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AVERAGE VALUE OF INVESTED FUNDS, OASI TRUST FUND, FISCAL YEAR 1962¹

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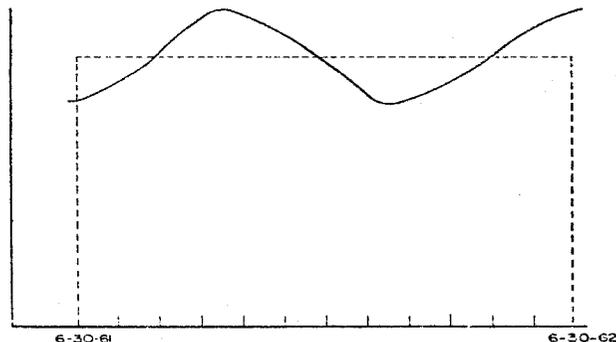
This note presents the average value of the invested portion of the OASI Trust Fund for the fiscal year 1962 (which has not previously been available), and some comparisons with certain approximations. It is believed to be of interest to (1) anyone interested in estimating or analyzing investment income and (2) anyone interested in the cash management of this fund. No specialized knowledge of mathematics is assumed, although there is an appendix which gives formulas and other details of the computation and data collection.

In summary, the analysis indicates that in fiscal year 1962, the true average size of the interest-earning, invested assets of the OASI Trust Fund computed from daily data was about 2½% higher than the average size computed from the readily available, published end-of-month data.

I. Average Value of Invested Funds - Concept and Approximations Defined

We shall refer to three concepts of Average Amount of Invested Funds.

The first concept will be called Exact Average Amount of Invested Funds (sometimes called Average Exposure, or even Exposure). It can be understood most easily by referring to the following diagram:



If the unbroken wavy line represents the actual amount of invested funds (which varies with time), the Exact Average Amount of Invested Funds is represented by the height of the rectangle that has an area equal to the figure bounded on top by the wavy line (both figures having the same width).

Like other averages, this is useful because it compresses a whole year's experience into a single figure. Like other averages (for example, the "Average American Family"), it also can lead to distortions if good judgment does not accompany its use. From here on, we assume that the reader is familiar with the limitations of the concept of Exact Average Amount of Invested Funds, and we shall examine the problem of computing the quantity by various methods and of evaluating the accuracy of these methods.²

There are no data regularly published which permit calculation of the Exact Average Amount of Invested Funds. In order to compute this quantity the author recorded and summarized the investment ledger for the OASI Trust Fund for the fiscal year 1962, obtaining the exact day of each transaction. The Exact Average Amount of Invested Funds was calculated from this record by the formula given in Item (a) of the Appendix.

¹The author wishes to thank Mr. John Turner, Chief, and Miss Michelina Falcone of the Investments Branch, Division of Deposits and Investments, Treasury Department, for their permission to use the investment ledgers of the Trust Fund and their help in using those records.

²The basic ideas that were used to develop the formula are set forth in Harry Gershenson, *Measurement of Mortality*, Chicago, Society of Actuaries, 1961. The complicated coefficient of "i" on page 24 thereof is similar to our concept of Exact Average Value, but our formula is the arithmetic mean of a formula that works "with traffic" and one that works "against traffic."

The second concept is the Arithmetic Mean of Invested Funds, Beginning and End of Fiscal Year. The method of computing this is quite simple. The source for this calculation is a regular Treasury Department statement of assets of the Trust Fund, issued monthly.

The third concept is Mean Value of Invested Funds based on End-of-Month Data. The mean value for each month is computed as the mean of the values for invested funds for the beginning and end of the month. Then, the mean of the 12 monthly means is calculated. The source is the same monthly statement of assets referred to previously.

It will be instructive for our analysis in the next sections to point out here some features of the process of computation. From the point of view of making the computation, the three concepts are closely related. The Exact Average Amount of Invested Funds is usually calculated in two steps:³ (1) the Arithmetic Mean of Invested Funds, Beginning and End of Fiscal Year is calculated, and (2) a complicated Adjustment Term is calculated and added on. There is another way of calculating the Exact Average Amount of Invested Funds, which shows the relation between Exact Average Amount of Invested Funds, and Mean Value of Invested Funds based on End-of-Month Data:⁴ (1) the Mean Value of Invested Funds, based on End-of-Month Data is calculated; (2) twelve adjustment terms are calculated, using in turn each calendar month of the fiscal year as the appropriate unit interval; (3) the Arithmetic Mean of the twelve adjustment terms is calculated; and (4) the Arithmetic Mean of the adjustment terms is added to the Mean Value of Invested Funds based on End-of-Month Data. The number thus computed will be identical with that computed under the first method.⁵

³Refer to Item (a) of the Appendix. The fiscal year is taken to be the unit interval.

⁴Refer to Item (a) of the Appendix. The calendar month is taken to be the unit interval, and the calculation is made separately for each month of the fiscal year.

⁵The second method would be inefficient as a practical procedure of computation. It is given here to show the relation between the two concepts.

II. Results of Calculations Under Three Concepts

(a) Arithmetic Mean of Invested Funds, Beginning and End of Fiscal Year.

\$18,978,459,825.25

(b) Adjustment to (a) due to transactions in securities during the year.

\$46,148,788.98

(c) Exact Average Amount of Invested Funds.

(a) + (b) \$19,024,608,614.23

(d) Mean Value of Invested Funds based on End-of-Month Data.

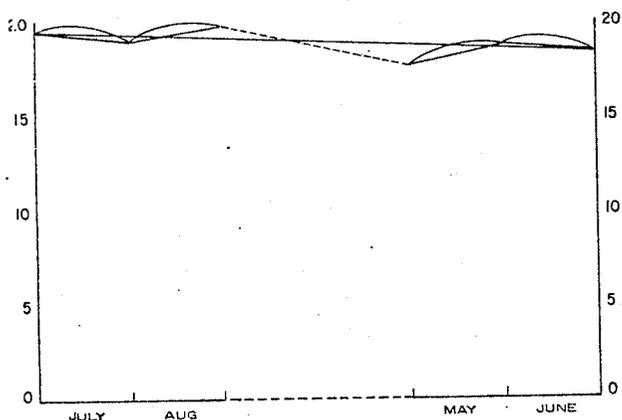
\$18,582,231,506.76

III. Discussion of Results of Calculations

A look at the results of the calculations, in Section II, for the Exact Average value (first concept) and Arithmetic Mean, Beginning and End of Fiscal Year (second concept) suggests that it may not be important to have detailed information on daily transactions within a month in order to calculate accurately the Exact Average value. The Adjustment Term increases the Arithmetic Mean of Invested Funds by \$46.1 million, which is equivalent in interest income to \$1.8 million per year at the current rate of 3-7/8% for new investments. Compared to the total invested funds, the \$46.1 million Adjustment is 1/4% of the \$19,025 million average invested portion of the Trust Fund.

However, if we consider the Mean Value of Invested Funds based on End-of-Month Data (third concept), we see that it is about 2-1/2% lower than the Exact Average Amount of Invested Funds. This is important because the former, using much more data than is used in Arithmetic Mean of Invested Funds, Beginning and End of Fiscal Year, would seem to have claim to greater accuracy. Thus, variation within each month is much more important than the analysis would at first lead us to believe--about 10 times as important for this fiscal year. The situation can be graphed as follows:

AMOUNT OF INVESTED FUNDS, BILLIONS



The single straight line represents the assumption of the mean of book value at the beginning and end of the fiscal year. The connected series of straight lines represents the use of all end-of-month data and thus can apparently lay claim to more credibility (although we know in this case that it is not so close to the true value) in the absence of "actual" data on average invested funds. The wavy line schematically represents the actual situation.

The data and the diagram so far show that the area of the wavy-line figure is very close to that bounded on top by the single straight line, but it is considerably different from the figure bounded on top by the connected series of straight lines. Prudence suggests that the closeness of the areas of the first two figures should be regarded as coincidence, unless and until better established, either by more years of empirical evidence, or by a sturdy rationale. On the other hand, it seems evident that the area under the series of straight lines will always be significantly less than that under the wavy lines, although we cannot be certain as to the precise amount of the differential.

This latter surmise is given further support if we consider that under present Trust Fund investment procedure, securities are generally redeemed on the last day of each month to provide cash for the payment of benefits.

IV. Comparison Within Months

The following table compares the Arithmetic Mean of Invested Funds, Beginning and End of Month with the Exact Average Amount of Invested Funds:

Month	(1)	(2)	(3)
	Average amount of invested funds for month (millions)		
	Mean of beginning and end	Actual	Ratio of (2) to (1)
July	\$19,254.4	\$19,936.2	1.0354
August	19,373.7	19,494.2	1.0062
September	19,580.7	20,039.9	1.0235
October	19,066.9	19,641.9	1.0302
November	18,753.3	19,078.3	1.0173
December	18,588.4	19,102.5	1.0277
January	17,989.7	18,675.4	1.0381
February	17,754.4	18,121.3	1.0207
March	17,962.8	18,514.9	1.0307
April	17,832.8	18,219.9	1.0217
May	18,224.5	18,378.4	1.0084
June	18,605.1	19,111.4	1.0272
Total	222,986.8	228,314.3	1.0239

In every month the actual average invested funds exceeds the mean of the beginning and end of the month. The average ratio for the year was 1.0239, which ratio was calculated in a different way in Section III.

Technical Appendix:

(a) Formula for Exact Average Amount of Invested Funds.

To show the computation in a compact formula, the following notational scheme will be used:

Standardized time scale. We can refer to the Exact Average Amount of Invested Funds over any given interval of time. For this formula, whatever the interval, we shall introduce a *standardized time scale* t where the beginning of the interval is identified as 0 and the end of the interval is identified as 1. "t" is used to represent the proportion of the interval that has elapsed. Only two time intervals are used in this note, the fiscal year and the calendar month.

Identifying transactions. Let each of the n transactions in the given interval be assigned one of the numbers 1, 2, ..., n for identification. The letter "i" will represent this identifying number.

Symbols used in the formula.

Let F_t be the Book Value of the Invested Funds at standard time t .

Let t_i be the standardized time of the i -th transaction.

Let β_i be the Book Value of the i -th transaction at t_i .

Let A_i have the value -1 if the i -th transaction is an acquisition, and +1 if the i -th transaction is a disposition.

The formula for the Exact Average Value of Invested Funds is as follows:

$$\frac{F_0 + F_1}{2} + \sum_{i=1}^n a_i (t_i - .5) \beta_i$$

(b) Formula for Mean Value of Invested Funds, based on End-of-Month Data.

Using the same notation as in (a), taking the unit time interval as one month in the fiscal year, the formula is

$$\frac{1}{12} \sum_{i=0}^{11} \frac{1}{2} (F_i + F_{i+1})$$

(c) Data sources.

All data for calculation of the Adjustment Terms came from direct examination of the investment ledgers of the Trust Fund (Treasury Department, Division of Deposits and Investments, Investments Branch).

(d) Terminology.

"Book value," "value," and "amount of invested funds" are synonyms in a broad sense, with variations specified by context.

"Average" and "mean" are synonyms in a broad sense with variations specified by context.

(e) Rounding of calculations.

Calculated numbers were deliberately carried to more places than are known to be accurate, to ensure that rounding error is negligible compared to other types of error.