2018 SOA Annual Meeting Session 003:
Stochastic Modelling for Social Security
Actuarial Valuations

Office of the Chief Actuary
U.S. Social Security Administration

October 15, 2018
Stochastic Model Basics

• Exploring uncertainty for 75-year projections
  – In addition to deterministic alternatives
  – In addition to sensitivity analysis
• Probabilistic illustration based on past variation
• 5,000 Monte Carlo simulations
• Reflects basic projection methods
  – But with somewhat more limited parameters
Stochastic Model Basics, cont.

- Demographic assumptions:
  - Total fertility rate
  - Mortality rates by sex/age-group
  - Total legal permanent resident (LPR) new immigrants
  - Total LPR and citizen emigrants
  - Rate of transfers of status from other-than-LPR stock
  - Total other-than-LPR immigration
Stochastic Model Basics, cont.

• Economic assumptions:
  – Unemployment rate
  – % change in CPI
  – Real interest rate
  – % change in real wage

• Disability assumptions:
  – Total male and female disability incidence rate
  – Total male and female disability recovery rate
Stochastic Model Basics, cont.

• Based on historical data, we estimate an equation (generally AR or ARMA, depending on the fit) for each assumption.

• Years of data used vary by assumption, depending on which years of data are available and appropriate.
The total fertility rate (TFR) equation is an ARMA(4,1):

\[ u_t + 1.84X_{t-1} - 1.29X_{t-2} + 0.83X_{t-3} - 0.41X_{t-4} + 1.00e_t - 0.47e_{t-1} \]

where \(e_t\) is the error term at time \(t\), \(u_t\) is the alt 2 value at time \(t\), and \(X_t\) is the deviation from the mean at time \(t\).
Stochastic Results of Total Fertility Rate

Note: The grey mark is the low-cost ultimate assumption and the green mark is the high-cost ultimate assumption.
Adding Parameter Uncertainty (PU)

\[ u_t + 1.84X_{t-1} - 1.29X_{t-2} + 0.83X_{t-3} - 0.41X_{t-4} + 1.00e_t - 0.47e_{t-1} \]

- Under PU, the parameters 1.84, -1.29, 0.83, -0.41, and 0.47 would change in each simulation.
- \( u_t \) would also change for each simulation.
Issues in Implementing Parameter Uncertainty

1. Demographic bounds reached more often.
2. Negative other-than-LPR population can occur.
3. Disability rates can go under 0% and over 100%.
4. Economic values can have extremely high variance.
Results of Total Fertility Rate
Summary Results – Cost Rate
Summary Results– Trust Fund Balance

![Graph of Trust Fund Ratio Stochastic Medians and 95% Confidence Intervals and Alternatives I-III](image)
# Summary Results – Actuarial Balance

<table>
<thead>
<tr>
<th>Alt / Percentile</th>
<th>Deterministic</th>
<th>2017TRStoch</th>
<th>PU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt 1 / 97.5</td>
<td>0.12</td>
<td>-1.07</td>
<td>0.14</td>
</tr>
<tr>
<td>Alt 2 / 50.0</td>
<td>-2.83</td>
<td>-2.81</td>
<td>-2.82</td>
</tr>
<tr>
<td>Alt 3 / 2.5</td>
<td>-6.63</td>
<td>-5.01</td>
<td>-6.74</td>
</tr>
</tbody>
</table>
Conclusions and Questions

• Usefulness of stochastic simulations
  – For Social Security
    • Is risk aversion same in both directions?
  – For private insurance
    • Is risk aversion one-sided?

• How is stochastic used in private insurance/pensions/other areas?
  – Univariante or multiple variables?
  – Parameter uncertainty?