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A RE-EXAMINATION OF THE LINK BETWEEN SOCIAL SECURITY AND SAVING

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by

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Recently much attention has been devoted to the economics of social insurance.[1/] A major area of disagreement concerns the relationship between pay-as-you-go social security and private saving. This debate emerged in 1974 with the publication of conflicting research. The traditional life-cycle model of saving was extended by Feldstein (1974) and Munnell (1974 a,b) to incorporate social security. In this model saving during the working years provides for consumption during the retirement years. When a pay-as-you-go social security program is introduced, the workers are taxed and the proceeds are distributed as benefits to the retired. It was argued that the retired would consume their social security benefits and the workers would reduce their private saving since they would anticipate receiving benefits themselves paid by the next generation of workers.[2/] Hence, private saving would fall with the introduction of social security.

A crucial assumption of this analysis is that saving is motivated solely by a desire for retirement consumption. Barro (1974) and Miller and Upton (1974) relaxed this assumption by analyzing the effect of social security in models where saving is also possibly motivated by a desire to leave a bequest. Under certain conditions they demonstrated that the introduction of pay-as-you-go social security would have no effect on private saving.[3/] This neutrality proposition followed because the start-up old generation rationally perceived their social security benefits as imposed negative bequests.[4/] If the bequest process was operative before the program, then the imposed negative bequests would be undesired and thus offset by increased private bequests.[5/] It followed that private saving was unaffected.

Feldstein (1976 a,b) argued that this rational bequest analysis is based on restrictive assumptions. Even if one accepts the basic framework, he claimed that the neutrality proposition follows only if two conditions are satisfied. First, the present value of the imposed future liabilities from the program (the negative bequests) must exactly equal the present value of the benefits received by the start-up old generation. If the present value of the liabilities is less than the present value of the benefits, then the start-up old increase private bequests by only a fraction of their benefits and private saving falls. We call this the social security wealth effect. Feldstein argued that this effect was absent in Barro's analysis because he assumed that there was no output growth in the economy.

Secondly, Feldstein (1978) reiterated that the neutrality proposition hinged on the existence of an operative bequest process. If, for example, the process was not operative because the start-up old generation actually desired to make negative bequests but was constrained by law to make non-negative bequests, then the introduction of social security would reduce saving as the old would not fully offset the imposed negative bequests of the program. We shall call this the endowment redistribution effect of social security.

Barro (1978a) agreed with Feldstein's discussion of the endowment redistribution effect. He disagreed, however, with Feldstein's contention that a social security wealth effect arises when there is output growth (see Barro (1976)). Only when the output growth rate is at least as large as the interest rate does the introduction of social security generate a wealth effect.[6/]

This paper attempts to make two contributions to this research. The first one is expositional. A simple overlapping generation's model is developed and we re-investigate the wealth and endowment redistribution effects from the introduction of pay-as-you-go social security. A single diagram capsulizes the analysis and hopefully adds to the reader's intuition. Our second contribution is substantive and extends the analysis of the endowment redistribution effect. Instead of thinking of social security as a program exogenously imposed on society and then analyzing how society responds to it, we model the program as a device that society implements to cause the endowment redistribution effect.[7/] Specifically, assuming a non-negative private bequest constraint exists,[8/] we argue that society may turn to pay-as-you-go social security in order to achieve its desired intergenerational allocation of wealth. The model we develop allows us to derive an explicit equation for the demand curve for social security. We show how certain events like a depression or an unanticipated inflation shift this demand curve through their effect on the intergenerational endowment of wealth. Events that tilt this distribution toward future generations increase the demand for social security. We specify a polar case model of social security supply to analyze the development of the program over time. Finally, we attempt to place the social security private saving disagreement in perspective.

I. The Model

The model developed in this section exhibits and hopefully clarifies the issues surrounding the debate on the effect of social security on private saving. We first briefly describe the economy and then turn to individual behavior. In this section it is assumed that pay-as-you-go social security is exogenously imposed on the society unexpectedly at some point in time. We will focus on the behavior of the start-up old generation as it was seen to be crucial in the analysis of the effects of the program on private saving.

A. The Economy

Time is divided into periods, say t, t+1, and so on. During any period the economy is composed of two overlapping generations à la Samuelson (1958). All individuals live for two periods. Consider a typical member of generation t. In period t, the individual's first period, he receives an inheritance from generation t-1, receives a wage for supplying a fixed quantity of labor (set at one unit), pays a social security tax on his wage (if a program exists), and allocates his after tax wealth between first period consumption and saving. In period t+1, the individual's second period, he is retired. His saving matures with interest and also he receives a social security benefit payment (again, if there is a program). He allocates this wealth between second period consumption and a private bequest to generation t+1.

The population is assumed to grow at rate n, so in any period there are 1+n persons in the first period of their lives (we refer to them as the workers, the young, etc.) for each person in the second period of his life (the retired, the old, etc.).

The two prices, the real wage rate w and the real one period interest rate r, are assumed to be constant through time.[9/]

B. Individual Behavior

1. Preferences

To allow for a bequest motive, we follow Barro (1974) and Miller and Upton (1974) by assuming that the utility of the representative member of generation t+1 is an argument of the utility function of the representative member of generation t:

$$U^t = U(c_1^t,\ c_2^t,\ U^{t+1})$$

where U^τ, c_1^τ, and c_2^τ are the utility, first period consumption, and second period consumption of the representative member of generation τ. For concreteness, we assume further that U has the following form:

$$U^t = \beta \log c_1^t + (1-\beta) \log c_2^t + \rho U^{t+1}$$

where ρ can be interpreted as the coefficient of altruism.

A reduced form utility function can be derived by substituting in for U^{t+1}, and then for U^{t+2}, and so on:

$$(1) \quad U^t = \sum_{\tau=t}^{\infty} \rho^{\tau-t} \left[\beta \log c_1^\tau + (1-\beta) \log c_2^\tau\right]$$

2. The Budget Set

The utility function in equation (1) has an infinite number of arguments. Write this consumption sequence as:

$$\left\{ (c_1^\tau,\ c_2^\tau) \right\}_{\tau=t}^{\infty}$$

The budget set includes all such sequences that are affordable. A sequence is affordable when its population-weighted present value is no greater than the individual's wealth (to be defined). To see this, consider an arbitrary element in the sequence, say (c_1^τ, c_2^τ), the first and second period consumption for a member of generation τ. Recall that there are $(1+n)^{\tau-t}$ members of generation τ for each member of generation t. Hence, the representative individual of generation t must plan for the larger consumption bundle:

$$\left((1+n)^{\tau-t}\ c_1^\tau,\ (1+n)^{\tau-t}\ c_2^\tau\right)$$

if the representative individual of generation τ is to enjoy (c_1^τ, c_2^τ). Weighting each element in the sequence in this way and appropriately discounting the sum back to period t we get c^t, the population-weighted present value:

$$(2) \qquad c^t = \sum_{\tau=t}^{\infty} \left(\frac{1+n}{1+r}\right)^{\tau-t} \left(c_1^\tau + \frac{c_2^\tau}{1+r}\right)$$

The appropriate definition of wealth for this problem is the population-weighted present value of the individual's "endowed" consumption sequence.[10/] Before the introduction of social security this sequence is the following:

$$\{(a_i^t + w,\ o),\ (w,o),\ (w,o),\ldots\}$$

Each element in the sequence has two arguments: first period endowment and second period endowment. The first element in the sequence is generation t's own endowment: an inheritance from generation t-1, a_i^t, and its wage, w, both received in period t (the individual's first period). The following elements are the own endowments of individuals in generations t+1, t+2, and so on. Population-weighting and discounting back to period t as before we get a^t, generation t's pre-social security wealth;

$$(3) \qquad a^t = (a_i^t + w) + \sum_{\tau=t+1}^{\infty} \left(\frac{1+n}{1+r}\right)^{\tau-t} .w$$

Putting equations (2) and (3) together and adding non-negativity constraints on consumption, we have the pre-social security budget set:

$$(4) \qquad c^t \leq a^t; \; c_i^\tau \geq o \quad \tau \geq t; \; i = 1, 2$$

A re-expansion of equation(4) in the following way facilitates graphical analysis:

$$(5) \qquad c^t = c_o^t + c_f^t \leq a_o^t + a_f^t = a^t \quad \text{where:}$$

$$c_o^t = c_1^t + \frac{c_2^t}{1+r}$$

$$c_f^t = \sum_{\tau=t+1}^{\infty} \left(\frac{1+n}{1+r}\right)^{\tau-t} \left(c_1^\tau + \frac{c_2^\tau}{1+r}\right)$$

$$a_o^t = a_i^t + w$$

$$a_f^t = \sum_{\tau=t+1}^{\infty} \left(\frac{1+n}{1+r}\right)^{\tau-t} .w$$

Equation (5) states that the present value of <u>own</u> lifetime consumption, c_o^t, plus the population-weighted present value of the lifetime consumption of individuals in all <u>future</u> generations, c_f^t, must not exceed the individual's <u>own</u> wealth (inherited plus human), a_o^t, plus the weighted present value of the human wealth of <u>future</u> generations, a_f^t.

The difference between the individual's own wealth and his own lifetime consumption is his <u>private</u> bequest:

(6) $b_p^t = a_o^t - c_o^t$

Figure 1 exhibits the individual's budget set. If private bequests are not constrained to be non-negative, then allocations within triangle OBC are feasible. Point A is the endowment and wealth a^t is measured along the c_o^t - axis. Budget line BC has a slope equalling -1.

If private bequests are constrained to be non-negative ($b_p^t \geq o$), then the budget set is constricted to trapezoid OBAD. Allocations within triangle DAC are attained only if the individual leaves a negative private bequest and, therefore, are not available to him.

3. Social Security and the Budget Set

We now show the effect of pay-as-you-go social security on the budget set. We investigate its effect on choice in the next sub-section. First, a brief description of a pay-as-you-go program is provided.[11/]

The basic feature of a pay-as-you-go program is that the payroll taxes collected from the workers equal the benefits paid to the retired population <u>in every period</u>. Thus, there is one generation, say the t th, that receives benefits (in period t+1 when they are retired) without paying taxes. Here the program is initiated in period t+1 and generation t is commonly referred to as the <u>start-up old generation.</u> Generation t+1, those workers who first pay social security taxes, are referred to as the <u>start-up young generation</u>. The program can be either perpetual or eventually liquidated (some generation $\tau > t$ pays taxes but receives no benefits). Finally, the program can be expressed simply as a sequence of payroll (wage) tax rates:

$$\left\{ g_{t+1}, g_{t+2}, \ldots \right\}$$

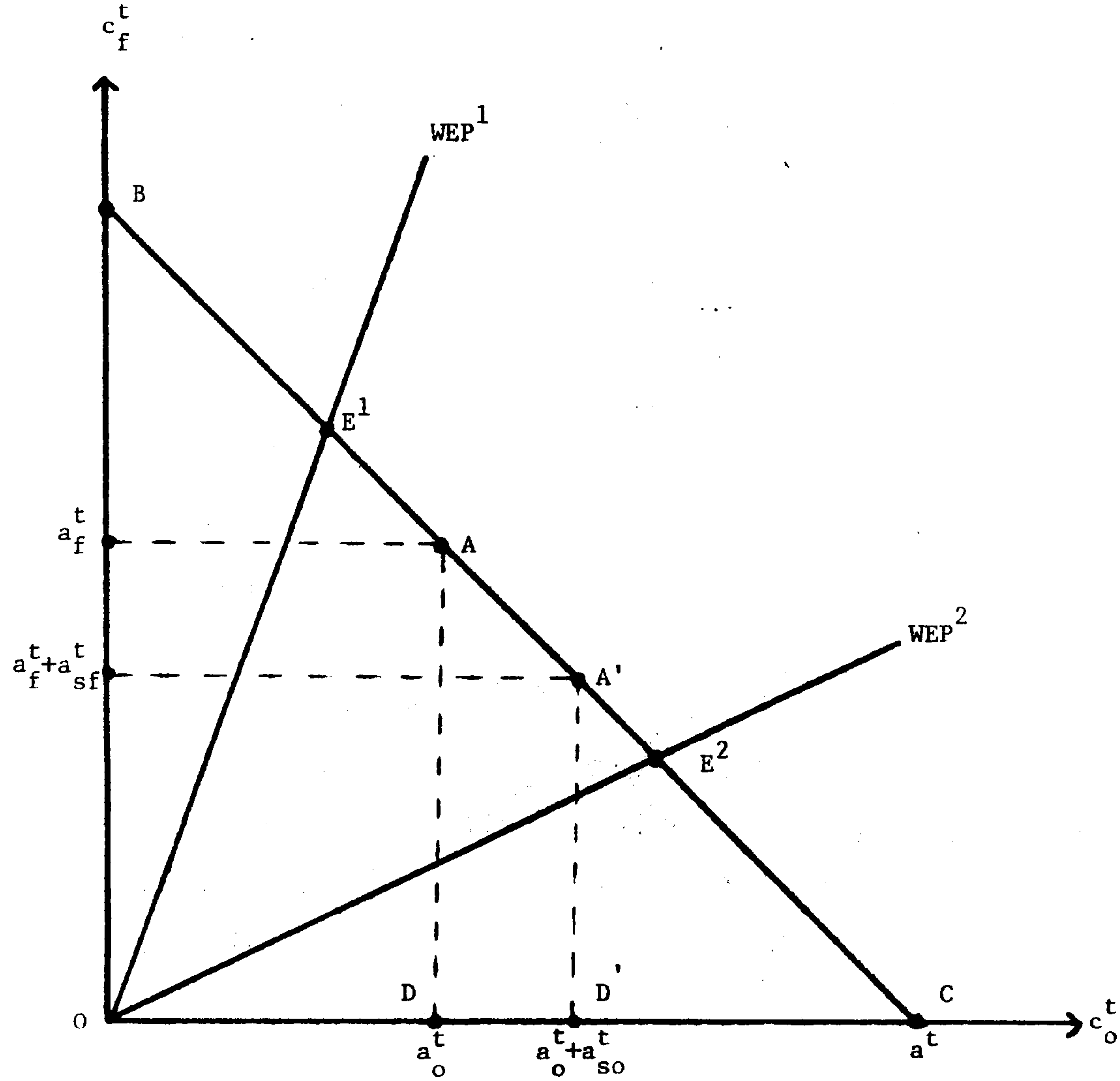

Figure 1

The review of the social security and saving debate revealed that the behavior of the start-up old generation was crucial. Therefore, we focus on the budget set effects of a perpetual program with constant tax rates (without loss of generality) instituted in period t+1. We set $g_\tau = g$ for all $\tau \geq t+1$.

The program generates the following sequence of social security endowments:

$$\{(o,\ gw(1+n)),\ (-gw,\ gw(1+n)),\ (-gw,\ gw(1+n)),\ldots\}$$

The first element is the endowment imposed on a member of generation t, the start-up old generation. The first argument is zero since no taxes are paid. The second argument is the social security benefit received when generation t is retired (in period t+1).12/ The following identical elements are the social security endowments imposed on the members of generations t+1, t+2, and so on. Each of these individuals pays gw in taxes during the working period and receives gw (1+n) in benefits during the retirement period.

The population-weighted present (period t) value of the social security endowment sequence is a_s^t, generation t's social security wealth:

$$a_s^t = a_{so}^t + a_{sf}^t \tag{6a}$$

where $$a_{so}^t = gw\ \frac{1+n}{1+r}$$

$$a_{sf}^t = \sum_{\tau=t+1}^{\infty} \left(\frac{1+n}{1+r}\right)^{\tau-t} \left(gw\left(\frac{1+n}{1+r}\right) - gw\right)$$

Equation (6a) states that generation t's social security wealth, a_s^t, equals <u>own</u> social security wealth, a_{so}^t, plus the population-weighted present value of the social security endowments imposed on <u>future</u> generations, a_{sf}^t. Note that a_{sf}^t can be thought of as the social security bequest for generation t.

By adding social security wealth given in equation (6a) to the right-hand side of equation (5), pre-social security wealth, we get the post-social security budget constraint for a member of the start-up old generation t:

$$(5') \qquad c^t = c_o^t + c_f^t \leq (a_o^t + a_{so}^t) + (a_f^t + a_{sf}^t) = a^t + a_s^t$$

Equation (6), the definition of private bequests, can also be expanded to include the social security program:

$$(6') \qquad b_p^t = (a_o^t + a_{so}^t) - c_o^t$$

Finally, we introduce the concept of generation t's total bequest, b^t, defined as the sum of private bequests and the social security bequest:

$$(7) \qquad b^t = b_p^t + a_{sf}^t$$

To visualize the effect of social security on the budget set of the start-up old generation refer to figure 1. When $a_s^t \neq o$ the budget line BC shifts. We call this the social security wealth effect. By algebraic manipulation of equation (6a) it can be shown that:[13/]

$$(8) \qquad a_s^t = \begin{cases} o & \text{when } r > n \\ gw & r = n \\ \infty & r < n \end{cases}$$

Equation (8) is a reproduction of Barro's no wealth effect proposition when r >n. The key to this result is the population-weighting of the imposed social security endowment sequence. A positive wealth effect arises when $r \leq n$. This "free-lunch" case is well-known in the literature at least since Samuelson's classic 1958 paper on consumption loans.[14/] We will therefore restrict our analysis to the case where there is no wealth effect: $a_s^t = o$.

When there is no wealth effect the present value of the benefits received by the start-up old generation, generation t's own social security wealth are exactly offset by the negative social security wealth (population-weighted present value) imposed on future generations: $a^t_{so} = -a^t_{sf}$. We call this the endowment redistribution effect on generation t's budget set. In figure 1 this effect is shown as the shift of the endowment along budget line BC from A to A'. If a non-negative bequest constraint exists, then the program expands the budget trapezoid from OBAD to OBA'D'. We now analyze this budget set effect on the consumption sequence choice of generation t.

4. Choice

The individual is assumed to choose the consumption sequence that maximizes utility subject to his budget constraint. Our technique is to consider two constrained maximization problems. Problem 1 is the maximization of equation (1), the individual's utility function, subject to the budget constraint given in equation (5') without the non-negative bequest constraint. Problem 2 is problem 1 with the non-negative bequest constraint added. Let $(\bar{c}^t_o, \bar{c}^t_f)$ and $(\bar{\bar{c}}^t_o, \bar{\bar{c}}^t_f)$ be the solutions 15/ to problems 1 and 2, respectively. It can be shown that the solutions are the following: 16/

$$(9) \qquad (\bar{c}^t_o, \bar{c}^t_f) = \left((1-\rho)(a^t + a^t_s),\ \rho(a^t + a^t_s)\right)$$

$$(\bar{\bar{c}}^t_o, \bar{\bar{c}}^t_f) = \begin{cases} (\bar{c}^t_o, \bar{c}^t_f) \text{ when } \dfrac{\rho}{1-\rho} \geq \dfrac{a^t_f + a^t_{sf}}{a^t_o + a^t_{so}} \\[2ex] (a^t_o + a^t_{so},\ a^t_f + a^t_{sf}) \quad \dfrac{\rho}{1-\rho} < \dfrac{a^t_f + a^t_{sf}}{a^t_o + a^t_{so}} \end{cases}$$

The solutions given in equation (9) are analyzed in figure 1. We have included two, alternative wealth expansion paths, WEP^1 and WEP^2.

Path WEP^1 corresponds to a relatively high coefficient of altruism, while WEP^2 reflects a lower coefficient. The intersection of the wealth expansion path and the budget line BC is the optimal allocation of wealth when there is no bequest constraint. The portion of wealth allocated to future generations rises with the coefficient of altruism, as one would expect. Note also that only the level and not the distribution of wealth (over a_o^t and a_f^t) is relevant, again an expected result.

Since the introduction of social security only affects the distribution of wealth (this is the endowment redistribution effect shown in figure 1 as the move from A to A') and not its level (since $a_s^t = o$ when $r > n$), it has no effect on the optimal wealth allocation when there is no bequest constraint. By substituting equation (9) into equations (6) and (7) we can see the effect of social security on optimal private and total bequests, $\bar{b}_p^t$ and $\bar{b}^t$, respectively:

		Before Social Security	After Social Security
Private Bequest	$\bar{b}_p^t$	$\rho a_o^t - (1-\rho)\, a_f^t$	$\rho a_o^t - (1-\rho) a_f^t + a_{so}^t$
Total Bequest	$\bar{b}^t$	$\rho a_o^t - (1-\rho) a_f^t$	$\rho a_o^t - (1-\rho) a_f^t$

The bottom row indicates that generation t's total bequest is unaltered by social security. Private bequests increase by generation t's social security benefit (see top row) exactly offsetting the negative social security wealth imposed on future generations (again, since $a_{so}^t = -a_{sf}^t$). To summarize, when $r > n$ and there is no bequest constraint, social security is neutral since the start-up generation's consumption is unaffected.[17/]

Turning to bequest-constrained problem 2, first consider the pre-social security situation with endowment A in figure 1. If WEP^1 is relevant $\left(\frac{\rho}{1-\rho} \geq \frac{a_f^t}{a_o^t}\right)$, then E^1 is the optimum--the unconstrained solution. Here the bequest constraint is not binding. The introduction of social security in this case is neutral. The set of allocations made available by social security (trapezoid DAA'D') only contains allocations inferior to E^1.

On the other hand, if path WEP^2 is relevant in figure 1 $\left(\frac{\rho}{1-\rho} < \frac{a_f^t}{a_o^t}\right)$, then social security is not neutral. Before social security the problem 1 optimum allocation E^2 lies outside the budget set (OBAD). The best allocation within OBAD is the endowment point A, a corner solution where the bequest constraint is binding. After social security is introduced, the new set of opportunities contains allocations superior to point A. In figure 1, the optimal allocation shifts with the endowment from A to A'. Generation t increases its own consumption by its own social security wealth. As we saw earlier, the reduced saving proposition follows in this case. To summarize, when $r > n$, the introduction of pay-as-you-go social security reduces saving if and only if there exists a binding non-negative bequest constraint.[18/]

5. Valuing Social Security

Since the effect of social security (the endowment redistribution effect) is the relaxation of the non-negative bequest constraint (assumed here to be present), a natural interpretation of the shadow value of the constraint is the individual's willingness-to-pay or demand price for social security.[19] Let λ^t be this demand price for a member of generation t, the start-up old generation. By determining how λ^t varies with the size of the social security program (measured by own social security wealth a^t_{so}), a demand curve for social security is obtained. For the utility function specified in equation (1) it can be shown that λ^t has the following form:[20]

$$(10) \quad \lambda^t = \lambda^t\,(a^t_{so}\,;\,a^t_o,\,a^t_f) = \text{Max}\left\{ o,\ 1-\frac{\rho}{1-\rho}\cdot\frac{a^t_o + a^t_{so}}{a^t_f - a^t_{so}}\right\}$$

Let us examine equation (10). First, a^t_o and a^t_f are demand curve shift parameters. We consider certain events in the next section that alter the intergenerational distribution of wealth and shift the demand curve for social security by changing a^t_o and a^t_f. Second, holding a^t_o and a^t_f fixed, we see that $\lambda^t = o$ when $\frac{\rho}{1-\rho} \geq \frac{a^t_f - a^t_{so}}{a^t_o + a^t_{so}}$. Recalling that $a^t_{so} = -a^t_{sf}$, this condition, from equation (9), is required for the coincidence of the solutions to problems 1 and 2. Here the non-negative bequest constraint is not binding and the demand price for social security is zero. In other words, social se-security is neutral when its demand price is zero. Third, when $\frac{\rho}{1-\rho} < \frac{a^t_f - a^t_{so}}{a^t_o + a^t_{so}}$ the demand price of social security is positive, the non-negative bequest constraint is binding, and social security reduces saving.

The relation between the endowment redistribution effect and the demand curve for social security is exhibited in figure 2. If the program measured by $\tilde{a}^t_{so}$ is introduced shifting the endowment from A to $\tilde{A}$, then all of the benefits are consumed by the start-up old since the bequest constraint

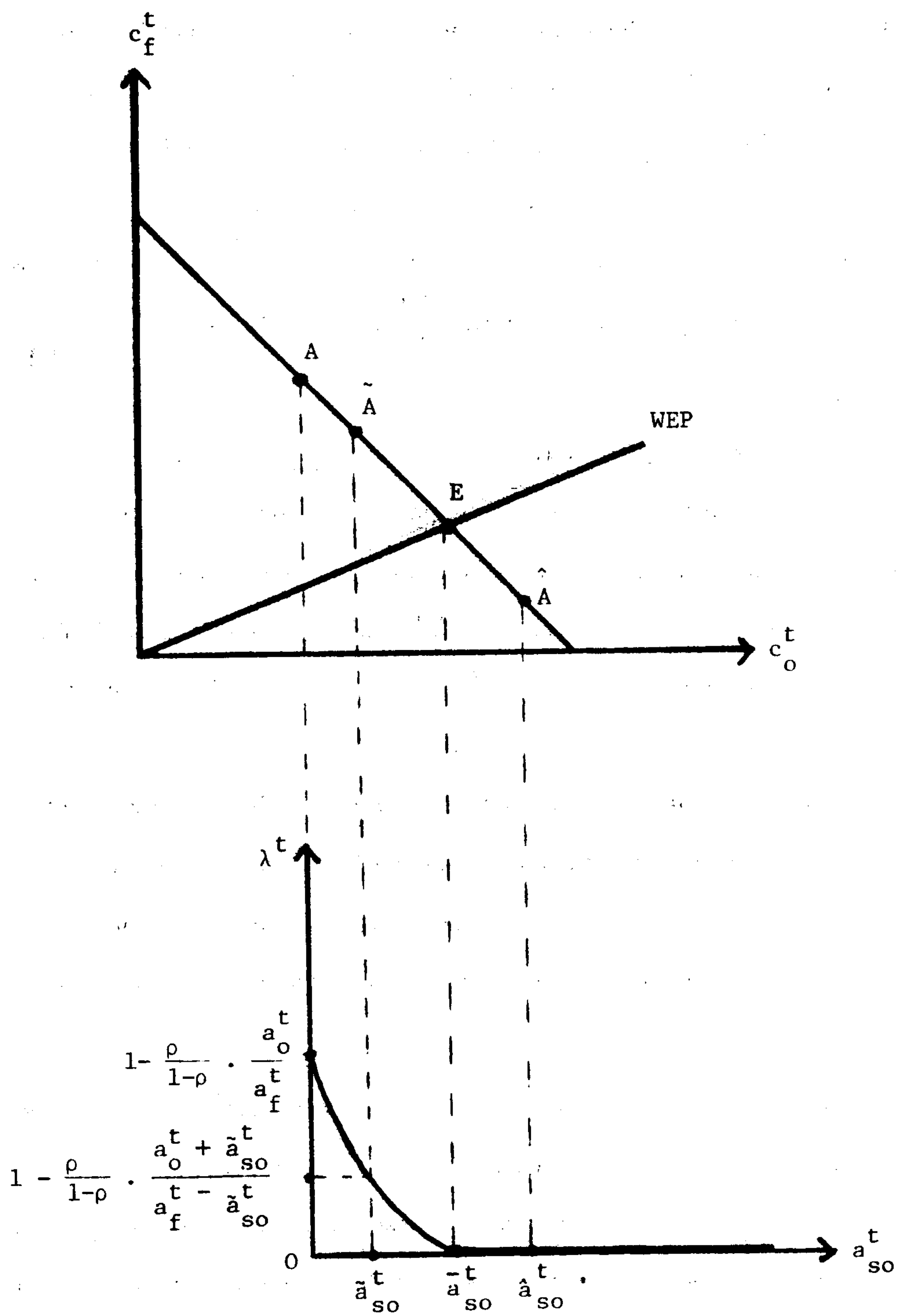
c_f^t
A
$\tilde{A}$
WEP
E
$\hat{A}$
c_o^t
λ^t
$1- \frac{\rho}{1-\rho} \cdot \frac{a_o^t}{a_f^t}$
$1 - \frac{\rho}{1-\rho} \cdot \frac{a_o^t + \tilde{a}_{so}^t}{a_f^t - \tilde{a}_{so}^t}$
0
$\tilde{a}_{so}^t$
$\bar{a}_{so}^t$
$\hat{a}_{so}^t$
a_{so}^t

Figure 2

is still binding: λ^t $(\tilde{a}^t_{so}; a^t_o, a^t_f) > o$. Any program at least as large as the one measured by $\bar{a}^t_{so}$ allows the individual to attain allocation E, the optimum. Here the bequest constraint is no longer binding and $\lambda^t = o$. If $\hat{a}^t_{so}$ is introduced, then the start-up old increase their own consumption by $\bar{a}^t_{so}$ and increase private bequests by $(\hat{a}^t_{so} - \bar{a}^t_{so})$. In the next section we use $\bar{a}^t_{so}$, the smallest program that allows the generation to achieve its optimal wealth allocation. We can solve for $\bar{a}^t_{so}$ by setting $\lambda^t = o$ in equation (10):

$$\text{(11)} \qquad \bar{a}^t_{so} = \bar{a}^t_{so}(a^t_o, a^t_f) = \text{Max} \left\{ o, -(\rho a^t_o - (1-\rho) a^t_f) \right\}$$

To understand equation (11), we first substitute the problem 1 solution in equation (9) into equation (7), giving us desired total bequests:

$$\text{(12)} \qquad \bar{b}^t = \rho a^t_o - (1-\rho) a^t_f$$

Using equation (12), then equation (11) is easily understood. When optimal bequests are negative ($\bar{b}^t < o$), then the private bequest process does not allow the generation to achieve its optimal allocation since bequests are constrained to be non-negative. Hence, in this case negative social security bequests, $-a^t_{so} = a^t_{sf} = \bar{b}^t$, are required. This insight is explored in the next section.

II. An Endogenous Model of Social Security

A potential criticism of much of the research on social security and saving is the way the social security program is treated. Typically, the program is assumed to be exogenously imposed on the private sector. Analysis then focuses on the rational response of the private sector to this imposition.[21/] As we saw in Section I, under certain conditions the private sector neutralizes the imposed intergenerational transfer by offsetting private transfer changes. Under other conditions, the neutralization does not occur.

An alternative perspective is to view social security as an available policy instrument that the private sector, through the political process, may utilize under certain conditions.[22/] We saw in Section I that the introduction (or expansion) of pay-as-you-go social security nominally transfers wealth from future generations to the current generation (the endowment redistribution effect). We also derived the demand curve for these transfers in Section I (equation (10)). We shall now consider different events that shift the demand curve for social security and, by specifying a model of social security supply, trace the effect of the events on the equilibrium quantity of social security. As we stated earlier, the demand curve shift parameters are a_o^t and a_f^t, own and future wealth endowments.

A. Shifts in the Demand for Social Security

Refer to the demand curve for social security in figure 2. Our earlier discussion implicitly assumed an exogenously determined vertical supply curve. Here, shifts in the demand curve have no effect on the equilibrium level of social security (where supply equals demand). Events that reduce the ratio of own wealth to future wealth, $\frac{a_o^t}{a_f^t}$, shift the demand for social security to the right. But under the exogenous vertical supply assumption, the effect is simply to increase the demand price. We will specify an alternative supply model after we more carefully characterize an event and its effect on demand.

Consider figure 3. The endowment point is A and the optimal allocation of wealth is point E, attained by generation t making positive private bequests. Here the non-negative bequest constraint is not binding. Assume initially there is no social security system. Since the constraint is not binding, the demand price of social security is zero for all quantities (see discussion of equation (10) in section I). Generally, an event is an endowment change:

$$(\Delta a_o^t, \Delta a_f^t)$$

For example, suppose generation t experiences a transitory period of extra properity. This event increases their own wealth, but has no effect on the wealth of future generations: $\Delta a_o^t > o$, $\Delta a_f^t = o$. The endowment in figure 3 shifts from A to A' and the optimal allocation of wealth moves up along the wealth expansion path to point E'. From equation (9), generation t's own consumption rises by a fraction of its own wealth increase:

$$\Delta c_o^t = (1-\rho)\ \Delta a_o^t \tag{13}$$

Also, part of generation t's good fortune is shared with future generations since their private bequests increase. From equations (6) and (13) we have:

$$\begin{aligned} \Delta b_p^t &= \Delta a_o^t - \Delta c_o^t \\ &= \rho\ \Delta a_o^t \end{aligned} \tag{14}$$

The higher generation t's coefficient of altruism, the larger the fraction of their good fortune they pass on to the future. Since the non-negative bequest constraint is not binding at A', this event has no effect on the demand for social security.

As another example, suppose generation t experiences a transitory period of depression, their own wealth is reduced and future wealth remains

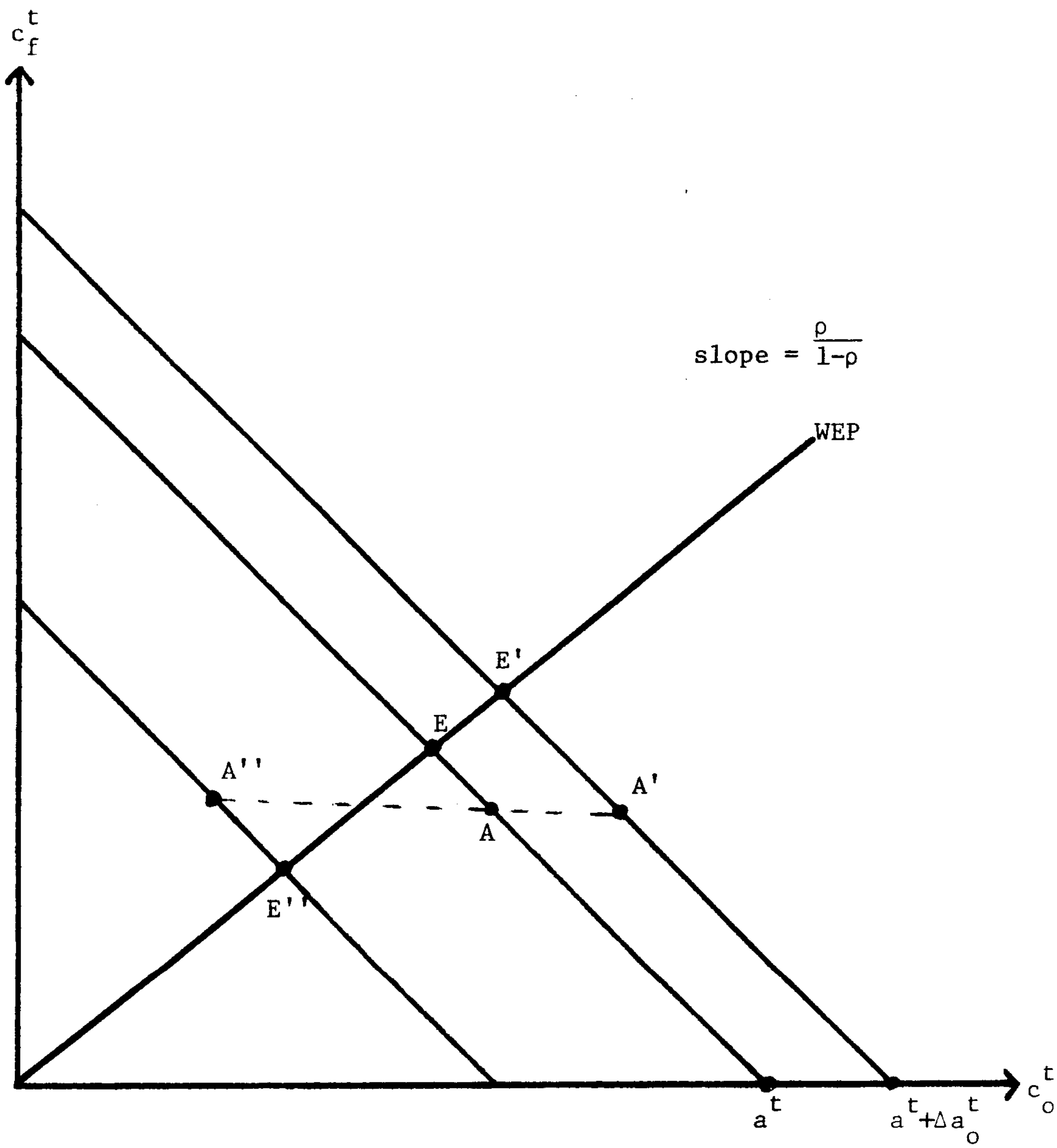

Figure 3

intact: $\Delta a_o^t < o$, $\Delta a_f^t = o$. Their endowment shifts left in figure 3. Using equation (13), we can see t's response: only a fraction of their bad fortune is endured by themselves in the form of lower own consumption. The remainder, from equation (14), is shared with future generations as they lower private bequests. Their initial level of private bequests, then, can be considered a cushion that generation t can use to partially absorb bad fortune.

If the endowment shifts to a point to the left of the wealth expansion path in figure 3, say to A'', then matters are not so simple. Here the private bequest cushion is not enough. Generation t optimally prefers to leave negative bequests and move from A'' to E''. Assuming there is a non-negative private bequest constraint, however, they must settle with allocation A'', making zero private bequests. But at A'', generation t hits the bequest constraint and, therefore, they have a positive demand price for social security.

In general, any event that shifts the initial endowment to a point above the wealth expansion path increases the demand for social security. We now specify a supply model.

B. Supply of Social Security

We specify a polar case model of social security supply. Assume that the government supplies just enough social security to saturate demand. In equilibrium, then, generation t's demand price is pushed to zero. From our discussion in Section I, $\bar{a}_{so}^t$ given in equation (11) can now be reinterpreted as the equilibrium level of social security.[23/] This specification of supply, from our previous discussion, boils down to the following: Social security is a device that a generation uses to get around the non-negative bequest constraint and achieve its optimal wealth allocation.

C. Examples

We now look at the impact effect of four events on the social security system. For simplicity, we assume that the mechanism is inactive initially, but the discussion easily extends to changes in an existing system.

1. Transitory Depression

The transitory depression we considered earlier and now shown in figure 4 causes an endowment shift from A to A'. The demand for social security increases as the event reduces the ratio of own wealth to future wealth. Thus the social security system is activated causing an endowment redistribution effect A' to E'. This allows the depression generation to achieve its optimal wealth allocation. It leaves a negative bequest via social security, thus passing to future generations part of the cost of depression.[24/]

2. Increase in Rate of Technical Change

It is easy to amend our model to allow for labor-augmenting technical change. An increase in the rate of technical change increases future wealth, a_f^t. This event shifts the endowment point A northward in figure 5. If the shift leaves the endowment below the wealth expansion path, then generation t responds by reducing private bequests. In this way generation t shares in the good foturne of future generations. If the endowment shifts to A', above the WEP, then the bequest constraint is binding, increasing the demand for social security. The program is activated shifting generation t's endowment from A' to E'.[25/]

3. Unanticipated Inflation

Suppose an unanticipated inflation occurs that transfers resources from creditors to debtors. Also suppose that the average creditor

Transitory Depression

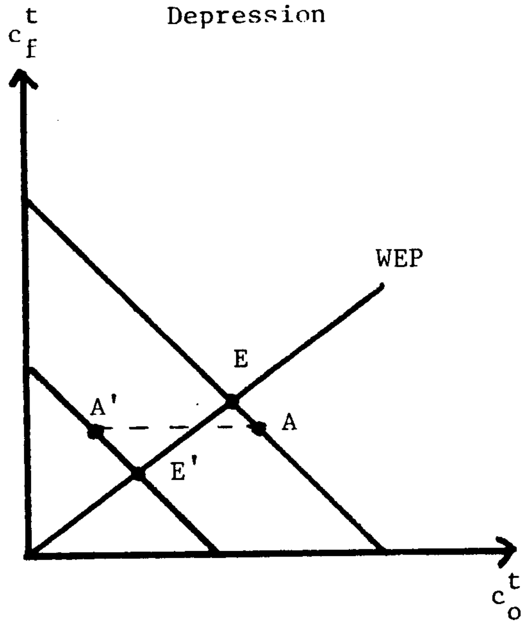

Figure 4

Technical Change Increase

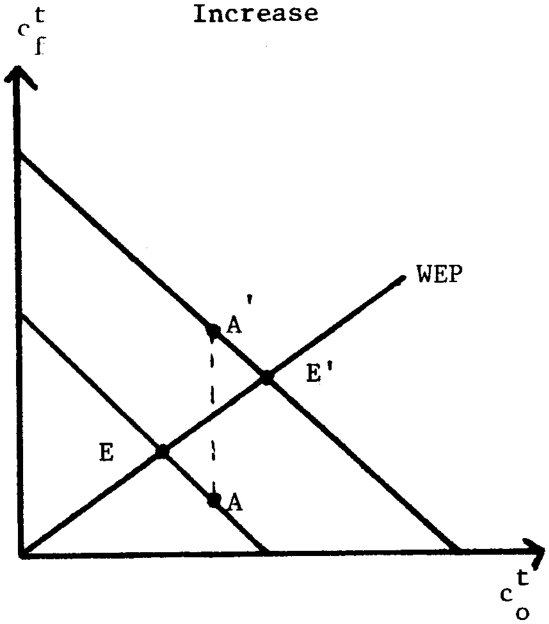

Figure 5

Unanticipated Inflation

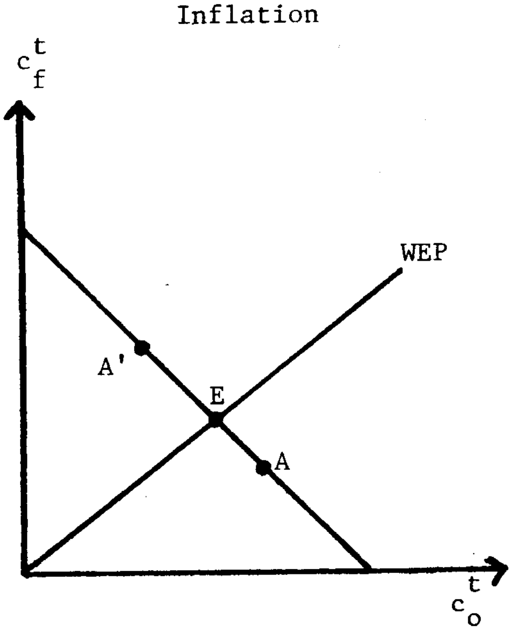

Figure 6

Human Capital Investment

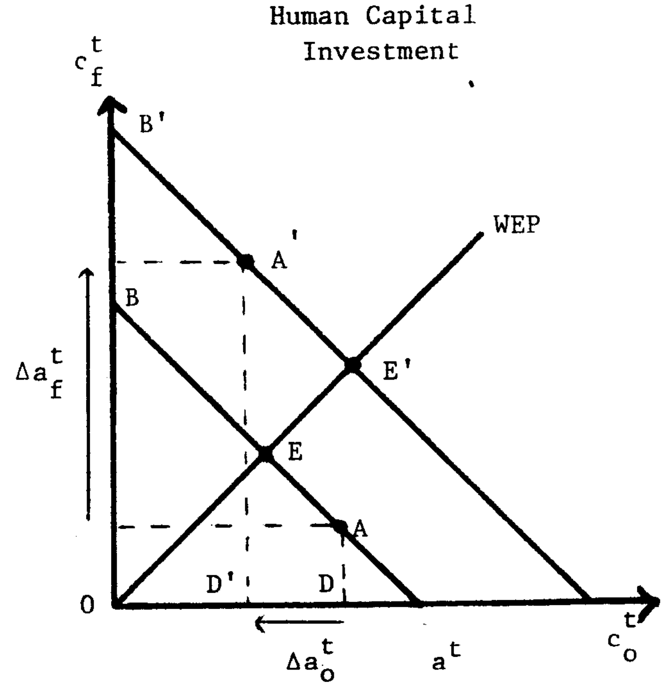

Figure 7

is older than the average debtor. In figure 6 this event is shown as an endowment shift along the initial budget line (no wealth is created or lost because of unanticipated inflation here) from initial endowment A to the northwest. If the new endowment is below WEP, then the private bequest cushion allows the old to offset the inflation induced transfer by reducing private bequests. But if the endowment shifts to A' (a severe inflation), then the demand for social security increases since the bequest constraint is binding. The system is activated, pushing the endowment to optimal allocation point E.[26/]

4. Human Capital Investment

It has been argued by Pogue and Sgontz (1977) and Drazen (1978) that social security is a means of obtaining the returns from investment in human capital. This insight fits into our analysis of social security. Consider figure 7. The endowment point is A with initial wealth a^t. Positive private bequests are made initially. Now suppose a new opportunity arises for generation t. By increasing their investment in the human capital of generation t+1, t's wealth increases by Δa^t. Their own wealth is reduced by Δa_o^t, the cost of the investment, but the increase in future wealth, Δa_f^t, more than offsets this cost: $\Delta a_f^t + \Delta a_o^t = \Delta a^t > o$. But since this wealth increment is not privately appropriable for generation t, their non-negative private bequest constrained budget set after the human capital investment is OB'A'D' rather than OBAD. With the new budget set, the bequest constraint is binding and social security is activated pushing the endowment to E'. At E', both generation t's own consumption and the consumption of future generations are higher than at E. Hence, social security in this case is a device used by human capital investors to secure part of the return.

III. Concluding Remarks

This paper has intended to place the social security and saving debate in perspective. Our major point is that society may use social security to achieve an intergenerational redistribution of wealth. The desired endowment redistribution effect from a pay-as-you-go program was seen to be a possible response by the current generation to events that tilt the intergenerational distribution of wealth toward future generations. This response arises when the current generation is altruistic toward future generations. Just as the current generation attempts to share good fortune with future generations by increasing bequest saving, they symmetrically attempt to partially pass on bad fortune, such as a depression, to future generations. When the private bequest cushion is inadequate, the negative public bequest device, social security, is activated. Similarly, events that increase the wealth of future generations such as exogenous technical change, endogenous human capital investment, or unanticipated inflation, may cause an activation of social security so that the current generation can enjoy part of the future generation's good fortune.

While a complete analysis of the current debate on funding social security [27] using our framework requires another paper, we might briefly speculate. There are essentially two kinds of funding plans. The first one is temporary. The current generation increases its tax contribution which accumulate in the trust funds. When the current generation retires, part of their benefits are financed by a decumulation of the funds. Essentially, temporary funding is a device the current generation could use to save for retirement consumption. This plan does not constitute an intergenerational redistribution of wealth, however, and our analysis has little to say about its effects.

The second type of funding plan that has been discussed is permanent. The current generation again increases its tax contribution which accumulates in the trust fund. But these funds are not used for retirement benefits for the current generation. Rather, they are passed on to future generations. Thus, current movement toward a permanently funded social security system is one way the current generation can increase its bequest to future generations. Our analysis would lead us to expect this public policy response to occur when events tilt the intergenerational distribution of wealth toward the current generation. If, however, the permanent funding proposal was enacted without public consensus, then the current generation may resist this positive social security bequest by reducing private bequests. The form of the reduction (human capital investment, for example) may actually lead to a net impoverishment of future generations.

FOOTNOTES

*This paper was written while I was a Brookings Economic Policy Fellow at the Division of Economic and Long Range Studies, Office of Research and Statistics, Social Security Administration. My thanks are extended to B. Bridges, J. Hambor, S. Lesnoy, M. Packard, and especially J. Turner. A. Pellechio made helpful comments on the first draft of this paper.

1/ For a readable discussion of social insurance see Feldstein (1977a). Also see Boskin (1977) for a number of survey articles on social security.

2/ Due to the earnings test that social security beneficiaries face, there may also be an induced retirement effect which increases saving. For more on this effect see Burkhauser and Turner (1978). We neglect the earnings test in our discussion.

3/ This argument, which has come to be called the Ricardian equivalence proposition, was made less rigorously by Goldin (1971), who adopted the approach developed by Head (1967) in his study of the incidence of the public debt.

4/ As we shall see, the computation of the present value of the string of imposed negative bequests is complex. There has been some debate on whether or not complexity per se generates a computation bias. If it does, then saving may be affected by social security. See Feldstein (1976a, p. 335) and Barro (1976, p. 346).

5/ Barro (1974, p. 1104) states that this point could also be made in a model where the intergenerational transfer goes from children to parents as gifts rather than parents to children. Our examination of the problem in section I only includes the parent to child transfer.

6/ Empirical work has not resolved the social security and saving debate. There have been time series studies using aggregate U.S. data by Feldstein (1974, 1978), Munnell (1974 a,b), Barro (1978a), and Darby (1978). See Esposito (1978) for a review of this work. Work using cross-country data has been done by Aaron (1967), Feldstein (1977b), and Barro and Mac Donald (1977). There is also work using cross-section household data by Kotlikoff et al. (1976), Munnell (1976), and Feldstein and Pellechio (1977).

7/ Other researchers have attempted to make social security endogenous, thus rationalizing its existence and searching for factors that help in an explanation of its development. Some of these contributors are Samuelson (1958) and Aaron (1966), who demonstrated how pay-as-you-go social security can be used to move an economy from an inefficient to an efficient growth path; Musgrave (1967), who discussed the social benefits of a forced saving program via funded social security; Green (1977), who considered whether or not society could pareto optimally use social security as a device to reduce demographic risk; Pogue and Sgontz (1977) and Drazen (1978), who argued that social security might be a mechanism human capital investors use to secure the unappropriable part of the return to their investment, and Flemming (1977), who demonstrated how pay-as-you-go social security might be introduced along with an intra-generational transfer program in order to offset the capital losses imposed on the retired (the capital owners) from the negative labor supply effect from the latter program.

8/ Drazen (1978, p. 506) also assumes the existence of a non-negative bequest constraint. See his paper for a defense. Pogue and Sgontz (1977) also give justifications for this assumption.

9/ This assumption may seem unreasonable in an analysis of the effect of social security on saving. For example, Feldstein (1976b) argued that the saving reducing effect of social security would lower the capital/labor ratio, raising the return to capital and lowering the wage rate. One justification is that factor prices are mostly determined in international markets and are affected only slightly by the behavior of one country. Barro (1974) avoids the problem by using the assumption of static price expectations which turn out to be correct when the neutrality proposition holds.

10/ See Becker (1974) for a discussion of this concept which he calls social income.

11/ We focus exclusively on the intergenerational redistribution aspect of social security in this paper since it is fundamental to the social security-saving debate. Other neglected components are the earnings test, the wage base ceiling, insurance aspects, and intrageneration redistributive aspects.

12/ Recall that there are 1+n young persons paying taxes gw for each old person receiving benefits.

13/ From equation (6) we have:

$$a_s^t = gw\left(\frac{1+n}{1+r}\right) + \sum_{\tau=t+1}^{\infty}\left(\frac{1+n}{1+r}\right)^{\tau-t}\left(gw\left(\frac{1+n}{1+r}\right) - gw\right)$$

By inspection it is easy to see that $\Sigma=0$ and $a_s^t = gw$ when $n=r$. Also, $\Sigma=\infty$ and $a_s^t = \infty$ when $n > r$. By factoring we can rewrite the equation as:

$$a_s^t = gw\left(\frac{1+n}{1+r}\right) + gw\left(\frac{1+n}{1+r}\right)\cdot\left(\frac{1+n}{1+r} - 1\right)\sum_{\tau=0}^{\infty}\left(\frac{1+n}{1+r}\right)^{\tau}$$

When $r > n > o$, $o < \frac{1+n}{1+r} < 1$. Thus we have

$$a_s^t = gw\left(\frac{1+n}{1+r}\right) + gw\left(\frac{1+n}{1+r}\right)\cdot\left(\frac{1+n}{1+r} - 1\right)\left(1 - \frac{1+n}{1+r}\right)^{-1} = 0$$

This proof is essentially identical to Barro's proof (1976) that government bonds do not represent net wealth (p. 344).

14/ The free lunch case arises when the steady-state capital/labor ratio lies in the inefficient range above the golden rule ratio. Given our assumption that r and n are constant, this case cannot be ruled out endogenously. Sidrauski (1967), as pointed out by Barro (1976, p. 345) has shown why a utility-maximizing immortal family [our utility function in equation (11) has this interpretation] model makes this case untenable, however.

15/ The solutions are actually infinite consumption sequences. We transform them via equation (5) to this form.

16/ The author can supply the reader with the tedious derivation. The derivation uses the assumption that $o \leq \rho < 1$.

17/ Similarly the system is neutral through time (no effect on saving). The start-up old return their benefits to the start-up young. Hence, the start-up young effectively pay no tax, but still anticipate receiving benefits. Thus, their position is identical to the initial position of the start-up old and they behave similarly. This argument follows with each generation.

18/ Evidence on the existence of a binding constraint must be considered carefully. A small (or even non-existent) bequest process is insufficient evidence since the WEP may lie quite close to the endowment making the problem 1 optimum bequest small.

19/ Other provisions of social security (insurance, intragenerational redistribution, etc.) would also effect the demand price.

20/ Again, the author will supply the reader with this derivation. Essentially λ^t is the difference between the marginal rate of transforming future consumption c_f^t into own consumption c_o^t (which equals 1) and the marginal rate of substituting own consumption for future consumption (which equals $(\rho/1-\rho)\left(\frac{a_o^t + a_{so}^t}{a_f^t + a_{sf}^t}\right)$).

21/ Feldstein (1976b, p.77) states: "social security wealth is an exogenous variable that can in principle provide a better test of the life-cycle hypothesis than is possible with the traditional measure of wealth." In empirical work on social security and saving, some measure of the social security system is treated as an independent variable in regression equations attempting to explain saving (or consumption). See some of the work cited in footnote 6. Earlier work by Aaron (1967) on cross-country comparisons of social security treated benefit payments as a dependent variable, however. Barro (1978b) has developed an endogenous model of the public debt to fill in "an embarrassing absence of a theory of public debt creation" (p.1) in his earlier work. Barro's model does not revolve around intergenerational transfers. Rather, debt and taxes are used in a proportion that minimizes the cost of collecting an amount of revenue determined by exogenous government expenditures.

22/ Browning (1975) analyzed the use of a social security program in a majority rule setting where interests conflict. The median voter determines the size of the system. Browning did not consider events that change the individual's demand for social security (other than age), as we do. Thus only changes in the age distribution of the population cause endogenous changes in the program in his model. See the exchange between Bridges (1978) and Browning (1978) and also the work by Greene (1974) and Hu (1977) for more on this type of model.

23/ Our model is a polar case because the young are assumed to have zero weight in the political process (or the policy for period t+1 is made by the young in period t). If the young, who incur the negative social security bequest have some weight, then our story is incomplete. In a more complete model where U^{t-1} as well as U^{t+1} enter as arguments to U^{t}, then the young may voluntarily make transfers to the old and the need for social security is unclear. Possibly the insurance aspect of social security must be introduced. The public good aspect of transfers and the associated free-rider problem might motivate the young to make transfers to the old through social security. It seems to me that events that change the intergenerational distribution of wealth would have the same qualitative effects in this more general model on intergenerational transfers, public or private. I have been unsuccessful in fully developing this model, however. See Marglin (1963), Sen (1967), and Hochman and Rogers (1969) for discussions of the public good aspect of transfers.

24/ Feldstein (1977a, p. 78) has made this point. He stated: "the Great Depression had a major effect on the design of social security. The length of the depression and the failure of financial institutions had destroyed a lifetime of savings.... A social security program that paid benefits to retirees would replace lost saving,..."

25/ Drazen (1978, p. 510) makes this point: "Considerations such as wage growth,..., all serve to reduce the likelihood of positive bequests."

26/ I want to thank A. Pellechio for this example. My colleague, Max Horlick, Chief, Comparative Studies Staff, Office of Research and Statistics, Social Security Administration, has told me that many European pay-as-you-go programs began when inflation eroded the retirement saving of the elderly.

27/ See Lesnoy and Hambor (1975), Feldstein (1976c) and Pechman's discussion, and Kurz (1977).

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