration 8 average benefits to duration 9+ average benefits, both average benefits are given a cost of living adjustment and a post-entitlement adjustment (see section “Post-Entitlement Adjustments”). The resulting duration 9+ average benefit is the weighted average of the adjusted prior year duration 8 and 9+ average benefits, weighted by the prior year’s numbers of beneficiaries in current-pay status for durations 8 and 9+ respectively.

The only column that is not handled by this procedure is the duration 0 column. This column corresponds to newly entitled disabled-worker beneficiaries, and is handled below.

Average Benefits for Newly Entitled Disabled-Worker Beneficiaries

The potential AIME percentage values for newly entitled disabled-worker benefits (DPAPs) are obtained from the AWARDS subprocess. The two bendpoints of the PIA formula, BP1 and BP2, are indexed by the increase in the average wage index. In 1979 dollars, the values of BP1 and BP2 are \$180 and \$1,085 respectively. The AIME dollars between 0 and BP1 are divided into four intervals (each of length \$45 in 1979 dollars). The AIME dollars between BP1 and BP2 are divided into fourteen intervals (nine of length \$45 and five of length \$100, in 1979 dollars). Twelve additional intervals are added beyond BP2 (ten of length \$200 and two of length \$1,000, in 1979 dollars).

To determine the average PIA for newly entitled beneficiaries, the DPAP values for each of the thirty intervals of AIME dollars are multiplied by the dollar amounts attributable to each interval (the length of the interval) and by the associated PIA factors. The distribution of prior year disability onset and current year disability onset is taken into consideration. It is assumed that this distribution is 6 months for prior year disability onset and 6 months for current year disability onset. In the formulas below, $j = 1$ signifies prior year disability onset and $j = 2$ signifies current year disability onset.

Let:

- $Wage_Idx_j(sx, ag, yr) = \frac{avgwg(yr - \max(ag - 60, 1 + j))}{avgwg(nbase - 2)}$ for $j=1,2$.
- $Cum_COLA_1(ag, yr) = \begin{cases} (1 + COLA(yr - 1)) \times (1 + COLA(yr)) & ag < 64 \\ \prod_{k=61}^{ag} (1 + COLA(yr - (k - 62))) & 64 \leq ag \leq 66. \end{cases}$
- $Cum_COLA_2(ag, yr) = \begin{cases} 1 + COLA(yr) & ag < 63 \\ \prod_{k=62}^{ag} (1 + COLA(yr - (k - 62))) & 63 \leq ag \leq 66. \end{cases}$
- $w_j = \frac{6}{12} = \frac{1}{2}, j=1,2$.
- PIA_factor_i represent the PIA factor for interval i (equal to 0.90 for intervals $i=1,4$, 0.32 for intervals $i=5, \dots, 18$, and 0.15 for intervals $i=19, \dots, 30$).
- $AIME_dollars_i$ represent the length of interval i , expressed in 1979 dollars.
- $dpap_i(yr, sx, ag)$ represent the DPAP value for newly entitled disabled workers in year yr whose sex is sx and age is ag .

To take into account the workers compensation offset to disability benefits, administrative data is reviewed, from which a factor is developed and applied to the average award benefit. We now describe how this factor, $facm2p(yr, sx)$ is computed. The 1-A table supplement as of the end of December, 2008, contains total award PIA and MBA data for disabled workers, by sex, for beneficiaries both non-actuarially reduced and actuarially reduced. Let

$totmba_DIB_nar(yr, sx)$ and $totpia_DIB_nar(yr, sx)$ be the total MBA and PIA respectively for DIBs that are not actuarially reduced as found in the 1-A table. In the historical period 1988-*nim1* we define $facm2p(yr, sx)$ to be the ratio of the total MBA to the total PIA for those not actuarially reduced. In other words,

$$facm2p(yr, sx) = \frac{totmba_DIB_nar(yr, sx)}{totpia_DIB_nar(yr, sx)}.$$

In the period ni through $ns+9$, $facm2p(yr, sx)$ is defined as follows. Let

$$x = facm2p(yr-1, sx) \times \left(\frac{facm2p(yr-1, sx)}{facm2p(yr-11, sx)} \right)^{1/10}.$$

This value is further adjusted by the variable $facm2p_param$ to reflect the offset amounts that end within the first entitlement year. For the 2009 TR the data suggests this factor should be .28. As a result, for $yr = ni, \dots, ns+9$,

$$\begin{aligned} facm2p(yr, sx) &= x + (1-x) \times 0.28 \\ &= 0.28 + 0.72 \times x. \end{aligned}$$

The factor reaches its ultimate value in years $ns+10$ and later.

The preliminary average PIA for newly entitled disabled worker beneficiaries may now be defined. It is equal to

$$\begin{aligned} LR_awdpia(sx, ag, yr) &= \sum_{i=1}^{30} PIA_factor_i \times AIME_dollars_i \times dpap_i(yr, sx, ag) \\ &\quad \times \left(\sum_{j=1}^2 w_j \times Wage_Idx_j(sx, ag, yr) \times Cum_COLA_j(ag, yr) \right) \\ &\quad \times facm2p(yr, sx). \end{aligned}$$

Once these average PIAs of newly entitled disabled-worker beneficiaries are computed, their values are filled into the average PIA matrices for duration 0 for the appropriate entitlement age.

Average PIAs and MBAs for Disabled-Worker Beneficiaries

An overall average PIA of newly entitled disabled worker beneficiaries for each sex and projection year is computed by taking the weighted average of $awdpia(sx, ag, yr)$, the weights being the number of disabled workers in current payment status of duration zero. This value is denoted $awdpia(sx, yr)$.

In addition, an overall average PIA and MBA for all disabled worker beneficiaries in current-payment status is computed by finding the weighted average of the average PIAs for each age in current-pay and duration with the number of people in current pay for each of these

ages and durations. The average PIAs were already reduced by a workers compensation offset factor, as briefly described above; a more careful description is given in this section. To get the average MBAs, we apply a factor which reflects the differences in average MBAs and PIAs for disabled workers, isolating only the trend in cases with an actuarial reduction. We also provide a relatively small reduction to reflect offsets starting and stopping in the year of DI entitlement that are not captured by the current method.

Workers Compensation Offset Factors

For each duration $j = 1, \dots, 7$, and 8+ and sex we define a workers compensation factor. This factor is applied to the average worker PIA matrix as mentioned above. It is denoted $wkcomp(yr, sx, dur)$. Let $facm2p_pct(dur)$ be defined as in the following table.

Duration	Cumulative product above set at x% of way between original $facm2p$ and 1
1	0.418840
2	0.597068
3	0.680659
4	0.734195
5	0.789081
6	0.815767
7	0.836760
8+	0.861111

Then $wkcomp(yr, sx, dur)$ is defined so that

$$facm2p_pct(dur) = facm2p(yr - dur, sx) \times \prod_{j=1}^{dur} (1 + wkcomp(yr, sx, j)).$$

This is an iterative process which first computes $wkcomp(yr, sx, 1)$ by solving the above equation with dur set equal to 1. The remaining factors for higher durations are then computed recursively using the above formula.

Trend in Average MBA to Average PIA

This trend is captured in a factor denoted $Fam2p(yr, sx)$. The 1-A table supplement as of the end of December, 2008, contains total award PIA and MBA data for disabled workers, by sex, for beneficiaries both non-actuarially reduced and actuarially reduced. Let $totmba_nar(yr, sx)$ and $totpia_nar(yr, sx)$ be the total MBA and PIA respectively for DIBs that are not actuarially reduced as found in the 1-A table. Similarly, let $totmba_ar(yr, sx)$ and $totpia_ar(yr, sx)$ be the total MBA and PIA respectively for cases that are actuarially

reduced. In the historical period 1988-*nim1* we define $fam2p(yr, sx)$ to be the ratio of the total MBA to the total PIA for those not actuarially reduced. In other words,

$$Fam2p(yr, sx) = \frac{totmba_ar(yr, sx) + totpia_nar(yr, sx)}{totpia_ar(yr, sx) + totpia_nar(yr, sx)}$$

In the period *ni* through *ns+9*, $fam2p(yr, sx)$ is defined by

$$Fam2p(yr, sx) = Fam2p(yr - 1, sx) \times \left(\frac{Fam2p(yr - 1, sx)}{Fam2p(yr - 11, sx)} \right)^{1/10}$$

The factor reaches its ultimate value in years *ns+10* and later.

More Workers Compensation Offsets

As mentioned above, we also provide a relatively small reduction to retroactive benefits to reflect offsets starting and stopping in the year of DI entitlement that are not captured by the current method. Administrative data suggests that these factors should be

Duration	Percentage Reduction
0	0.2861%
1	0.2121%
2	0.0696%
3	0.0381%
4	0.0138%

and 0 for higher durations. We define $wkcomp_red(dur)$ to be 1 minus these percentage reductions.

By law disabled workers attaining age 65 are no longer subject to the workers compensation offset. Therefore, all DI worker benefit levels are adjusted at age 65 to eliminate the effect of the offset.

Adjustment to Short-Range Average Benefit Levels

Average PIAs are further adjusted by a factor, icp_adj , designed to be the factor that adjusts the long-range aggregate average disabled worker in current pay monthly benefit amount (MBA) to the value projected by short range in the short-range period. This is done for each year of the short-range period and for each sex with the adjustment in year *ns* linearly phased down to 0 by year *ns+10*.

Computation of Average MBA for DI Workers

The disabled worker PIA as presented in the average benefit matrix was already incremented by age and duration using a COLA and a workers compensation adjustment. The average PIA by year, age and duration, is denoted $avgpia(yr, ag, sx, dur)$. The overall average MBA

by year and sex is the weighted average of $avgpia(yr, ag, sx, dur) \times Fam2p(yr, sx, dur)$, the weights being the number of DI workers in current payment status by age, sex, and duration.

DI Conversions

Disabled-worker beneficiaries convert to retired-worker beneficiary status (called DI conversions) at normal retirement age (NRA). The average DI conversion benefit for a given sex and single age 65 through NRA is the weighted average of the average DI conversion benefits for that sex and age, weighted by the number of people in current pay in each duration. Both the average conversion benefit for each sex and age 65 through NRA and the number of people in current pay for these ages are used in the computation of average retired worker benefits (see subsection (2) below).

Post-Entitlement Adjustments

As a cohort of beneficiaries age, their average benefit level will likely change for reasons other than just the COLA increase. The two primary reasons for this are post-entitlement work, which could lead to a re-calculation of one's benefit, and a known correlation between greater lifetime earnings and lower mortality rates. The Cost process uses post-entitlement factors by sex and duration to account for the expected dynamic benefit levels. One percent December MBR data from the most recent 10 historical years are used to calculate post-entitlement factors.

For disabled Workers we calculate separate factors for those in current pay (ICP) who are younger than 50 and those ICP who are older or at age 50. We use separate factors for each sex and each duration (0-9⁺). Initial and Ultimate factors are calculated before being read into the cost program with initial factors set to a 3-year historical average and ultimate factors at a 10-year average.

For retired workers we calculate separate factors for those ICP who converted from DI status and those ICP who came on the rolls as a retired worker. We use separate factors for each sex and each duration (0-12⁺). Initial and ultimate factors are calculated before being read into the cost program with initial factors set to a 3-year historical average and ultimate factors at a 10-year average. For females the ultimate post-entitlement factors are adjusted further to reflect the trend that female retired workers are starting to have earnings and benefit levels more similar to men. Therefore female ultimate post-entitlement factors are calculated in the program as 90% of the male 10-year average plus 10% of the female 10-year average.

All initial factors are phased-in linearly to the ultimate factors over a 20 year period (reaching the ultimate values in $ni+19$).

(2) Retired-Worker Benefits

Retired-Worker Beneficiary Matrix

The number of retired-worker beneficiaries for a given year and sex is provided from subprocess 3.3. Two matrices are provided – one for males and one for females. The structure of each matrix is as follows:

- 10 columns. The first 9 columns are the age at entitlement, ages 62 through 70. The last column is the number of disabled workers who are projected to convert to retired-worker beneficiary status (DI conversions) at normal retirement age.
- 34 rows. These rows correspond to the age in current pay, ages 62 through 94 and ages 95+.

Note that the entries on the diagonal (where age in current-pay equals age at entitlement) is the number of new entitlements projected for that year.

Building the Average Benefit Matrices for Retired Workers

In each projection year, the COST subprocess produces four average benefit matrices. For each sex there are two matrices, an average monthly benefit amount (MBA) matrix and the average primary insurance amount (PIA) matrix. Each matrix has the same structure as the beneficiary matrices. In other words, each matrix is a 34 by 10 matrix whose entries are the average benefit amounts of retired worker beneficiaries whose age in current pay is indexed by the rows and whose original age at entitlement is indexed by the columns. The final column simply gives the average benefits for DI conversions at the various ages in current pay.

The 100% MBR extract is processed by a side model. This side model computes a starting matrix for year *ni-1*. This starting matrix contains the four initial *benefit* matrices, constructed using the most recent data. For a given year of the projection period, the average benefit matrix is updated from its previous year's value incrementing each benefit amount (PIA or MBA) by one year of age and increasing it by a cost of living adjustment (COLA). MBAs are increased by the appropriate post entitlement factor (see section "Post-entitlement adjustments") for males and females. Adjusted age 94 benefits and age 95+ benefits are averaged, based on the respective number of beneficiaries in current pay in the prior year, to get the new average benefit for age 95+. DI conversion benefits are handled similarly, except the average conversion benefit for each age 64 through 66 is combined (as a weighted average) between the DI conversion average benefits computed in subprocess 3.2 and the DI conversions of the corresponding age already receiving benefits (provided by subprocess 3.3).

The entries along the diagonal, the average benefits of newly entitled beneficiaries by age, must still be computed. The remainder of this section will explain how these average benefits are computed. Once these are computed, all entries are computed and the average benefit matrix for the year is complete.

Average Benefits for Newly Entitled Retired Worker Beneficiaries

The potential AIME percentage (OPAPs) values for newly entitled retired-worker benefits are obtained from subprocess 4.2. The OPAPs for the projection of average benefits for newly entitled retired-worker beneficiaries are modified by the COST subprocess. The reason is that the age distribution of newly entitled retired worker beneficiaries as determined in the awards sample is used for each projection year, i.e., the age-sex distribution matches that of the sample. These new potential AIME percentages are denoted OPAP1. To incorporate the projected change in the age distribution of projected newly entitled retired-worker beneficiaries for the purpose of determining their average benefits, we use a “shuttling method.” Additional details about the shuttling method are given in Appendix 4.3-1.

Average benefits for newly entitled retired-worker beneficiaries are calculated by sex for single year ages 62 through 69, and ages 70+. The two bendpoints of the PIA formula, BP1 and BP2, are indexed by the increase in the average wage index. In 1979 dollars, the values of BP1 and BP2 are \$180 and \$1,085 respectively. The AIME dollars between 0 and BP1 are divided into four intervals (each of length \$45 in 1979 dollars). The AIME dollars between BP1 and BP2 are divided into fourteen intervals (nine of length \$45 and five of length \$100, in 1979 dollars). Twelve additional intervals are added beyond BP2 (ten of length \$200 and two of length \$1,000, in 1979 dollars).

To determine the average PIA for newly entitled beneficiaries, the OPAP1 values for each interval of AIME dollars is multiplied by the dollar amounts attributable to each interval (the length of the interval). More precisely, let

- PIA_factor_i be the PIA factor for subinterval i (equal to 0.90 for intervals $i=1,4$, 0.32 for intervals $i=4, \dots, 18$, and 0.15 for intervals $i=19, \dots, 30$), $AIME_dollars_i$ be the length of subinterval i ,
- $opap1_i(yr, sx, ag)$ be the modified PAP value for retired workers newly entitled in year yr whose sex is sx and whose age is ag .
- $wff(yr, sx, age)$ be a reduction factor to account for the Windfall Elimination Provision.⁶¹
- $Wage_Idx(ag, yr) = \frac{avgwg(yr - (ag - 62))}{avgwg(nbase - 2)}$

Then the average PIA for these newly entitled retired worker beneficiaries is equal to

⁶¹ The Windfall Elimination Provision (WEP) reduces the first PIA formula factor from 90% to as low as 40% for individuals who receive a pension based on specified categories of non-covered employment, primarily non-covered state and local government employees and federal workers receiving a pension under the Civil Service Retirement System. The cost process uses initial factors by sex and age, ultimate factors, years in which ultimate factors are reached and phase-in trend lines to the ultimate factor, all supplied by a side model.

$$LR_awdpia(sx, ag, yr) = Wage_Idx(ag, yr) \times wff(yr, sx, age) \\ \times \sum_{i=1}^{30} PIA_factor_i \times AIME_dollars_i \times opap1_i(yr, sx, ag).$$

This formula incorporates the fact that the PAP values are the estimated cumulative distribution of AIME dollars. The average award MBA for a worker beneficiary is then the average newly entitled PIA multiplied by the appropriate actuarial reduction factors and delayed retirement credits, *arfdrc(ag, yr)*.

Once these average benefits of newly entitled retired-worker beneficiaries are computed, their values are filled into the appropriate average benefit matrices.

For summary purposes, the COST subprocess computes an average PIA and MBA for all male and female newly entitled retired-worker beneficiaries, as well as for all retired-worker beneficiaries in current-payment status. These are just the respective weighted averages of the average PIAs and MBAs by age and sex, the weights being the number of newly entitled retired-worker beneficiaries.

Adjustment to Short-Range Average Benefit Levels

Average PIAs are further adjusted by a factor, *icp_adj*, designed to be the factor that adjusts the long-range aggregate average retired worker in current pay monthly benefit amount (MBA) to the value projected by short range in the short-range period. This is done for each year of the short-range period and for each sex with the adjustment in year *ns* linearly phased down to 0 by year *ns+10*.

(3) Annualizing Benefits

Scheduled benefits are calculated by trust fund and projection year. For each year scheduled benefits for each trust fund are found by adding up the appropriate benefit categories. This section considers all benefit amounts except the “dual entitlement excess amount.” If a retired worker beneficiary is also entitled to spouse or widow(er) benefits and the auxiliary benefits are greater, than the amount by which the auxiliary benefit exceeds the worker’s MBA is the dual entitlement excess amount. The four categories of excess amounts (dually entitled wives, widows, husbands, and widowers) are projected separately. More information is found in subsection (4).

The first step is to annualize benefits by category. The average benefit for a specific auxiliary beneficiary category (*avgben*) is found by multiplying the linkage factor (the assumed relationship between an auxiliary beneficiary’s benefit and the corresponding worker benefit) by the average benefit of the primary account holder (the account on which the auxiliary beneficiary is entitled to receive the benefit; discussion of its computation appears in subsection (2) above). In order to annualize benefits, two values are used. The beginning-of-year average benefit for a specific auxiliary beneficiary category is found by using the average benefit of the worker

beneficiary for the prior year. The end-of-year benefit for that category is found by using the average benefit of the worker beneficiary for the current year and removing the cost of living adjustment. The average benefit by category for each month is found by taking a weighted average of the benefits at the beginning and end of the year, the weights being the fractions of the year the prior and current year's beneficiaries have been exposed. Since the new COLA takes effect in December of the year, the new COLA must be reflected in the December benefits. If $cp(cat, yr)$ is the number of beneficiaries in category cat for year yr , and $avgben(cat, yr)$ is the average benefit for category cat for year yr , then the amount of aggregate benefits paid in year yr is given by the formula

$$\text{AggBen}(yr, cat) = \sum_{i=0}^{11} \left[\frac{(12-i)}{12} \times cp(cat, yr-1) \times avgben(cat, yr-1) + \frac{i}{12} \times cp(cat, yr) \times avgben(cat, yr) \right].$$

For worker beneficiary categories, the amount is also increased by the retroactive payments that were projected to be paid during the year. See section (5), below. A description of beneficiary categories is as follows. An odd category number refers to the male account holder, while an even category number refers to the female account holder. As an example, for category 4, the aged married spouse is the aged married husband of the retired female worker.

Category # (cat)	<u>Beneficiary Type</u>
	<u>Old-Age Insurance Beneficiaries</u>
1 & 2	Retired worker (includes DI conversions)
3 & 4	Aged married spouse
5 & 6	Aged divorced spouse
9 & 10	Young spouse with child
11 & 12	Child < 18
13 & 14	Student child
15 & 16	Disabled adult child
	<u>Disability Insurance Beneficiaries</u>
17 & 18	Disabled worker
19 & 20	Aged married spouse
21 & 22	Aged divorced spouse
25 & 26	Young spouse with child
27 & 28	Young child
29 & 30	Student child
31 & 32	Disabled adult child
	<u>Survivors Insurance Beneficiaries</u>
33 & 34	Aged married widow
35 & 36	Aged divorced widow
39 & 40	Young married disabled widow
41 & 42	Young divorced disabled widow
43 & 44	Aged parent

45 & 46	Young married widow with child
47 & 48	Young divorced widow with child
49 & 50	Young child
51 & 52	Student child
53 & 54	Disabled student child
55 & 56	Lump sum death benefit (\$255)

(4) Dually Entitled Beneficiaries and Benefits

Number of Dually Entitled Beneficiaries

There are four categories of dually entitled beneficiaries. They are the dually entitled wives (1), widows (2), widowers (3), and husbands (4). The number of dually entitled husbands is very small (currently less than 0.05% of all dually entitled beneficiaries and are expected to remain at this level in the future). As a result, the number of dually entitled husbands and their benefits are not projected.

In past years, the equations used to project the number of dually entitled beneficiaries each have had three terms. Each equation has three coefficients, $a_j^{(k)}$, $j = 1, 2, 3$ (corresponding to each of the categories above). These coefficients are provided by the Economics process based on historical series of the other terms in the equations. These terms contain the basic quantities (a) projected weighted average AIME deviations of newly entitled retired worker beneficiaries for each sex [AIME_Dev(yr,sx)] and (b) average PIA of newly entitled retired worker beneficiaries by sex, wage indexed to the year of the sample [PIA(yr,sx)]. Therefore, the three equations

$$\text{PctExp}^{(k)}(yr) = a_1^{(k)} \frac{\text{AIME_Dev}(yr,M)}{\text{PIA}(yr,M)} + a_2^{(k)} \frac{\text{AIME_Dev}(yr,F)}{\text{PIA}(yr,F)} + a_3^{(k)} \frac{\text{PIA}(yr,M) - \text{PIA}(yr,F)}{\text{PIA}(yr,M)}$$

($k = 1, 2, 3$) project the percentage of the exposed population entitled to wife (1), widow (2), and widower (3) benefits.

However, for the 2009 TR, the a_1 and a_2 coefficients were generally negative and did not improve the fit of the regression so these coefficients and independent variables were dropped from the regression equations and “add-factor” adjustments after the regression (denoted “c” in below equation) were enacted to promote more reasonable values through part of the TR projection period. This adjustment was made in the year 2083 from the pure regression value to a “targeted value” of the percentage of exposure to the population of dually entitled, and was then phased in over the earlier years of the projection period in such a way that the overall trend was reasonable. The target values, “add factoring” phase-in years and a_3 coefficients for the 2009 Trustees Report are shown in the table below.

$$\text{PctExp}^{(k)}(yr) = a_3^{(k)} \frac{\text{PIA}(yr,M) - \text{PIA}(yr,F)}{\text{PIA}(yr,M)} + c^{(k)}(yr)$$

Suppose that $ult^{(k)}$ is the value obtained from the regression equation without add-factoring in the final year of the projection period. Therefore

$$ult^{(k)} = a_3^{(k)} \frac{PIA(ni + 74, M) - PIA(ni + 74, F)}{PIA(ni + 74, M)}.$$

Let $targ^{(k)}$ be the target value we estimate for the final year of the projection period. Let $phaseyrs$ be the number of years it takes to fully phase in the target value. Then we have

$$c^{(k)}(yr) = \min(yr - 2007, phaseyrs) \times \frac{targ^{(k)} - ult^{(k)}}{phaseyrs}.$$

The following table displays the coefficients, target values, and phase-in years for each type of beneficiary.

k	Type	$a_3^{(k)}$	Target Value $targ^{(k)}$	Add-factoring Phase-in Years $phaseyrs$
1	Wife	0.91458	0.25	40
2	Widow	1.70073	0.57	40
3	Widower	-0.09045	0.06	40

In the above equations, the average PIA of newly entitled beneficiaries by sex has already been computed (see subsection (2) above).

The remaining paragraphs discuss the methodology used to compute the AIME deviations that are used in the above equation, though no longer used directly in the regression analysis. This process requires two steps:

- obtain the AIME deviation in each year by sex and
- obtain a 10 year historical average of these AIME deviations.

The projection of a deviation of AIME values [$std_dev(yr, sx)$] is as follows. The projection uses the interval of AIME dollars for 1979. The 1979 bend points are \$180 and \$1,085. The entire interval of AIME dollars is divided into 30 smaller intervals [as was done in the projection of average benefits of newly entitled beneficiaries; see subsection (2)]. The left endpoints of these intervals (\$0, 45, 90, 135, 180, 280, 380, etc.) are the 1979 interval point values. These 1979 interval point values are wage indexed to the award sample year (2004 for the 2008 Trustees Report). Each interval point i , in 2004 wage indexed dollars, is

$$\text{Interval Point}_i = (1979 \text{ Interval Point}_i) \times \frac{\text{AverageWageIndex}(2002)}{\text{AverageWageIndex}(1977)}.$$

Each interval point is assigned a probability using the OPAP1 values [see subsection (2)]. Each interval is assigned probability

$$p_i(yr, sx, ag) = \begin{cases} 1.0 - opap1_i(yr, ag, sx) & i = 1 \\ opap1_i(yr, ag, sx) - opap1_{i+1}(yr, ag, sx) & i = 2, \dots, 29. \end{cases}$$

The average AIME of each newly entitled beneficiary aged $ag=62, \dots, 70$, by sex (sx) and projection year (yr) in sample year (2004) dollars is

$$\text{AvgAIME}(yr, sx, ag) = \sum_{i=1}^{10} p_i(yr, sx, ag) \times \text{Interval Point}_i.$$

The variance of AIME for each newly entitled beneficiary aged $ag=62, \dots, 70$, by sex (sx) and projection year (yr) in sample year (2004) dollars is

$$\text{VarAIME}(yr, sx, ag) = \sum_{i=1}^{30} p_i(yr, sx, ag) \times [\text{Interval Point}_i - \text{AvgAIME}(yr, sx, ag)]^2.$$

The standard deviation by year, sex, and age is

$$\text{std_dev}(yr, sx, ag) = \sqrt{\text{VarAIME}(yr, sx, ag)}.$$

For each year and sex, the standard deviation $\text{std_dev}(yr, sx)$ is a weighted average of the standard deviations by year, sex, and age, the weights being the percentages of newly entitled beneficiaries by sex and age.

Once the standard deviation of AIME [$\text{std_dev}(yr, sx)$] is computed for the given projection year and both sexes, a weighted average of the AIME deviations over the last 10 years (including the projection year) is computed. This weighted average for the projection year yr is given by

$$\text{AIME_Dev}(yr, sx) = \left(\sum_{i=yr-9}^{yr} \text{weight}(yr, i) \times \text{std_dev}(yr, sx) \right) \Bigg/ \sum_{i=yr-9}^{yr} \text{weight}(yr, i) \text{ with}$$

$$\text{weight}(yr, i) = 0.55 + 0.5 \times (9 + i - yr).$$

Note that the weight in year $i = yr$ equals 1.0, and that the weights decrease linearly to 0.55 over the 10 year look-back period. Note also that the sum of the weights (the denominator in the AIME_Dev expression), equals 7.75.

Average Excess Amount for Dually Entitled Beneficiaries

The projection of the average excess amounts for two categories of dually entitled beneficiaries (wives and widows) is similar to that of the number of dually entitled beneficiaries. The structure of the equations used to project these amounts is similar to the equations used to project the percentage exposures. While the coefficients provided by the Economics process are different, the underlying historical variables and their projections remain the same.

The equations used to project the average excess amount each have two terms. Each equation k has one coefficient, $b_1^{(k)}$, provided by the Economics process based on historical series of the other terms in the equations and an add factor adjustment (denoted “ $c^{(k)}$ ”) similar to the process for the number of dually entitled beneficiaries. A target value in the 75th year of the projection period is used in deriving the “add-factor” adjustment. The two equations

$$\text{AvgExcPct}^{(k)}(yr) = b_1^{(k)} \frac{\text{PIA}(yr,M) - \text{PIA}(yr,F)}{\text{PIA}(yr,M)} + c^{(k)}(yr)$$

($k = 1, 2$) project the average excess benefit amounts of wife (1) and widow (2) beneficiaries as a percentage of the male retired worker benefit.

The derivation of $c^{(k)}(yr)$ is similar to the case of the exposure percentages as described above. Suppose that $ult^{(k)}$ is the value obtained from the regression equation without add-factoring in the final year of the projection period. Therefore

$$ult^{(k)} = b_1^{(k)} \frac{\text{PIA}(ni + 74, M) - \text{PIA}(ni + 74, F)}{\text{PIA}(ni + 74, M)}$$

Let $targ^{(k)}$ be the target value we estimate for the final year of the projection period. Let $phaseyrs$ be the number of years it takes to fully phase in the target value. Then we have

$$c^{(k)}(yr) = \min(yr - 2007, phaseyrs) \times \frac{targ^{(k)} - ult^{(k)}}{phaseyrs}$$

The table below shows the regression coefficients and other relevant adjustments in the 2009 Trustees Report.

k	Type	$b_1^{(k)}$	Targeted Value (2083) in Nominal Dollars $targ^{(k)}$	Add-factoring Phase-in Years $phaseyrs$
1	Wife	0.43783	2,700	40
2	Widow	1.19425	8,700	40

The average excess amount of widower beneficiaries is estimated to be a fixed percentage (46%) of the average excess amount of widow beneficiaries. That is,
 $AvgExcPct^{(3)}(yr) = 0.46 \times AvgExcPct^{(2)}(yr)$.

Annualizing Excess Amounts

The process to annualize excess amounts is very similar to the process for annualizing auxiliary benefits.

For each dual entitlement category, the number of beneficiaries is simply $cp(yr, cat) = PctExp^{(cat)}(yr) \times ExposedPop^{(cat)}(yr)$. With this method, however, no linkage factor is used. Instead, the projected average excess amount, as explained above, is used. Therefore,

$$AggExcess(yr, cat) = \sum_{i=0}^{11} \left[\frac{(12-i)}{12} \times cp(cat, yr-1) \times AvgExcAmt^{(cat)}(yr-1) + \frac{i}{12} \times cp(cat, yr) \times AvgExcAmt^{(cat)}(yr) \right]$$

(5) Retroactive Payments

Disabled Workers

For each age and sex the DI area (subprocess 3.2) provides the cumulative distribution, by duration, of incurred but not reported cases. Denote this by $ibnr(sex, age, dur)$. The number of disability beneficiaries *currently entitled* (by age, sex, and duration) is $dibce(sex, age, dur) = dibcp(sex, age, dur) / ibnr(sex, age, dur)$.

The associated frequency distribution of incurred but not reported cases is $ibnr_freq(sex, age, dur)$. In other words, we have $ibnr_freq(sex, age, 0) = ibnr(sex, age, 0)$ (duration 0) and for higher durations, $1 \leq dur \leq 9$ and 10+, we have $ibnr_freq(sex, age, dur) = ibnr(sex, age, dur) - ibnr(sex, age, dur - 1)$.

Let dur be a duration, 0 through 9 or 10+. Define

- $Cum_COLA(dur) = \prod_{j=0}^{dur} (1 + COLA(yr - j))$.
- For $i = 0, \dots, 10$, $Num_Months(i) = \begin{cases} 2 & dur = 0 \\ 5 & dur > 0 \text{ and } i = 0 \\ 12 & dur > 0 \text{ and } 0 < i < dur \\ 6 & dur > 0 \text{ and } i = dur \end{cases}$

Then the aggregate retroactive payments for disabled workers, in millions, are defined to be

$Retro_DIB(sex, yr)$

$$= \sum_{age=20}^{66} \sum_{dur=0}^{10} \sum_{i=0}^{dur} dibpia(age, sex, dur) \times wkcomp_red(dur) \times \frac{dibce(sex, age, 0)}{10^6} \times \frac{Num_Months(i) \times ibnr_freq(sex, age, dur)}{Cum_COLA(dur)}.$$

This is simply added to the disabled worker benefit category by year and sex.

Retired Worker Beneficiaries

Retired Worker beneficiaries who are currently entitled but change to current payment status later are assumed to do so on average 0.9 months after they became entitled. Hence

$$retro_OAB(sex, yr) = \sum_{age=62}^{70} \frac{0.9 \times \frac{oabipc(yr, sex, age, age)}{1000} \times oabmba(yr, sex, age, age)}{1 + COLA(yr)}.$$

In the above formula, $oabipc(yr, sex, age, age)$ is the number of newly entitled beneficiaries at age equal to age (age in current pay equals age at entitlement equals age) and $oabmba(yr, sex, age, age)$ is the corresponding average benefit. The aggregate retroactive benefits for old-age beneficiaries are simply added to the retired worker benefit category by year and sex.

Auxiliary Beneficiary Categories

Retroactive payments for auxiliary beneficiaries are similarly linked to the primary worker beneficiary. The linkages are a loading of the aggregate annual benefits by auxiliary category. Each auxiliary benefit category has a loading factor, and the aggregate annual benefits by category are increased by this loading factor.

(6) Aggregate Scheduled Benefits (BEN)

Aggregate benefits by trust fund, $BEN(tf, yr)$, are computed as follows. For each year of the 75-year long-range projection period, the aggregate benefits by category (including retroactive payments, as described above) are summed up to give the annual scheduled benefit levels by trust fund. In the short-range period, the long-range values are overridden by the values estimated by the short-range office. The difference between long-range scheduled benefits and short-range benefits in the 10th year of the short-range period is called the scheduled benefits adjustment. In the 10 years after the end of the short-range period, the long-range scheduled benefits is adjusted by linearly grading the scheduled benefits adjustment to zero over the 10 year period. From the 20th year forward, the projection is the pure long-range value.

iii. Equation 4.3.3 – Taxation of Benefits (TAXBEN)

The taxation of benefits levels in the short-range period, by trust fund, are provided by the short-range office. These implicitly give, for each year, an estimated taxation of benefits factor, by trust fund, equal to the estimated taxation of benefits as a percentage of benefits scheduled to be paid. The long-range office projects these factors independently for every year of the projection

period, also by trust fund. (See subprocess 4.1.) The difference in the factors from both offices at the end of the short-range period is phased out linearly over the next ten years. The long-range projection of taxation of benefits is estimated by multiplying the projected taxation of benefits factors by the benefits scheduled to be paid, by trust fund. If $taxben_factor(tf, yr)$ is the percentage of scheduled benefits for the year, by trust funds, estimated to be collected as taxation on benefits, then

$$TAXBEN(tf, yr) = taxben_factor(tf, yr) \times BEN(tf, yr)$$

for $yr \geq ns + 10$.

iv. Equation 4.3.4 – Administrative Expenses (ADM)

Administrative expenses are estimated separately by trust fund. In the short-range period, the short-range office provides the estimates of administrative expenses by trust fund. Thereafter, administrative expenses are computed by multiplying the previous year's administrative expenses by three factors: annual changes in total beneficiaries, annual changes in AWI, and one minus annual productivity growth. As a formula, if $ticip(tf, yr)$ is the total estimated number of beneficiaries in current-pay status by trust fund and year, $AWI(yr)$ is the average wage index in year yr , and $prod$ is the ultimate assumed annual growth in productivity, then

$$ADM(tf, yr) = ADM(tf, yr - 1) \times [ticip(tf, yr) / particip(tf, yr - 1)] \times [AWI(yr) / AWI(yr - 1)] \times (1 - prod)$$

for $yr > ns$.

v. Equation 4.3.5 – Railroad Interchange (RR)

Railroad interchange is disaggregated by trust fund and projection year. The long-range office does a projection for each year in the 75-year period. In the short-range period (first 10 years of the 75-year projection period), the short-range office provides the estimates of railroad interchange by trust fund and the long-range projection is overridden in these years. Over the next five years of the projection period, the estimate of the railroad interchange is a linear interpolation between the short-range projection at the end of the short-range period and the long-range projection five years hence. During the final 60 years of the projection period, the projection is as estimated by the long-range office.

By trust fund, the total cost in calendar year yr , $rr_cost(tf, yr)$ is the sum of railroad benefits and railroad administrative expenses, less the sum of railroad contributions and railroad taxation of benefits. By trust fund, the railroad interchange component of total cost is then equal to

$$RR(tf, yr) = [0.25 \times rr_cost(tf, yr - 2) + 0.75 \times rr_cost(tf, yr - 1)] \times [1 + 1.25 \text{ yield}(yr)].$$

Note that the railroad cost is increased by the yield rate on the combined OASDI trust funds for year yr . This formula takes into account the fact that there is a delay from the end of the fiscal year to the time of valuation.

The total cost in year yr is broken down into four components: railroad benefits in year yr , railroad administrative expenses in year yr , railroad contributions in year yr , and railroad taxation of benefits in year yr .

Projections of numbers of newly entitled retired workers are determined by analyzing the ratio of new entitlements to previous levels of railroad employment using 1997-2006 new entitlement data in the analysis. After initial entitlement, a mortality rate of 6.1% for 2007 is assumed based on analysis of recent Railroad Retirement Board financial interchange data, with mortality assumed to improve thereafter. For projections of newly entitled disabled workers a similar trend analysis based on prior employment is used. A “mortality” rate (deaths plus recoveries plus conversions to retired worker benefits) is determined using Railroad Retirement Board (RRB) data, with the rate assumed to be constant thereafter.

Assuming a 90/10 male/female split, the OASI railroad worker average benefit level is calculated as a ratio to the OASI retired worker benefit. This ratio is constant throughout the projection period and is derived by comparing MBR and Railroad Board data. Additionally a constant auxiliary loading factor based on the same data is multiplied in. This same approach is used to determine similar constants for DI railroad benefits. The aggregate disabled worker railroad benefits (and beneficiaries) are estimated in the same way.

It is assumed that the ratio of OASI taxation of benefits to OASI benefits and DI taxation of benefits to DI benefits are both the same for railroad taxation of benefits. The railroad taxation of benefits is estimated by multiplying the railroad benefits by these ratios.

Administrative expenses for railroad are computed separately by trust fund. They are set at levels determined by short range in the short-range period and for years $ni+10$ to nf they are computed similarly to OASDI administrative expenses. This is administrative expenses in $yr-1$, multiplied by the change in the total number of worker beneficiaries, the annual change in average wage, and (1 minus the ultimate productivity factor).

Railroad contributions are estimated, by trust fund, to be total railroad employment, multiplied by average railroad earnings, multiplied by the combined OASDI employer/employee tax rate. Railroad earnings are assumed to grow with the increase in the average wage index, and railroad employment is assumed to decrease over time, both of which are in line with the Railroad Retirement Board’s own “most likely” projections.

vi. Equation 4.3.6 – Interest Income (INT)

In the short-range period, the projection of interest income by trust fund is provided by the short-range office. In each year of the short-range period, the annual yield rate is defined as the ratio of interest earned by a fund to the average level of assets held by the fund during the year.

The ultimate annual yield rate on each trust fund is equal to the nominal yield, which is the real interest rate increased for inflation. As a formula,

$$\text{ultimate yield rate} = (1 + \text{real interest rate}) \times (1 + \text{inflation rate}) - 1.$$

To get the yield rate for each year between the end of the short-range period and 5 years later when the ultimate yield rate is assumed to be reached, the program linearly interpolates between the values for these two years.

The projection of interest income in a given year is the yield rate for that year multiplied by the average level of assets. As a formula,

$$INT(tf, yr) = \text{yield}(tf, yr) \times \text{avg_assets}(tf, yr).$$

The average level of assets in a trust fund, for a given year, is estimated to be the beginning of the year assets, increased by the tax contributions and taxation of benefits income (each exposed to the point in the year in which they are estimated to be received, on average), and decreased by scheduled benefits, railroad interchange, and administrative expenses (each exposed to the point in the year in which they are estimated to be disbursed, on average).

For all years of the projection period, tax contributions are given an exposure of 0.525, taxation of benefits are given an exposure of 0.5, railroad interchange is given an exposure of 0.583, and administrative expenses are given an exposure of 0.5. For scheduled benefits, an exposure of 0.5 is ultimately given. The exposure, $ben_exp(yr)$, is larger than 0.5 in the early years of the projection period and grades down smoothly to 0.5. The reason is that in the past, benefits were always paid at the beginning of each month. Now benefits are paid out throughout the month, based on the birth date of the beneficiary. The average assets held by the trust funds for a given year is estimated by the formula

$$\begin{aligned} \text{avg_assets}(tf, yr) = & ASSETS_{BOY}(tf, yr) + 0.525 \times CONTRB(tf, yr) \\ & + 0.5 \times TAXBEN(tf, yr) - ben_exp(yr) \times BEN(tf, yr) \\ & - 0.583 \times RR(tf, yr) - 0.5 \times ADM(tf, yr). \end{aligned}$$

vii. Equations 4.3.7, 4.3.8 and 4.3.9 – Annual Values

The annual income rate for a trust fund is computed as the employer/employee tax rate for that fund, plus taxation of benefits as a percentage of taxable payroll.

$$ANN_INC_RT(tf, yr) = taxrate(tf, yr) + \frac{TAXBEN(tf, yr)}{payroll(yr)}.$$

The annual cost rate for a trust fund is computed as the total cost of providing scheduled benefits from that fund as a percentage of taxable payroll. If

$$COST(tf, yr) = BEN(tf, yr) + RR(tf, yr) + ADM(tf, yr),$$

then

$$ANN_COST_RT(tf, yr) = \frac{COST(tf, yr)}{payroll(yr)}.$$

The trust fund ratio measures the amount of beginning of year assets that can be used to pay scheduled benefits. It is expressed as a percentage.

$$TFR(tf, yr) = \frac{ASSETS_{BOY}(tf, yr)}{BEN(tf, yr)}.$$

viii. Equations 4.3.10, 4.3.11, 4.3.12, and 4.3.13 – Summarized Values

Present values of cash flows during the year are computed using the yield rate on the combined OASDI trust fund for that year. Each component of trust fund operations is exposed, with interest, to the point in the year in which, on average, it is received or disbursed. These exposure levels, $ben_exp(yr)$, are the same as described above. These exposed levels are then discounted to January 1 of the year of the Trustees Report, ni . If $yield(j)$ is the annual yield rate on the combined OASDI trust funds for year j and $v(yr)$ is the discounting factor for the year, then

$$v(yr) = \prod_{j=ni}^{yr} \frac{1}{[1 + yield(j)]}.$$

For a given year, and trust fund,

$$\begin{aligned} PV_TAX(tf, yr) &= (1 + 0.525 \times yield(yr)) \times TAX(tf, yr) \times v(yr), \\ PV_TAXBEN(tf, yr) &= (1 + 0.5 \times yield(yr)) \times TAXBEN(tf, yr) \times v(yr), \\ PV_BEN(tf, yr) &= (1 + ben_exp(yr) \times yield(yr)) \times BEN(tf, yr) \times v(yr), \\ PV_RR(tf, yr) &= (1 + 0.58\bar{3} \times yield(yr)) \times RR(tf, yr) \times v(yr), \\ \text{and } PV_ADM(tf, yr) &= (1 + 0.5 \times yield(yr)) \times ADM(tf, yr) \times v(yr). \end{aligned}$$

The target fund for a year is next year's cost. Its present value is computed as

$$PV_TARG(tf, yr) = [BEN(tf, yr + 1) + RR(tf, yr + 1) + ADM(tf, yr + 1)] \times v(yr),$$

Taxable payroll is exposed to the middle of the year when computing present values:

$$PV_PAYROLL(yr) = (1 + 0.5 \times yield(yr)) \times payroll(yr) \times v(yr).$$

We also define

$$PV_INC(tf, yr) = PV_TAX(tf, yr) + PV_TAXBEN(tf, yr)$$

and

$$PV_COST(tf, yr) = PV_BEN(tf, yr) + PV_RR(tf, yr) + PV_ADM(tf, yr).$$

Summarized rates are calculated using beginning of period assets and a target fund.

$$SUMM_INC_RT(tf, yr_1, yr_2) = \frac{ASSETS_{BOY}(tf, yr_1) + \left(\sum_{j=yr_1}^{yr_2} PV_INC(tf, j) \right)}{\sum_{j=yr_1}^{yr_2} PV_PAYROLL(j)}.$$

The summarized cost rate is similarly computed:

$$SUMM_COST_RT(tf, yr_1, yr_2) = \frac{\left(\sum_{j=yr_1}^{yr_2} PV_COST(tf, j) \right) + PV_TARG(tf, yr_2)}{\sum_{j=yr_1}^{yr_2} PV_PAYROLL(j)}.$$

The 75-year actuarial balance is computed for a period beginning January 1 of the Trustees Report year, ni . It includes both beginning of period assets and a target fund. Therefore,

$$ACTBAL_{75, yr}(tf) = SUMM_INC_RT(tf, ni, ni + 74) - SUMM_COST_RT(tf, ni, ni + 74).$$

In general, an actuarial balance may be computed for any given subperiod of the projection period. In general, actuarial balances for a subperiod beginning on January 1 of year ni and continuing through the end of year yr are computed using

$$ACTBAL_{ni, yr}(tf) = SUMM_INC_RT(tf, ni, yr) - SUMM_COST_RT(tf, ni, yr).$$

The unfunded obligation of a trust fund for a given period is the excess of the present value of the net cash deficits for each year of that period over the trust fund balance at the beginning of the period. The unfunded obligation for the period beginning on January 1 of year ni and continuing through the end of year yr is computed using

$$UNF_OBL(tf, yr) = \sum_{j=ni}^{yr} [PV_COST(tf, j) - PV_INC(tf, j)] - ASSETS_{BOY}(tf, ni).$$

ix. Equation 4.3.14—Closed Group Unfunded Obligation

The closed group is defined as individuals who attain specified ages in the first year of the projection period (*ni*). The Statement of Social Insurance displays unfunded obligations for closed groups (1) attaining 15 or later in 2009, (2) attaining 62 or older in 2009, and (3) attaining 15 to 61 in 2009. For each year of the projection period, closed group calculations attribute a portion of the items in equations 4.3.1 through 4.3.6 to individuals falling in the defined closed group. The calculation of the closed-group unfunded obligation, then, uses the equation above but only considering the present values of cost and income attributable to the closed group.

The following information, developed elsewhere in the “Cost” process, is used for developing closed group unfunded obligation amounts:

- Total number of workers and total taxable earnings by single year of age 15-74 and sex, years 1951 through *nf*, updated yearly
- Taxable payroll, years *ni* through *nf*, updated yearly
- Payroll tax income, years *ni* through *nf*, updated yearly
- Income from taxation of benefits, years *ni* through *nf*, updated yearly
- Scheduled benefits by beneficiary category, years *ni* through *nf*, updated yearly
- Railroad interchange, years *ni* through *nf*, updated yearly
- Administrative expenses, years *ni* through *nf*, updated yearly
- Yield rate on the combined OASDI trust funds, years *ni* through *nf*, updated yearly

It is important to note that, for dependent beneficiaries, the age of the worker, on whose account the benefits are based, determines whether that beneficiary would fall in the closed group. For instance, if the closed group were defined as individuals attaining age 15 or later in 2009 and in 2009 a 3-year-old minor child was receiving benefits on the account of a retired worker aged 63, the minor child beneficiary would be considered as part of this closed group because the account holder was at least age 15 in 2009. The following describes how the various components of income and cost are allocated to the defined closed group in question:

Payroll Tax Contributions

Closed group taxable payroll is defined as the percentage of OASDI taxable payroll attributable to the closed group in question. An input file of *closed group payroll factors*, containing these percentages by year from 2009 through 2108, is used by the cost program to compute payroll tax contributions attributable to the closed group. For each year, the *closed group payroll factors* are determined as follows:

- The number of projected workers by single year of age (ages 1-99) and sex are multiplied by the associated average earnings by age/sex.

- Then, the portion of total taxable earnings attributable to the closed group is calculated.

For each year of the projection period, the number of workers and average taxable earnings by single year of age and sex are determined as follows:

- For ages 15-74, the number of projected workers comes directly from Economics group projections, and the average earnings by age and sex comes directly from the AWARDS subprocess.
- For ages 1-14, the number of projected workers is obtained by averaging the percentage of workers at each age relative to the total 1-14 age group, using 15 years of earnings data (1991-2005) in the averaging calculations. Average taxable earnings for ages 1-14 are obtained by analyzing historical 1991-2005 data of the average earnings at each age relative to the age 15 average earnings, and judgmentally assigning a ratio (to age 15 average earnings) for each age.
- For ages 75-99, the number of projected workers by age and sex is obtained through first developing the ratio of workers in 2005 at each age to the projected population in 2005 at that age and then applying that ratio to projected population by age in years 2008 and later. Then, the resulting estimated workers are adjusted by an equivalent percentage to match the total number of projected workers aged 75 and later, as provided by the Economics group. This process is done separately by sex. To obtain the average taxable earnings for single years of age 75-99, ratios of average earnings by age 75-99 to overall average earnings for all ages is computed for each age 75-99, for earnings years 1991 through 2005. These ratios by age are then averaged for all 15 years, from which earnings ratios to overall average earnings are developed judgmentally. This process is done separately by sex.

Benefits

Methodologies for computing benefits attributable to the closed groups differs among benefit categories, as described below:

Retired Workers

For each age in current pay, the number of beneficiaries is multiplied by the corresponding average benefit amount across all ages of entitlement. The same applies for DI conversion cases. Retroactive benefits for the current year, by age, are then calculated. The closed-group factors for old-age benefits for each year are found by summing the benefit amounts attributable to the specified closed group, as a proportion of total retired worker benefits for all ages. This process is done separately by sex.

Disabled Workers

For each age from 20 to the year before normal retirement age, the program adds the products of the number of beneficiaries for each duration and the PIA for that duration. Retroactive benefits for the current year, by age, are then calculated. The closed-group factors for disability benefits are found by summing the total benefit amounts attributable to the closed group, as a proportion of total disabled worker benefits for all ages. This process is done separately by sex.

Aged Spouses

The number of aged wife beneficiaries in current pay status (no dual entitlement) is provided by single year of age. Then, for each single year of age, the program allocates total numbers of husbands by age, from 5 years younger to 12 years older than the aged wife using an assumed distribution. Next, for each age of husband in current pay, the number of husbands is multiplied by the weighted average retired worker benefit across all ages of entitlement. The closed group factors, then, are obtained by determining the proportion of total benefit amounts attributable to the given closed group (based on the husband's age).

The same closed group factors are applied to the excess benefit amount for aged wives in dual entitlement status (i.e. aged wives with a smaller worker benefit). Aged husbands, a very small beneficiary group, get the same closed group benefit factors as do the aged wives, described above.

Aged Widows

The number of aged widow beneficiaries in current pay status (no dual entitlement) is provided by age from 60 to 95. For each single year of age, the program allocates total number of aged widows by age of the deceased husband (age the husband would have been if he had not died), from 10 years younger to 15 years older than the aged widow using an assumed distribution. For each age of deceased husband aged 95 or younger, a real wage growth factor is applied to reflect ultimate real wage growth taken to the power to the number of years younger than age 96.

$$Benefit_{adj} = (1.039 / 1.028)^{(96 - \text{deceased worker age})}$$

This exponent is intended to reflect differences in average levels of benefits, with younger deceased husbands having higher benefits based on real wage growth. The closed group factors, then, are obtained by determining the proportion of total benefit amounts attributable to the closed group

The same closed group factors are applied to excess benefit amounts for aged widows in dual entitlement status (i.e. aged widows with a smaller worker benefit). Aged widowers receive the same closed group benefit factors as do aged widows (based on the deceased husband's age).

Other Beneficiary Categories

For the 20 other dependent beneficiaries of retired workers, disabled workers, and deceased workers, an input file of closed group benefit factors is created, which represents the proportion of total (open-group) projected benefits in that category attributable to the given closed group age and year. This file is used by the cost program to compute amounts from each beneficiary category attributable to the closed group. The file, separately created for each closed group run, contains closed group benefit factors for ages 0 through 150 for each of the 20 beneficiary categories by sex of the account holder (worker). These input files are created by examining a recent sample of Master Beneficiary Record (MBR) data¹ for each of the beneficiary categories by age of the worker, and projecting future distributions by age of the worker based on population and, for survivor benefits, projected deaths by age. Then, adjustments are made for real wage growth to reflect different benefit levels by birth cohort.

Taxation of Benefits

Since taxation of benefits is related to benefits, the closed-group taxation of benefit amounts are computed by multiplying the total (open-group) taxation of benefit amounts by Trust Fund, by the corresponding total closed-group benefit factors by Trust Fund.

Administrative Expenses

Since administrative expenses are also assumed to be related to benefits, the closed-group administrative expense amounts are computed by multiplying the total (open-group) administrative expenses by Trust Fund, by the corresponding total closed-group benefit factors by Trust Fund.

Railroad Interchange

Since the railroad interchange has both a payroll tax and benefit component, each component is multiplied by its corresponding closed-group factor. That is, total payroll tax contributions arising from railroad interchange are multiplied by the closed group payroll factor discussed above in the “Tax on Contributions” section. Total railroad benefits, by Trust Fund, are multiplied by the aggregate closed-group benefit factors by Trust Fund. Closed-group railroad administrative expenses and closed-group railroad taxation of benefits are also estimated by applying aggregate closed group benefit factors by Trust Fund. The final amount is then the difference in the components (closed group railroad income less closed group railroad cost).

Appendix 4.3-1 Shuttling Method

In this appendix we discuss the “shuttling method”.

The shuttling method as presented in the COST model attempts to reorganize the PAPs obtained from subprocess 4.2, which maintains a static age-sex distribution of newly entitled beneficiaries, in such a way that captures the changing age-sex distribution of newly entitled beneficiaries provided by subprocess 3.3. The age-sex distribution of the sample (subprocess 4.2) and those newly entitled from subprocess 3.3 are aligned in the sample year. This alignment persists throughout all years in the long-range period. When we refer to the age-sex distribution from subprocess 3.3 in what follows below, we refer to the aligned age-sex distribution.

Let $oadsrs$ be the age/sex distribution of the sample from subprocess 4.2. Let $oabicip$ be the number of newly entitled beneficiaries by age and sex from subprocess 3.3. Let $total$ be the total number of newly entitled beneficiaries by sex. The ratio

$\frac{oabicip(sex, age)}{total(sex)}$, for $age = 62, \dots, 70$ gives the age-sex distribution from subprocess 3.3.

The array $rsb_oadscp_sampleyr$ is the age-sex distribution from subprocess 3.3 from the sample year (2004 for the 2009 Trustees Report).

The value of $oadscp$ is defined, by age and sex, to be:

$$oadscp(sex, age) = \frac{oabicip(sex, age)}{total(sex)} + oadsrs(sex, age) - rsb_oadscp_sampleyr(sex, age).$$

We will call this the *aligned* age-sex distribution as obtained from subprocess 3.3.

For each sex, we construct a matrix $oads$. Construction of this matrix uses two different age-sex distributions: the age-sex distribution of the awards sample and the aligned age-sex distribution of newly entitled beneficiaries from subprocess 3.3. The matrix $oads$ is a 9×9 matrix whose rows and columns are indexed consecutively by the ages 62, ..., 70. We index the rows of this matrix by the age of entitlement of a worker in the projection ($ageentRSB$) and the columns of this matrix by the age of entitlement of a worker in the Awards sample ($ageentAWD$).

More precisely, for a given sex, let $oadsrs$ be the age distribution of the sample; $oadscp$ be the aligned age distribution of newly entitleds from subprocess 3.3. Both arrays $oadsrs$ and $oadscp$ are indexed by age, $ages = 62, \dots, 70$. For a given sex, the matrix $oads$ is constructed with the following properties:

$$oadscp(ageentRSB) = \sum_{ageentAWD=62}^{70} oads(ageentRSB, ageentAWD)$$

and

$$oadsrs(ageentAWD) = \sum_{ageentRSB=62}^{70} oads(ageentRSB, ageentAWD).$$

In other words, the matrix *oads* has the following properties:

- The sum of the entries in any row of the matrix is the value of the distribution of the projection for the age corresponding to the row.
- The sum of the entries in any column of the matrix is the value of the distribution of the Awards sample for the age corresponding to the column.

Let *opap* be the original potential AIME percentages (PAPs) passed to subprocess 4.3 from subprocess 4.2. This is a 9×10 matrix whose rows are indexed by ages at entitlement 62, ..., 70 and whose columns are indexed by benefit interval 1, ..., 10 (represented by the variable *i* in the formulas below). The values of *opap* are modified and the results are the PAPs used by process 4.3, called *opap1*. As a formula,

$$opap1(ageentRSB, i) = \frac{\sum_{ageentAWD=62}^{70} opap(ageentAWD, i) \times oads(ageentRSB, ageentAWD)}{\sum_{ageentAWD=62}^{70} oads(ageentRSB, ageentAWD)}.$$

This formula may be rewritten as follows.

$$opap1(ageentRSB, i) = \sum_{ageentAWD=62}^{70} opap(ageentAWD, i) \times \frac{oads(ageentRSB, ageentAWD)}{\sum_{ageentAWD=62}^{70} oads(ageentRSB, ageentAWD)}.$$

It follows that *opap1* may be interpreted as a reweighting of *opap*. We have

$$opap1(ageentRSB, i) = \sum_{ageentAWD=62}^{70} w_{ageentRSB, ageentAWD} \times opap(ageentAWD, i)$$

with weights $w_{ageentRSB, ageentAWD} = \frac{oads(ageentRSB, ageentAWD)}{oadscp(ageentRSB)}$.

As matrices, $opap1 = w \times opap$ (and is another 9×10 matrix).

Consider the following example. In this example, the projection year is 2040, and the sex is males.

The *oadsrs* vector (from subprocess 4.2) is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.4615	0.1456	0.0673	0.2543	0.0558	0.0058	0.0036	0.0030	0.0030

The unaligned age-sex distribution (from subprocess 3.3) is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.3116	0.1386	0.0502	0.1266	0.1239	0.2145	0.0093	0.0134	0.0118

The age-sex distribution (from subprocess 3.3) in 2004, the year of the sample is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.4348	0.1513	0.0703	0.2476	0.0797	0.0060	0.0018	0.0024	0.0062

Hence the aligned age-sex distribution (from subprocess 3.3), that is, the *oadscp* vector, is as follows.

	62	63	64	65	66	67	68	69	70
Male	0.3383	0.1330	0.0472	0.1333	0.1000	0.2143	0.0112	0.0140	0.0087

The matrix *oads*, computed in this subprocess (4.3) is as follows. An explanation of how this matrix is generated appears below.

<i>ageentRSB</i> \ <i>ageentAWD</i>	62	63	64	65	66	67	68	69	70	Total
62	0.3383	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3383
63	0.1232	0.0098	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1330
64	0.0000	0.0472	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0472
65	0.0000	0.0886	0.0447	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1333
66	0.0000	0.0000	0.0226	0.0774	0.0000	0.0000	0.0000	0.0000	0.0000	0.1000
67	0.0000	0.0000	0.0000	0.1769	0.0374	0.0000	0.0000	0.0000	0.0000	0.2143
68	0.0000	0.0000	0.0000	0.0000	0.0112	0.0000	0.0000	0.0000	0.0000	0.0112
69	0.0000	0.0000	0.0000	0.0000	0.0072	0.0058	0.0010	0.0000	0.0000	0.0140
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0026	0.0030	0.0030	0.0087
Total	0.4615	0.1456	0.0673	0.2543	0.0558	0.0058	0.0036	0.0030	0.0030	1.0000

Note that the column total is *oadsrs* and the row total is *oadscp*. The *oads* matrix is determined using these row and column sum constraints. The nonzero entries of the *oads* matrix zig-zag down and to the right. Starting at the upper left hand corner, the lesser of *oadsrs* and *oadscp* is put there. So $oads(62, 62) = 0.3383$. In this case (as is usually the case) *oadscp* is less. The difference $oadsrs(62) - oadscp(62) = 0.4615 - 0.3383 = 0.1232$ is placed one lower, in $oads(63, 62)$. Since the sum of the first column is $oadsrs(62)$ and the sum of the first row is $oadscp(62)$, we move to the next entry on the right. The difference $oadscp(63) - oads(63, 62) = 0.1330 - 0.1232 = 0.0098$, is $oads(63, 63)$. Now we have that the row sum is $oadscp(63)$. We want the column sum to be $oadsrs(63)$. We move down to $oads(64, 63)$. Since $oadsrs(63) - oads(63, 63) = 0.1456 - 0.0098 = 0.1358 > 0.0472 = oadscp(63)$, by the row sum constraint we are forced to have $oads(64, 63) = 0.0472$. Now move one lower. Since $0.1456 - 0.0472 - 0.0098 = 0.0886 < oadscp(65) = 0.1333$, the entry $oads(65, 63) = 0.0886$. The column sum is now $oadsrs(63)$, and we move one over to the right. We want to fill in this entry. We compute $oadscp(65) - oads(65, 62) = 0.1333 - 0.0886 = 0.0447 < 0.0673$, by the column sum constraint we have $oads(65, 64) = 0.0447$. The row constraint is now met, and we move down to $oads(66, 64)$. We look at $oadscp(66) = 0.1000$. By the column constraint, we cannot place this value in $oads(66, 64)$. So to meet the column constraint, we set

$oads(66, 64)$ equal to the difference $oadsrs(64) - oads(65, 64) = 0.0673 - 0.0447 = 0.0226$. Now that this column constraint is met, we move one column over to $oads(66, 65)$. Note that we cannot put $oadsrs(65) = 0.2543$ there since then the row constraint would be violated. In order to meet the row constraint, $oads(66, 65) = oadscp(66) - oads(66, 64) = 0.1000 - 0.0226 = 0.0774$. Since the row constraint is met, we move one row down to $oads(67, 65)$. We cannot put $oadscp(67)$ there since then the column constraint would be violated. Hence $oads(67, 65) = oadsrs(65) - oads(66, 65) = 0.2543 - 0.0774 = 0.1769$. Now that the column constraint is met, we move one column over to $oads(67, 66)$. We cannot put $oadsrs(66) = 0.0558$ there or else the row constraint would be violated. So we put $oads(67, 66) = oadscp(67) - oads(67, 65) = 0.2143 - 0.1769 = 0.0374$ there. Now the row constraint is met and we move one row down to $oads(68, 66)$. Since $oads(67, 66) + oadscp(68) < oadsrs(66)$, we have $oads(68, 66) = oadscp(66) = 0.0112$. Now move one row down to $oads(69, 66)$. We cannot put $oadscp(69) = 0.0140$ there or else the column constraint would be violated. Hence $oads(69, 66) = oadsrs(66) - oads(67, 66) - oads(68, 66) = 0.0558 - 0.0374 - 0.0112 = 0.0072$ and the column constraint is met. Now move one column over to $oads(69, 67)$. We put $oads(69, 67) = oadsrs(67) = 0.0058$ there since that meets the column constraint and the row constraint remains unsatisfied. So move over to the next column, $oads(69, 68)$. We cannot put $oadsrs(68)$ there since that would violate the row constraint. It follows that $oads(69, 68) = oadscp(69) - oads(69, 66) - oads(69, 67) = 0.0140 - 0.0072 - 0.0058 = 0.0010$. Now the row constraint is satisfied and we move down to the last row and $oads(70, 68)$. By the column constraint, this entry is forced to be 0.0026. Moving to column 69, again, by the column constraint, the entry $oads(70, 69)$ is forced to be 0.0030. Finally, by the row and column constraints, the last entry, $oads(70, 70)$, is 0.0030.

To obtain the w matrix, normalize the rows by dividing by the row sum.

<i>age</i> entRSB \ <i>age</i> entAWD	62	63	64	65	66	67	68	69	70
62	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
63	0.9263	0.0737	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
64	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
65	0.0000	0.6647	0.3353	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
66	0.0000	0.0000	0.2260	0.7740	0.0000	0.0000	0.0000	0.0000	0.0000
67	0.0000	0.0000	0.0000	0.8255	0.1745	0.0000	0.0000	0.0000	0.0000
68	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
69	0.0000	0.0000	0.0000	0.0000	0.5143	0.4143	0.0714	0.0000	0.0000
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2989	0.3448	0.3448

Then, as matrices, $opap1 = w \times opap$, as one may verify. For display purposes the transposes of $opap$ and $opap1$ are shown.

opap	62	63	64	65	66	67	68	69	70
1	0.9994	0.9993	0.9990	0.9985	0.9987	0.9966	0.9970	0.9894	0.9890
2	0.9899	0.9903	0.9856	0.9873	0.9838	0.9319	0.9364	0.9064	0.9109
3	0.9650	0.9696	0.9624	0.9679	0.9575	0.8248	0.7890	0.7381	0.8301
4	0.9348	0.9453	0.9323	0.9479	0.9352	0.7209	0.6623	0.5752	0.7383
5	0.9053	0.9182	0.9076	0.9281	0.9165	0.6464	0.5708	0.4788	0.6876
6	0.8781	0.8940	0.8861	0.9093	0.8946	0.6190	0.5153	0.4281	0.6425
7	0.8510	0.8700	0.8615	0.8905	0.8732	0.5779	0.4924	0.4009	0.6020
8	0.8246	0.8457	0.8373	0.8713	0.8562	0.5380	0.4590	0.3726	0.5662
9	0.7987	0.8201	0.8166	0.8525	0.8396	0.5071	0.4136	0.3495	0.5349
10	0.7720	0.7917	0.7943	0.8341	0.8216	0.4823	0.3883	0.3377	0.5087
11	0.7458	0.7651	0.7710	0.8142	0.8042	0.4643	0.3678	0.3313	0.4740
12	0.7194	0.7367	0.7459	0.7927	0.7878	0.4484	0.3487	0.3117	0.4382
13	0.6915	0.7065	0.7172	0.7696	0.7666	0.4355	0.3289	0.2889	0.4145
14	0.6430	0.6524	0.6702	0.7268	0.7299	0.3998	0.2968	0.2563	0.3834
15	0.5744	0.5800	0.5990	0.6630	0.6743	0.3622	0.2764	0.2305	0.3398
16	0.5019	0.5018	0.5272	0.5961	0.6168	0.3328	0.2391	0.2029	0.3089
17	0.4301	0.4235	0.4531	0.5269	0.5522	0.3027	0.2139	0.1925	0.2836
18	0.3590	0.3518	0.3850	0.4638	0.4865	0.2740	0.1955	0.1837	0.2606
19	0.2610	0.2591	0.2979	0.3734	0.4026	0.2416	0.1668	0.1649	0.2294
20	0.1530	0.1624	0.1950	0.2693	0.2972	0.1961	0.1381	0.1323	0.1923
21	0.0750	0.0851	0.1099	0.1711	0.2023	0.1509	0.1020	0.0979	0.1434
22	0.0238	0.0284	0.0413	0.0711	0.0956	0.0850	0.0632	0.0640	0.0829
23	0.0037	0.0049	0.0096	0.0164	0.0230	0.0233	0.0259	0.0192	0.0267
24	0.0000	0.0000	0.0000	0.0000	0.0002	0.0009	0.0019	0.0008	0.0017
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

opap1	62	63	64	65	66	67	68	69	70
1	0.9994	0.9994	0.9993	0.9992	0.9986	0.9985	0.9987	0.9977	0.9802
2	0.9899	0.9899	0.9903	0.9887	0.9869	0.9867	0.9838	0.9589	0.9065
3	0.9650	0.9653	0.9696	0.9672	0.9667	0.9661	0.9575	0.8905	0.7766
4	0.9348	0.9356	0.9453	0.9409	0.9444	0.9457	0.9352	0.8269	0.6509
5	0.9053	0.9063	0.9182	0.9146	0.9235	0.9261	0.9165	0.7799	0.5728
6	0.8781	0.8793	0.8940	0.8914	0.9041	0.9067	0.8946	0.7533	0.5232
7	0.8510	0.8524	0.8700	0.8671	0.8839	0.8875	0.8732	0.7237	0.4930
8	0.8246	0.8262	0.8457	0.8429	0.8636	0.8687	0.8562	0.6960	0.4609
9	0.7987	0.8003	0.8201	0.8189	0.8444	0.8502	0.8396	0.6714	0.4286
10	0.7720	0.7735	0.7917	0.7926	0.8251	0.8319	0.8216	0.6501	0.4079
11	0.7458	0.7472	0.7651	0.7671	0.8044	0.8125	0.8042	0.6322	0.3876
12	0.7194	0.7207	0.7367	0.7398	0.7821	0.7918	0.7878	0.6158	0.3628
13	0.6915	0.6926	0.7065	0.7101	0.7578	0.7691	0.7666	0.5982	0.3408
14	0.6430	0.6437	0.6524	0.6584	0.7140	0.7273	0.7299	0.5622	0.3093
15	0.5744	0.5748	0.5800	0.5864	0.6485	0.6650	0.6743	0.5166	0.2793
16	0.5019	0.5019	0.5018	0.5103	0.5805	0.5997	0.6168	0.4722	0.2479
17	0.4301	0.4296	0.4235	0.4334	0.5102	0.5313	0.5522	0.4247	0.2281
18	0.3590	0.3585	0.3518	0.3629	0.4460	0.4678	0.4865	0.3777	0.2116
19	0.2610	0.2609	0.2591	0.2721	0.3563	0.3785	0.4026	0.3191	0.1858
20	0.1530	0.1537	0.1624	0.1733	0.2525	0.2742	0.2972	0.2440	0.1532
21	0.0750	0.0757	0.0851	0.0934	0.1573	0.1765	0.2023	0.1738	0.1137
22	0.0238	0.0241	0.0284	0.0327	0.0644	0.0754	0.0956	0.0889	0.0695
23	0.0037	0.0038	0.0049	0.0065	0.0149	0.0176	0.0230	0.0233	0.0236
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0006	0.0014
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000