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OPTIMAL AND MAJORITY-VOTING EQUILIBRIUM LEVELS OF SOCIAL SECURITY

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I. INTRODUCTION

In the recent economic literature on social security, much attention has been focused on its welfare implications (e.g., Samuelson [1975]), and its impacts on individual retirement decisions (e.g., Boskin [1977], Sheshinski [1978], Diamond and Mirrlees [1978]) and capital accumulation (e.g., Feldstein [1974], Munnell [1974], and Kotlikoff [1979]). In all these works, the level of social security is assumed to be exogenous although it is often determined in the real world by the desire of the majority of voters and thus is an endogenous variable of the economic system. While Browning [1975] and Hu [1978] did consider the determination of social security by a majority-voting process, they used the partial-equilibrium approaches in the sense that wages and the interest rate were assumed exogenous and independent of social security. The present paper constructs a simple three-period life-cycle model in which social security is determined by the majority-voting process, and the rate of interest by the demand for and supply of capital. In this framework, the tax rate voted by each person depends on the market rate of interest, which in turn is affected by the prevailing tax rate. It is assumed that social security is financed by a pay-as-you-go plan.

It is well-known that in a life-cycle model the competitive equilibrium rate of interest could converge to a level lower than the growth rate of population, thus resulting in intertemporal inefficiency (see Diamond [1965]), and that an appropriate level of social security could lead to the golden rule of capital accumulation (see Samuelson [1975]). The question that can be raised is then whether such an optimal level of social security can be brought about by a majority-voting process. This paper shows that the outcome of the majority-voting process depends crucially on whether the current tax rate is expected by voters to remain effective over their life spans. Among publicly provided goods (public or private goods), social security has least synchronized tax payments and benefits. Current taxpayers will not receive their benefits until quite a few years later when they are retired. Thus, under the pay-as-you-go system in which pension benefits received by each retiree are not directly related to his past contributions to social security, unless voters are convinced that the current tax rate will remain effective over their entire life spans (i.e., there will not be another voting opportunity before their death), there is an incentive for them to misrepresent their preferences by voting now for a lower tax rate and later for a higher tax rate. If no revoting opportunities were expected in the future and the true preferences of voters were revealed, the majority-voting process could lead to oversupply of social security, but the economy would never be intertemporally inefficient. But if there are revoting opportunities in the future, it is possible for the level of social security to be higher or lower than the socially optimal level. With a properly designed voting process, the social optimum can be achieved by a majority-voting equilibrium.

II. THE MODEL

(i) The Production Sector. Consider an economy in which the production function is given by

$$Q(t) = F(K(t), L(t)), \tag{1}$$

where $Q(t)$ = total output, $K(t)$ = capital sotck, and $L(t)$ = the total supply of labor. The production function satisfies the neoclassical conditions: i.e., it is homogeneous of degree 1 with respect to capital and labor, and exhibits positive but diminishing marginal products. Moreover, capital is perfectly durable.

Assume that perfect competition prevails in the economy. The equilibrium conditions for the factor markets require that the wage rate be equal to the marginal product of labor and the real rate of interest be equal to the marginal product of capital.

$$w(t) = F_L(K(t), L(t)), \tag{2}$$

$$r(t) = F_K(K(t), L(t)). \tag{3}$$

Because of the homogeneity of the production function, these marginal products are homogeneous of degree 0 with respect to K and L. Therefore, (2) and (3) can be solved for [2/]

$$k(t) = \Lambda(r(t)) \qquad \Lambda' = L/F_{KK} < 0, \tag{4}$$

$$w(t) = \Omega(r(t)) \qquad \Omega' = -k < 0. \tag{5}$$

where $k = K/L$. The right-hand side of (4) denotes the demand for capital per unit of labor which in equilibrium must equal the supply of capital (i.e., the left-hand side). (5) is the well-known factor-price frontier.

(ii) The Household Sector. Assume that the population of the economy grows at a constant rate g. All individuals are alike except in their ages. Each person born at the beginning of period t lives for three periods. During the first two periods (i.e., t and t+1), he works full time, earning a wage income of $w(t+n-1)$ and paying a social security tax of $x(t+n-1)$ when he is of age n. In the last period, he retires completely and receives from the government pension benefits in the amount of $z(t+2)$. For simplicity, assume that his lifetime utility function is of the Cobb-Douglas form and there is no bequest motive: i.e.,

$$U_1(t) = \alpha_1 \log c_1(t) + \alpha_2 \log c_2(t+1) + \alpha_3 \log c_3(t+2), \qquad \sum_{n=1}^{3} \alpha_n = 1, \tag{6}$$

where $c_n(t+n-1)$ denotes his consumption in period t+n-1, when he is aged n (n=1, 2, 3). His allocation problem is to choose $c_1(t)$, $c_2(t+1)$ and $c_3(t+2)$ so as to maximize his lifetime utility subject to his life-time budget constraint, which can be written as

$$c_1(t) + \frac{c_2(t+1)}{1+r(t+1)} + \frac{c_3(t+2)}{(1+r(t+1))(1+r(t+2))} = y_1(t) , \tag{7}$$

$$y_1(t) = w(t) - x(t) + \frac{w(t+1) - x(t+1)}{1+r(t+1)} + \frac{z(t+2)}{(1+r(t+1))(1+r(t+2))} . \tag{8}$$

Solving this maximization problem gives

$$\begin{aligned} c_1(t) &= \alpha_1 y_1(t), \\ c_2(t+1) &= \alpha_2 (1+r(t+1)) y_1(t), \\ c_3(t+2) &= \alpha_3 (1+r(t+1))(1+r(t+2)) y_1(t). \end{aligned} \tag{9}$$

Upon substituting these consumption functions into (6), we obtain the following indirect utility function:

$$V_1(t) = \log y_1(t) + \nu_1(r(t+1),\ r(t+2)), \tag{10}$$

where

$$\nu_1 = (\alpha_2 + \alpha_3)\log(1 + r(t+1)) + \alpha_3 \log\ (1+r(t+2)) + \sum_{n=1}^{3} \alpha_n \log\ \alpha_n .$$

His saving in the first period is given by

$$\begin{aligned} a_1(t) &= w(t) = x(t) - c_1(t) \\ &= (1-\alpha_1)(w(t)-x(t)) - \alpha_1 \left[\frac{w(t+1)-x(t+1)}{1+r(t+1)} + \frac{\ddot{z}(t+2)}{(1+r(t+1))(1+r(t+2))}\right] . \end{aligned} \tag{11}$$

This constitutes his terminal asset in the current period or the initial asset in the following period (t+1). Note that $a_1(t)$ can be either positive or negative because this model does not impose any constraint on borrowing.

The allocation problem for an individual who is aged 2 at the end of period t is to

$$\text{maximize } U_2(t) = \alpha_2 \log\ c_2(t) + \alpha_3 \log\ c_3(t+1) \tag{12}$$

$$\text{subject to } c_2(t) + \frac{c_3(t+1)}{1+r(t+1)} = y_2, \tag{13}$$

$$y_2(t) = (1+r(t))a_1(t-1) + w(t) - x(t) + \frac{z(t+1)}{1+r(t+1)} . \tag{14}$$

where $a_1(t-1)$ is his initial asset in the current period (t) or his terminal asset in the last period (t-1), and $y_2(t)$ is his lifetime income for the remaining horizon. Solving this maximization problem to obtain

$$c_2(t) = \beta_2 y_2(t), \tag{15}$$
$$c_3(t) = \beta_3(1+r(t+1))y_2(t) ,$$

where $\beta_n = \alpha_n/(\alpha_2 + \alpha_3)$, $n = 2, 3$. And the corresponding indirect utility function can be expressed as

$$V_2(t) = (\alpha_2 + \alpha_3) \log y_2(t) + \nu_2(r(t+1)), \tag{16}$$
$$\nu_2 = \alpha_3 \log (1 + r(t+1)) + \sum_{n=2}^{3} \alpha_n \log \beta_n.$$

The terminal asset for this person is given by

$$\begin{aligned} a_2(t) &= a_1(t-1)(1+r(t)) + w(t) - x(t) - \beta_2 y_2(t) \\ &= (1-\beta_2)[a_1(t-1)(1+r(t)) + w(t) - x(t)] - \beta_2 \frac{z(t+1)}{1+r(t+1)} . \end{aligned} \tag{17}$$

Given the assumption that there is no bequest motive, the consumption of an individual aged 3 equals his entire asset at the end of the period,

$$c_3(t) = a_2(t-1)(1+r(t)) + z(t).$$

Thus his terminal asset is equal to zero.

(iii) Financing of Social Security. Let us consider only the simple case where the social security system is financed by a pay-as-you-go plan under which incomes are transferred from the working population to retirees, and the government maintains its (social security) budget balanced each period. This implies that

$$L_3(t)z(t) = (L_1(t) + L_2(t))x(t).$$

Here, $L_n(t)$ denotes the population aged n in period t. Because the population growth rate is g, $L_{n-1}(t) = (1+g)L_n(t)$. Thus, the above budget constraint can be rewritten as

$$z(t) = \Pi x(t), \tag{18}$$

with

$$\Pi = (L_1 + L_2)/L_3 = (1+g)^2 + (1+g) = (1+g)(2+g).$$

To facilitate our discussions, consider first the impact of social security on the market equilibrium relations and derive the optimal tax rate.

III. MARKET EQUILIBRIUM

(i) Short-Run Equilibrium. The total supply of labor in period t consists of the net assets held by individuals aged 2 and 3 at the beginning of the period:

$$K(t) = L_2(t)a_1(t-1) + L_3(t)a_2(t-1).$$

Therefore, the capital-labor ratio is given by

$$k(t) = K(t)/L(t) = \theta_1 a_1(t-1) + \theta_2 a_2(t-1), \tag{19}$$

where

$$\theta_1 = L_2(t)/(L_1(t) + L_2(t)) = 1/(2+g),$$

$$\theta_2 = L_3(t)/(L_1(t) + L_2(t)) = 1/((2+g)(1+g)) = 1/\Pi.$$

The rate of interest in period t is determined by

$$\Lambda(r(t)) = \theta_1 a_1(t-1) + \theta_2 a_2(t-1) \tag{20}$$

and the current wage rate is in turn obtained from (5). Current consumption $c_n(t)$ and the change in the social security tax, if any, are made at the end of period t when incomes are received. Thus they affect the supply of capital and thereby the interest rate only in the next period. It is assumed that the individual has short-run perfect foresight with respect to r(t+1) and w(t+1) when making his consumption decisions at the end of period t. The equilibrium condition at the end of period t is then given by

$$\Lambda(r(t+1)) = \theta_1 a_1(t) + \theta_2 a_2(t) = k(t+1) \ .$$

Upon substituting from (11) and (17),

$$\Lambda(r(t+1)) = \theta_1\left[(1-\alpha_1)(w(t) - x(t)) - \alpha_1\left(\frac{w(t+1) - x(t+1)}{1+r(t+1)} + \frac{z(t+2)}{(1+r(t+1))(1+r(t+2))}\right)\right] + \theta_2\left[(1-\beta_2)(a_1(t-1)(1+r(t)) + w(t) - x(t)) - \beta_2 \frac{z(t+1)}{1+r(t+1)}\right] \ . \tag{21}$$

The endogenous variable in this equation is of course r (t+1). $a_1(t-1)$, w(t) and r(t) are historical data. x(t), x(t+1) and z(t+2) are policy instrument variables. r(t+2) is the expected future interest rate. The supply behavior is affected by the manner in which expectations regarding r(t+2) are formed. Withour further complications, let us assume that expectations are adaptive so that

$$r(t+2) = r(t) + \delta(r(t+1) - r(t)) = \delta r(t+1) + (1-\delta)r(t) \tag{22}$$

Taking this and equation (5) into consideration, we obtain upon differentiating (21) with respect to r(t+1):

$$\frac{\partial k(t+1)}{\partial r(t+1)} = -\theta_1\alpha_1\left(\frac{(1+r(t+1))\Omega'(r(t+1)) - (w(t+1) - x(t+1))}{(1+r(t+1))^2} - \frac{z(t+2)(1+r(t+2)+\delta(1+r(t+1))}{(1+r(t+1))^2(1+r(t+2))^2}\right) - \theta_2\beta_2\left(\frac{-z(t+1)}{(1+r(t+1))^2}\right) > 0 \ .$$

Thus, while the demand for capital (Λ) is downward sloped, the supply of capital is upward sloped with respect to the real rate of interest. This ensures the existence of a unique short-run equilibrium, as illustrated in Figure 1a.

Consider now the short-run effect of a permanent change in the social security tax. Setting $x(t+1) = x(t)$ and $z(t+2) = z(t+1) = z(t) = \Pi x(t)$ in equation (21) and differentiating the resultant expression with respect to $x(t)$ yields

$$\frac{\partial k(t+1)}{\partial x(t)} = -\theta_1\left[1-\frac{\alpha_1 r(t+1)}{1+r(t+1)}+\frac{\alpha_1\Pi}{(1+r(t+1))(1+r(t+2))}\right] -\theta_2\left[1-\beta_2 + \frac{\beta_2\Pi}{1+r(t+1)}\right]$$

which is negative because $\alpha_1 r(t+1)/(1+r(t+1))<\alpha_1<1$ and $\beta_2<1$. The effect of a permanent increase in the social security tax is therefore to shift the capital supply curve to the left and thereby raise the short-run equilibrium rate of interest. This positive relationship between the equilibrium interest rate and the social security tax is shown in Figure 1b by the RR curve.

(ii) Long-Run Equilibrium. In the long-run equilibrium where $w(t) = w^*$, $r(t) = r^*$, and $a_n(t) = a_n^*$ are all constant, (21) is reduced to

$$\Lambda(r^*) = \left\{[\theta_1 + \theta_2(1-\beta_2)(1+r^*)]\left(1-\frac{2+r^*}{1+r^*}\alpha_1\right) + \theta_2(1-\beta_2)\right\}(w^*-x) - \left\{\alpha_1[\theta_1+\theta_2(1-\beta_2)(1+r^*)] + \theta_2\beta_2(1+r^*)\right\}\frac{\Pi x}{(1+r^*)^2} = k^* \tag{23}$$

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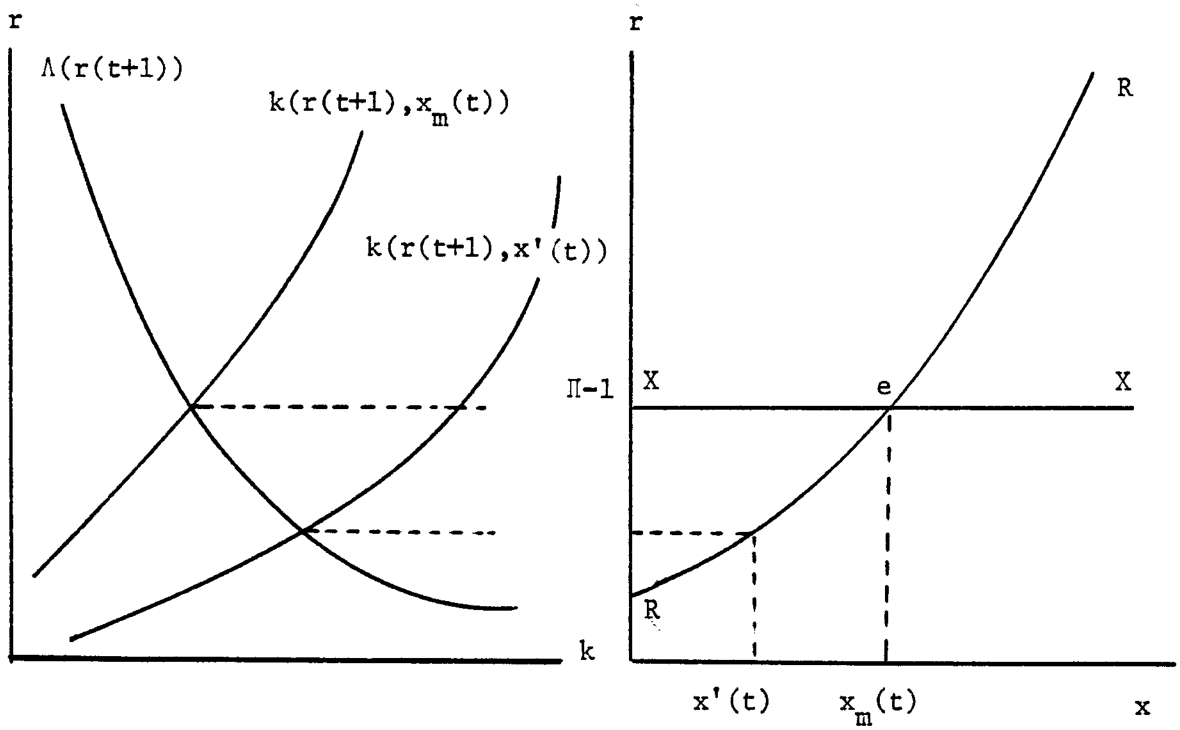

Figure 1.--Determination of the Short-Run Majority-Voting Equilibrium Tax Level

with $w^* = \Omega(r^*)$. The long-run supply of capital (i.e., the right-hand side) is no longer necessarily increasing with respect to r^*. But in view of the long-run stability conditions, it may be assumed that

$$dk^*/dr^* > \Lambda'(r^*).$$

That is, the supply of capital is either upward sloped or, if downward sloped, steeper than the demand for capital. This ensures the existence of a unique equilibrium. As can be seen, the second term in (22) is negative; thus, for the equilibrium k^* to be positive, the coefficient of $(w-x)$ must be positive. It follows directly that k^* is decreasing with respect to x. In other words, an increase in the social security tax shifts the long-run supply curve of capital to the left, and thereby raises the equilibrium interest rate:

$$r^* = r^*(x), \quad r^{*\prime}(x) > 0. \quad \text{[4/]} \tag{23'}$$

Let us now derive the optimal tax rate which maximizes the lifetime utility of a representative individual in the long-run equilibrium. Substituting (23') into (10) and maximizing V_1 with respect to x, we obtain the following first-order condition:

$$\frac{\partial V_1}{\partial x} = \frac{1}{y_1^*}\left\{\frac{2+r^*}{1+r^*}(\Omega' r^{*\prime}-1) + \frac{\Pi}{(1+r^*)^2} - \frac{r^{*\prime}}{(1+r^*)^2}(w^*-x) - \frac{2\Pi x r^{*\prime}}{(1+r^*)^3} + \frac{\alpha_2 + 2\alpha_3}{1+r^*} r^{*\prime} y_1\right\} = 0. \tag{24}$$

Noting that $\Omega' = -k = -\Lambda$, this condition can be shown to be satisfied when $r^* = g$. Thus the optimal tax rate obeys the golden rule of capital accumulation.[5/] Upon substituting $r^*=g$ in (22) the optimal level of the social security tax can be obtained:

$$x_g = \frac{\theta_1(1-\alpha_1-\alpha_1/(1+g)) + \theta_2\alpha_3(2+g) - \Lambda(g)/\Omega(g)}{\theta_1+\theta_2(2+g)}\,\Omega(g) \quad \underline{6}/ . \qquad (25)$$

The numerator on the right-hand side is simply the gap between the long-run supply of capital that would have taken place in the absence of social security (i.e., $[\theta_1(1-\alpha_1-\alpha_1/(1+g)) + \theta_2\alpha_3(2+g)]\,\Omega(g)$) and the demand for capital ($\Lambda(g)$) on the golden rule path. The optimal social security tax is equal to the ratio of this gap to the marginal propensity of capital supply with respect to social security (i.e., the denominator). The question now arises as to whether the golden rule can be achieved by a majority-voting process.

IV. IDEAL MAJORITY-VOTING EQUILIBRIUM

In a democratic society, the value of $x(t)$ is not exogenous but rather determined by the desires of voters. Each voter chooses at the end of each period the tax level $x(t) \in [0, w(t)]$ which maximizes his utility over the remaining life span. Let us consider first the case in which the voter knows that the benefit schedule is given by (18) and he expects no revoting opportunities in the future so that the current tax level, once determined, will remain effective over his lifetime. Obviously, this last assumption is rather restrictive and will be relaxed in the following section. Given these assumptions, the problem for a voter aged 1 is to choose the level of $x(t)$ which maximizes (10) subject to condition (18), with $x(t+n) = x(t)$, $n = 1, 2$. Under the previous assumption of adaptive expectations on $r(t+2)$, the lifetime utility (V_1) of the young voter is maximized when the following expression (i.e., the present value of his net benefits from social security) is maximized:

$$\left[-1 - \frac{1}{1+r(t+1)} + \frac{\Pi}{(1+r(t+1))(1+r(t+2))}\right] x(t)$$

Here, the bracketed terms are positive or negative as

$$\Pi \gtrless (1+r(t+2)) + (1+r(t+1))(1+r(t+2)) \quad .$$

In the former case, he votes for as much tax as possible because his lifetime utility increases with an increase in the tax. In the converse case, he prefers no tax. If the interest rate is expected to remain constant, which, as will be seen below, is the relevant case for the present analysis, the former (resp. latter) case holds if g is higher (resp. lower) than r(t+1). Thus his voting behavior has the following property:

$$x(t) \left\{ \begin{array}{l} = w(t) \\ \in [0,\ w(t)] \\ = 0 \end{array} \right\} \text{ if } r(t+1) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} g \quad . \tag{26}$$

To put it another way, the hurdle rate of return that a young voter would demand of social security is the market rate of interest, while the marginal rate of return he expects from social security is g. If this marginal rate exceeds the hurdle rate, he votes for the maximum level of x(t). Conversely, if this rate is lower than the hurdle rate of return on social security, he votes for nothing. When the two rates are equal, he is indifferent among the tax levels.

For an individual aged 2, maximization of his lifetime utility (16) is realized when

$$-x(t) + \frac{z(t+1)}{1+r(t+1)} = \left[-1 + \frac{\Pi}{1+r(t+1)}\right] x(t)$$

is maximized. Because this expression is increasing or decreasing with respect to x(t) as r(t+1) is lower or higher than Π-1, his desired tax level has the following property:

$$x(t) \begin{cases} = w(t) \\ \varepsilon[0,\ w(t)] \\ = 0 \end{cases} \text{ if } r(t+1) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \Pi-1 \quad . \tag{27}$$

While the hurdle rate of return on social security for the middle-aged voter is still r(t+1), his expected marginal rate of return from social security is $\Pi - 1 > g$. If this marginal expected rate of return exceeds the hurdle rate, he would prefer a 100% tax rate, and vice versa. Because $\Pi - 1 > g$, a middle-aged voter expects a higher marginal rate of return from social security than a young voter. As a result, he tends to desire a higher tax rate on social security. In particular, when $g < r(t+1) < \Pi-1$, a middle-aged voter desires a 100% tax rate while a young voter a 0% tax rate. It needs no saying that a retired voter always desires a 100% tax rate.[7/]

In Figure 1b, the XX line denotes the marginal rate of return on social security for a middle-aged voter. As can be seen, for any tax rate lower than x_m, the market equilibrium rate of interest (the hurdle rate on social security) determined by the RR curve is less than the marginal expected rate of return on social security for a middle-aged voter, and therefore he prefers an increase in the tax rate; so does a retired voter. Thus, the majority of voters prefer an increase in the tax rate (provided that the population growth rate is less than 100%).[8/] If the tax rate is above x_m, then the marginal expected rate of return from social security for a middle-aged voter is less than the hurdle rate, thus he desires a fall in the tax rate; so does a young voter. In other words, the majority of voters prefer a lower tax rate. It follows that the short-run majority-voting equilibrium is determined by the intersection (e) of RR and XX, with [9/]

$$x = x_m(t) \ , \tag{28}$$

$$r = r_m = \Pi - 1 \ . \tag{29}$$

Here, x_m is obtained by setting $r(t+1) = r_m$ in equation (21), i.e.,

$$\Lambda(r_m) = \theta_1 \left[(1-\alpha_1)(w_m - x_m(t)) - \alpha_1 \left(\frac{w_m - x_m(t)}{1+r_m} + \frac{\Pi \ x_m(t)}{(1+r_m)^2} \right) \right]$$

$$+ \theta_2 \left[(1-\beta_2)((1+r_m)a_1(t-1) + w_m - x_m(t)) - \beta_2 \frac{\Pi \ x_m(t)}{(1+r_m)} \right] \ ,$$

or

$$x_m(t) = \frac{1}{D} \left\{ \left[\theta_1(1-\alpha_1-\alpha_1\theta_2) + \theta_2(1-\beta_2) \right] w_m + (1-\beta_2)a_1(t-1) - \Lambda(r_m) \right\} \tag{30}$$

where $w_m = \Omega(r_m)$ and $D = \theta_1(1-\alpha_1) + \theta_2$. The value of $x_m(t)$ so determined is an increasing linear function of $a_1(t-1)$. This relationship between $x_m(t)$ and $a_1(t-1)$ is depicted in Figure 2 by the MM curve. On the other hand, given $x_m(t)$, the terminal wealth of a youngster in period t is given by

$$a_1(t) = (1-\alpha_1)(w_m - x_m(t)) - \alpha_1 \left(\frac{w_m - x_m(t)}{1+r_m} + \frac{\Pi x_m(t)}{(1+r_m)^2} \right)$$

$$(31)$$

$$= (1-\alpha_1-\alpha_1\theta_2)w_m - (1-\alpha_1)x_m(t) \ .$$

This equation determines $a_1(t)$ as a linear decreasing functions of $x_m(t)$, which is depicted in Figure 2 by AA. In this figure, given any $a_1(t-1)$, the value of $x_m(t)$ is determined by the MM line (point a), which in turn determines $a_1(t)$ (point b) through the AA line. The time path of the economy is characterized by a sequence of cyclic movements (a-b-c-d...), in which the tax rate desired by the public goes up and down continually. Such cyclic movements can be either convergent toward or divergent from the long-run majority-voting equilibrium (e).[10/] However, if the partial derivative of $a_1(t)$ with respect to $x_m(t)$ (in equation 31) and that of $x_m(t)$ with respect to $a_1(t-1)$ (in equation 30) are such that

$$\left(\frac{\partial a_1(t)}{\partial x_m(t)}\right)_{(31)} \times \left(\frac{\partial x_m(t)}{\partial a_1(t-1)}\right)_{(30)} > -1 \quad ,$$

then the long-run majority-voting equilibrium is stable. Expanding the above inequality yields

$$\alpha_3 / \; \theta_1(1-\alpha_1) + \theta_2 \;\; < 1 \quad . \tag{32}$$

This inequality can be shown to be satisfied if the subjective discount rate is positive and the population growth rate is less than 100%.[11/]

Setting $x_m(t) = x_m^*$ and $a_1^m(t) = a_1^{m*}$ in equations (30) and (31) and solving these two equations, we obtain the long-run equilibrium level of x_m.

$$x_m^* = \frac{\theta_1(1-\alpha_1-\alpha_1\theta_2) + \alpha_3(1+\theta_2) - \Lambda(r_m)/w_m}{\theta_1(1-\alpha_1) + \theta_2 + \alpha_3} \; w_m \tag{33}$$

Because $r_m > g$ and by (23) the long-run equilibrium r is increasing with respect to x x_m^* is higher than the socially optimal tax level given by (25). Thus, although the majority-voting equilibrium does not achieve the golden rule and thereby the social optimum, neither does it lead to dynamic inefficiency in the sense that there exists another path which provides more consumption for some periods but not less consumption for any period.[12/]

We have assumed that while voting takes place each period, voters always expect that there will be no revoting opportunities in the future so that the current tax rate will remain effective over their lifetimes. As a result, their true preferences are revealed. The tax rate so determined may be called the "ideal" majority-voting equilibrium tax rate. However, among publicly provided goods, social security has least synchronized taxes and benefits. Current taxpayers will not reap their benefits until quite a few years later when they are retired. Moreover, under the pay-as-you-go plan, their retirement benefits are not directly related to their current contributions. Thus, if voters have any reason to believe that there will be another voting opportunity before their death, then their expected benefits from voting for a higher tax rate in the current period are much smaller and possibly zero. Consequently, there is an incentive for them to under-represent their true preferences. In other words, such an expectation alters their preferences regarding current tax and benefit levels. And the actual outcome can be quite different from the ideal majority-voting equilibrium solution.

V. POSSIBILITY OF REVOTING

For expository convenience, let us imagine an economy where voting takes place by random drawing with a probability of p that voting will take place and the probability of (1-p) that it will not take place in period t.

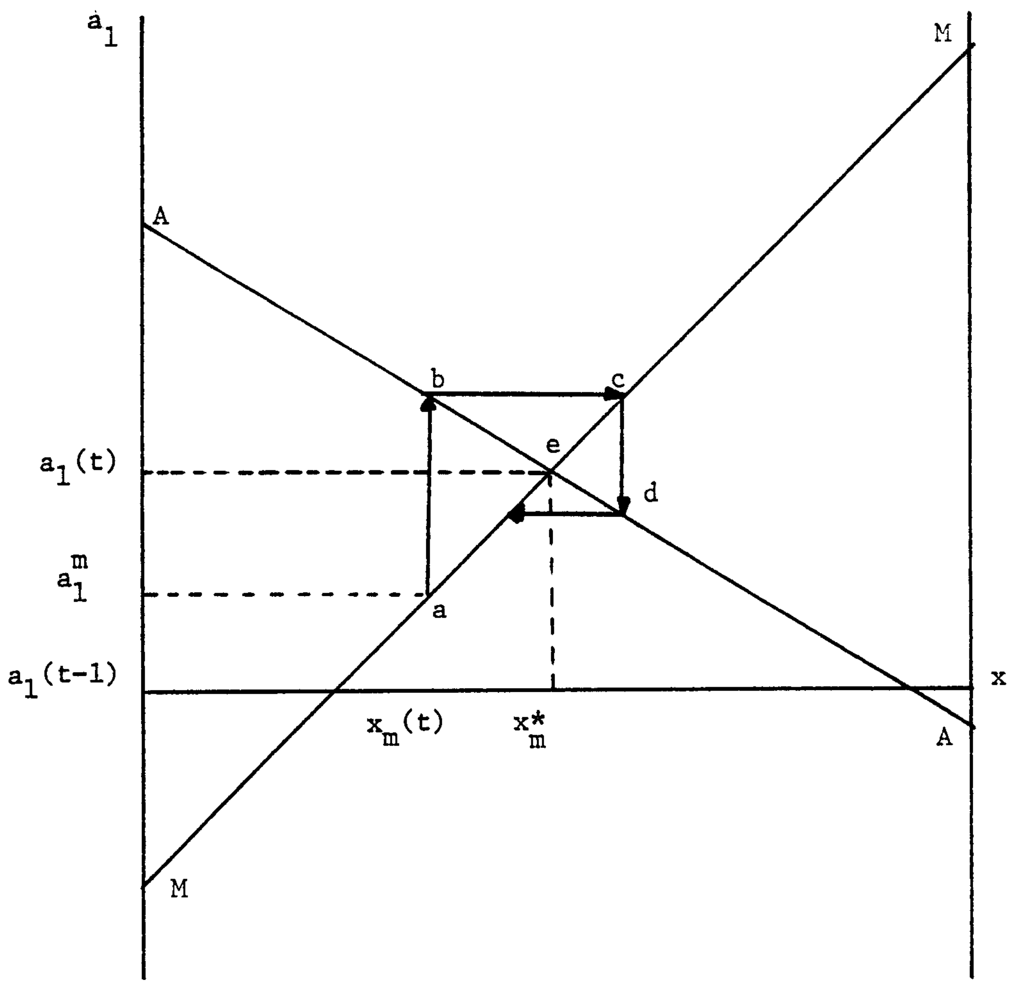

Figure 2.--Dynamic Adjustment Process of the Majority-Voting Equilibrium

Consider first the behavior of an individual aged 2. Assume that he expects that if revoting does take place next period, the resulting tax rate is given by

$$x(t+1) = x(t-1) + \sigma(x(t) - x(t-1)) = \sigma x(t) + (1-\sigma)x(t-1) \ , \tag{34}$$

and his future consumption will be

$$c_3^2(t+1) = (1+r(t+1)) \ [a_1(t-1)(1+r(t))+w(t)-x(t)-c_2(t)] + \Pi(\sigma x(t)+(1-\sigma)x(t-1)) \ .$$

If, however, there is no revoting next period, then $x(t+1)=x(t)$ and his future consumption will be

$$c_3^b(t+1) = (1+r(t+1)) \ [a_1(t-1)(1+r(t)) + w(t) - x(t) - c_2(t)] + \Pi x(t) \ .$$

Thus his expected utility is

$$EV_2 = p[\alpha_2 \log c_2(t) + \alpha_3 \log c_3^a(t+1)] + (1-p)[\alpha_2 \log c_2(t) + \alpha_3 \log c_3^b(t+1)] \ . \tag{35}$$

Maximizing EV_2 with respect to c_2 yields

$$\frac{\alpha_2}{c_2(t)} - \frac{p\alpha_3(1+r(t+1))}{c_3^a(t+1)} - \frac{(1-p)\alpha_3(1+r(t+1))}{c_3^b(t+1)} = 0$$

or

$$\frac{\alpha_2}{c_2(t)} - \frac{p\alpha_3}{y_2^a(t) - c_2(t)} - \frac{(1-p)\alpha_3}{y_2^b(t) - c_2(t)} = 0 \ . \tag{36}$$

Here,

$$y_2^i(t+1) = (1+r(t))a_1(t-1) + w(t) - x(t) + \frac{x^i(t+1)}{1+r(t+1)} \quad i=a,b,$$

$$x^a(t+1) = \sigma x(t) + (1-\sigma)x(t-1) \ ,$$

$$x^b(t+1) = x(t) \ .$$

Solving equation (36) gives $c_2(t)$ as a function of current after-tax income, the interest rate as well as the probability of revoting, the past tax level and the marginal impact of current tax level on future taxes.

$$c_2(t) = c_2^p((1+r(t))a_1(t-1) + w(t) - x(t),\ r(t+1), \sigma,\ p,\ x(t-1)) \quad (37)$$

It can be shown that the left-hand side of (36) is positive for $c_2 = \beta_2 \min\{y_2^a,\ y_2^b\}$ and is negative for $c_2 = \beta_2[p\ y_2^a + (1-p)\ y_2^b]$. Thus the desired level of consumption must be between 13/

$$\beta_1 \min\{y_2^a,\ y_2^b\} < c_2^p(t) < \beta_2[p\ y_2^a + (1-p)y_2^b] \quad . \qquad (37')$$

It follows directly from equation (15) that if the tax level were permenent and were such that $y_2 = py_2^a + (1-p)y_2^b$, then the desired level of consumption for the middle-aged person would have been $c_2 = \beta_2(py_2^a + (1-p)y_2^b)$. However, as can be seen from (37), because of risk aversion, uncertainty about future tax levels leads to a reduction of consumption.

Substituting the solution $c_2^p(t)$ into (16), we can write the resultant indirect utility function as

$$V_2 = V_2(x(t);\ (1+r(t))a_1(t-1) + w(t),\ r(t+1),\ \sigma,\ p,\ x(t-1)) \quad . \qquad (38)$$

The level of $x(t)$ voted by a middle-aged person maximizes V_2, and satisfies the following first-order condition:

$$\frac{\partial V_2}{\partial x(t)} = \frac{p\alpha_3(1+r(t+1) - \Pi\sigma)}{c_3^a(t+1)} + \frac{(1-p)\alpha_3(1+r(t+1) - \Pi)}{c_3^b(t+1)} = 0 \quad . \qquad (39)$$

Solving for the desired tax level $x(t)$,

$$x(t) = x_p((1+r(t))a_1(t-1) + w(t),\ r(t+1),\ x(t-1),\ \sigma,\ p). \qquad (40)$$

As can be seen, (39) is positive if $r(t+1) > \Pi - 1$ and is negative if $r(t+1) < \sigma\Pi - 1$. Thus it is possible for the solution x_p to be higher than $w(t)$ or negative. Taking into consideration the appropriate constraint, the tax level voted by the middle-age person has the following property:

$$x(t) = \begin{Bmatrix} w(t) \\ x_p(\cdot) \\ 0 \end{Bmatrix} \quad \text{if} \quad x_p(\cdot) \quad \begin{Bmatrix} >w(t) \\ \varepsilon[0,\ w(t)] \\ <0 \end{Bmatrix} \quad , \tag{41}$$

where $x_p(\cdot)$ is given by equation (40).

Consider next the behavior of an individual aged 1. Assume that he follows the same expectations as described in equation (34). There are four possibilities regarding future voting opportunities: (i) There is a probability of p^2 that revoting will take place in each of periods t+1 and t+2 with the resulting tax levels given by

$$x^a(t+1) = \sigma x(t) + (1-\sigma)x(t-1) \ ,$$

$$x^{aa}(t+2) = \sigma x^a(t+1) + (1-\sigma)x(t-1) = \sigma^2 x(t) + (1-\sigma^2)x(t-1) \ .$$

(ii) There is a probability of p(1-p) that revoting will take place in t+1 but not in t+2, with the resulting tax levels expected to be

$$x^a(t+1) = \sigma x(t) + (1-\sigma)x(t-1),$$

$$x^{ab}(t+2) = x^a(t+1) = \sigma x(t) + (1-\sigma)x(t-1) \ .$$

(iii) There is a probability of (1-p)p that revoting will take place in t+2 only; the resulting tax levels are expected to be

$$x^b(t+1) = x(t),$$

$$x^{ba}(t+2) = \sigma x^b(t+1) + (1-\sigma)x(t-1) = \sigma x(t) + (1-\sigma)x(t-1) \quad .$$

(iv) There is a probability of $(1-p)^2$ that revoting will not take place in either period t+1 or t+2; the resulting tax levels are

$$x^b(t+1) = x(t),$$

$$x^{bb}(t+2) = x(t) \ .$$

Here, the first superscript for x(t+2) denotes the voting opportunity in period t+1 and the second superscript denotes that for period t+2. An "a" means that revoting is expected to take place, and a "b" means the converse. The superscripts for c_3 below are to be interpreted in the same manner.

In each of the above four cases, his consumption in the third period is given by

$$\begin{aligned} c_3^{ij}(t+2) &= [w(t) - x(t) - c_1)]\ (1+r(t+1))(1+r(t+2) \\ &\quad + [w(t+1) - x^i(t+1) - c_2(t)]\ (1+r(t+1)) + z^{ij}(t+2), \\ &\quad i,j = a,b\ . \end{aligned}$$

His optimal current consumption therefore solves the following maximization problem:

$$\begin{aligned} \max_{c_1} V_1 &= \alpha_1 \log c_1 \\ &\quad + p\{\max_{c_2}[\alpha_2 \log c_2 + \alpha_3(p \log c_3^{aa} + (1-p)\log c_3^{ab})]\} \\ &\quad + (1-p)\{\max_{c_2}[\alpha_2 \log c_2 + \alpha_3(p \log c_3^{ba} + (1-p)\log c_3^{bb})]\} \end{aligned}$$

The solution c_1 to this maximization problem determines the current consumption of the youngster as a function of w(t), w(t+1), r(t+1), x(t), x(t-1) as well as p and σ.

$$c_1(t) = c_1^p(w(t),\ w(t+1),\ r(t+1),\ x(t),\ x(t-1),\ p,\ \sigma)\ . \tag{42}$$

And his terminal asset is given by

$$a_1^p(t) = w(t) - x(t) - c_1^p(t)\ . \tag{43}$$

Again, using the same method as before, we can show that because of risk aversion, uncertainty leads to a lower level of consumption; that is, $c_1^p(t)$ is less than α_1% of the expected lifetime income.

Consequently, other things being equal, a_1^p is less than that would have been realized in the certainty case.

For any given current tax rate, the capital market equilibrium condition is now given by

$$\Lambda(r(t+1)) = \theta_1[w(t) - x(t) - c_1^p(t))] + \theta_2[(1+r(t))a_1(t-1) + w(t) - x(t) - c_2^p(t)] , \tag{44}$$

where $c_1^p(t)$ and $c_2^p(t)$ are, respectively, given by (42) and (37). Let the solution r to (44) be denoted $r_p(t)$. As noted above, uncertainty about future voting opportunities reduces consumption; thus other things being equal, capital supply is larger and r_p is lower than that would have taken place if the tax rate is expected to remain constant at the expected level.

The time path of the economy in the present case is characterized by a random process. If there is no voting to take place in the current period, then the equilibrium r_p and a_1^p are determined by (43) and (44) with x(t) equal to that in the previous period, i.e.,

$$x(t) = x(t-1) \quad .$$

If voting does take place, then the tax level x(t) is given by (4), i.e.,

$$x(t) = x_p(t) \; .$$

Equations (40), (43) and (44) now jointly determine the majority-voting equilibrium values of $(x(t), r_p(+1), a_1^p)$. These short-run majority-voting equilibrium values are dependent not only on the historical data such as $a_1(t-1)$, $x(t-1)$, $r(t)$, but also on whether voting takes place in the current period.[14/] To illustrate, let us consider the extreme case in which voting is expected in each period (i.e., $p = 1$). Setting $p = 1$ in equation (37) and making appropriate manipulations, we obtain

$$c_2^p = \beta_2 y_2^a(t) ,$$

$$y_2^a(t) = a_1(t-1)(1+r(t)) + w(t) - x(t) + \frac{\Pi(\sigma x(t) + (1-\sigma)x(t-1))}{1+r(t+1)} .$$

Correspondingly, in equation (39), the second term disappears while the first term is positive or negative as $\sigma\Pi - 1$ is higher or lower than $r(t+1)$. In the former (later) case the lifetime utility of a middle-aged person increases (decreases) with the tax level. Thus his voting behavior derived in (41) is reduced to

$$x(t) \begin{cases} = w(t) \\ \varepsilon[0, w(t)] \\ = 0 \end{cases} \quad \text{if} \quad r(t+1)\{\substack{<\\=\\>}\}\sigma\Pi - 1 . \tag{41'}$$

With expected future revoting opportunities, the marginal rate of return on social security for the middle-aged voter is decreased to $\sigma\Pi - 1$, and it is lower the lower is the correlation between future and current tax levels. In the extreme case where $\sigma = 0$, the marginal return on tax payments is negative and the middle-aged person would vote for nothing. In the other extreme case where $\sigma = 1$, because the current tax rate is expected to prevail in the next voting process, he behaves as if the current tax rate were to remain permanent.

In the same manner, setting $p = 1$ in (42), the level of current consumption desired by a youngster is obtained

$$c_1^p(t) = \alpha_1 y_1^{aa}(t) , \tag{42'}$$

$$y_1^{aa}(t) = w(t) - x(t) + \frac{w(t+1) - (\sigma x(t) + (1-\sigma)x(t-1))}{1+r(t+1)}$$

$$+ \frac{\Pi(\sigma^2 x(t) + (1-\sigma^2)x(t-1))}{(1+r(t+1))(1+r(t+2))} .$$

The capital-market equilibrium condition (43) is reduced to

$$\Lambda(r(t+1)) = \theta_1\left\{(1-\alpha_1)(w(t)-x(t)) - \alpha_1\left(\frac{w(t+1) - \sigma x(t) - (1-\sigma)x(t-1))}{1+r(t+1)}\right.\right.$$
$$\left.\left. + \frac{\Pi(\sigma^2 x(t) + (1-\sigma^2)x(t-1))}{(1+r(t+1))(1+r(t+2))}\right)\right\} \tag{43'}$$
$$+ \theta_2\left\{(1-\beta_2)((1+r(t))a_1(t-1) + w(t) - x(t))\right.$$
$$\left. - \beta_2\frac{\Pi(\sigma x(t) + (1-\sigma)x(t-1))}{1+r(t+1)}\right\} \quad .$$

Comparing this and equation (43) it follows directly that if $x(t) >$ (resp. $<$) $x(t-1)$, the supply of capital is larger (resp. smaller) when there are revoting opportunities than when the current tax rate is expected to remain permanent, and as a result the equilibrium rate of interest determined by (43') is lower (resp. higher) than that by (43). In other words, the effect of the expected opportunities for revoting is to make the RR curve flatter. Let us label the RR curve for the present case by R'R'.

In Figure 3, the short-run majority-voting equilibrium determined by (41') and (43') is shown by point E, with

$$r_M(t) = \sigma\Pi - 1$$

and

$$x_M(t) = \frac{1}{D}\left\{\left[\theta_1\left(1-\frac{\sigma+\theta_2}{\sigma}\alpha_1\right) + \theta_2(1-\beta_2)\right]w_M+(1-\beta_2)\theta a_1(t-1)\right.$$
$$\left. -\left(\frac{\alpha_1\theta_1}{\sigma} + \beta_2\right)\frac{1-\sigma}{\sigma}\theta_2 x_M(t-1)-\Lambda(r_M)\right\} \quad ,$$

where $w_M = \Omega(r_M)$. In contrast to the case where the current tax rate is expected to remain permanent, the short-run equilibrium is dependent on both $a_1(t-1)$ and $x(t-1)$. In particular, the equilibrium tax level is

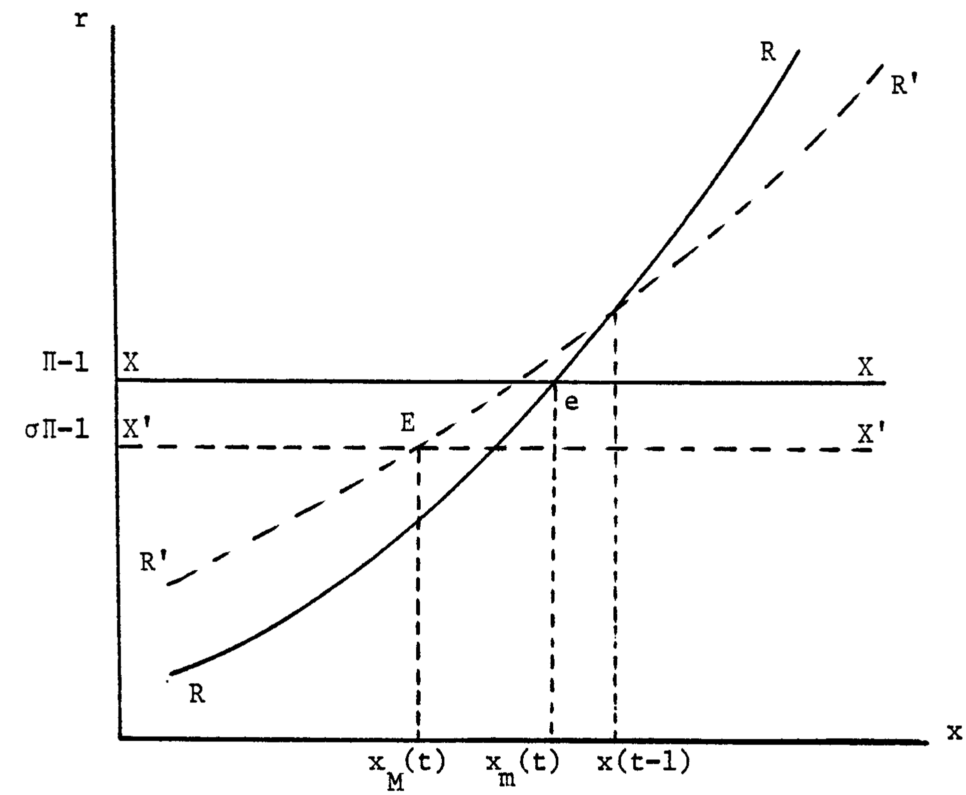

Figure 3.--Effects of Revoting Possibilities on the Majority-Voting Equilibrium Tax Level

higher the lower is the past tax level or the higher is the initial wealth of the middle-aged. It can also be seen from Figure 3 that a sufficient condition for $x_M(t)$ to be less than $x_m(t)$ is that $x_m(t) \lesseqgtr x(t-1)$. In the converse case, $x_M(t)$ can be either **larger or smaller than** $x_m(t)$.

Making use of (42'), the terminal wealth of a youngster is given by

$$a_1^M = (1-\alpha_1)(w_M - x_M(t)) - \alpha_1\left[\frac{w_M - (1-\sigma)x_M(t-1)}{1+r_M} + \frac{(1-\sigma^2)\Pi x_M(t-1)}{(1+r_M)^2}\right].$$

Setting a_1^M and x_M constant yields the long-run equilibrium social security tax:

$$x_M^* = \frac{\theta_1(1-\alpha_1-\alpha_1\theta_2/\sigma) + \alpha_3(\sigma+\theta_2) - \Lambda(r_M)/w_M}{\theta_1(1-\alpha_1\frac{\theta_2+\sigma}{\sigma}) + \frac{\theta_2}{\sigma}(\alpha_2+\alpha_1\frac{\theta_1+\sigma}{\sigma})+\alpha_3(\sigma+\theta_2)}\, w_M \quad . \tag{33'}$$

If $\sigma=1$, then voters expect the current tax level to prevail in the next voting process, and thus they behave as if there would be no future voting opportunities. In other words, (33') reduces to (33). Upon differentiating, x_M^* can be shown to be an increasing function of σ. Therefore, the tax level so determined is less than that determined by (33) where voters expect no future revoting opportunities. Moreover, the rate of return on capital in the present case is higher than that in the case of permanent tax rate. Depending on the value of σ, the rate of return can also be higher or lower than the growth rate of the population. It is not an ultimate outcome that the majority-voting equilibrium should lead to excessive provision of social security relative to the socially optimal level (x_g). Indeed, if $\sigma=\theta_1$, then

$x_M^* = x_g$ and $r_M = g$, and consequently the majority-voting equilibrium is optimal.

In the more general case where $0<p<1$, although the exact dynamic adjustment process cannot be characterized, the following relationship can be shown to be satisfied in the long-run equilibrium where the same tax rate prevails in each period whether voting takes place or not in that period:

$$\Lambda(r_p^*) = \left(\theta_1 + \theta_2(1+r_p^*)\right) \left(w_p^* - x_p^* - c_1^{p*}\right) + \theta_2\left(w_p^* - x_p^* - c_2^{p*}\right) . \tag{45}$$

Inspection of (36), (40) and (42) shows that the long-run equilibrium values of c_1^{p*} and c_2^{p*} are equal to

$$c_1^{p*} = \alpha_1\left(w_p^* - x_p^* + \frac{w_p^* - x_p^*}{1+r^*} + \frac{\Pi x_p^*}{(1+r^*)^2}\right) , \tag{46}$$

$$c_2^{p*} = \alpha_2(1+r_p^*)\left(w_p^* - x_p^* + \underline{w_p^* - x_p^*} + \frac{\Pi x_p^*}{(1+r_p^*)^2}\right) , \tag{47}$$

and x_p^* must be such that

$$r_p^* = p\sigma\Pi + (1-p)\Pi - 1 . \tag{48}$$

Combining equations (45)-(48), we obtain

$$x_p^* = \frac{\theta_1\left[1-\alpha_1-\alpha_1\theta_2/(1-p+\sigma p)\right] + \alpha_3(\theta_2+1-p+\sigma p) - \Lambda(r_p^*)/w_p^*}{\theta_1(1-\alpha_1 - \frac{\sigma p}{1-p_1+\sigma p}\alpha_1\theta_2) + \frac{\theta_2}{1-p+\sigma p}(1-\alpha_3 p(1-\sigma) - \frac{\alpha_1\theta_1}{1-p+\sigma p}) + \alpha_3(1-p+\sigma p)} \tag{33''}$$

Both the long-run equilibrium rate of interest and tax level are between those determined in the presence of annual voting opportunities and those under which the current tax rate is expected to be permanent. Moreover, the higher is p the lower is the equilibrium r_p^*. In particular, if $\sigma<1/(2+g)$, then setting

$$p = (1+g)/[(1-\sigma)(2+g)] < 1, \tag{49}$$

the resulting long-run equilibrium interest rate is equal to g, and the golden rule is attained. This suggests that with a properly designed voting process in which p is chosen so as to satisfy(49), the majority-voting equilibrium leads to the golden rule and is efficient in the ex ante sense.[15/]

VI. CONCLUDING REMARKS

This paper has shown that because contributions made to and benefits received from social security are not synchronized, underrepresentation of preferences by voters could be brought about by the expectations of revoting opportunities under the pay-as-you go system. Although with revelation of true preferences, the majority-voting equilibrium may lead to excessive provision of social security, such an equilibrium is never dynamically inefficient. In the case where future revoting opportunities are expected with respect to social security, the outcome of the majority voting process may depend on both the probability at which such an opportunity would take place and, should it take place, the marginal impact of the current tax rate on the future tax level. Obviously, the model constructed here is too simple. For example, care for the welfare of the past or future generations may lead to an alternative voting pattern (see, for example, Barro [1974]). Introduction of imperfect capital markets or endogenous retirement decisions may also lead to undersupply of social security even if the current tax rate is expected to remain effective in the future. But the conclusion remains that while the majority-voting equilibrium may lead to either over or under provision of social security, a properly designed voting process in the random sense could bring about the golden rule of capital accumulation. Finally, it should be noted that the population growth rate is assumed constant, thus the only uncertainty is about the possibility of revoting. With variable growth rate, there would be further uncertainty about the size of the future working population contributing to social security. Under the assumption of risk aversion, this double uncertainty would further reduce the desired tax rate.[16/]

FOOTNOTES

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2/ Because F_K is homogeneous of degree 0, (3) can be rewritten as

$$r = F_K(k, 1),$$

which, upon differentiating yields

$$\Lambda' = dk/dr = 1/F_{KK}(k, 1) = L/F_{KK}(K, L).$$

(Noting that F_{KK} is homogeneous of degree -1). In the same manner, write (4) as

$$w = F_L(k, 1)$$

Differentiating this equation yields

$$dw/dk = F_{LK}(k, 1) \quad .$$

Therefore

$$\Omega' = \frac{dw}{dr} = \frac{dw}{dk} \quad \frac{dk}{dr} = \frac{F_{LK}(k, 1)}{F_{KK}(k, 1)} = -k$$

because $F_{KK}(k, 1)k + F_{KL} = 0$.

3/ After several manipulations, this equation can be reduced to

$$L_1c_1 + L_2c_2 + L_3c_3 = F(K, L) - gK.$$

Which says that in the long-run equilibrium, the aggregate level of consumption equals the net national product of the economy.

4/ It should be noted that the monotonicity in the relationship between the short-run and long-run equilibrium r and x follows from our assumptions of (a) a Cobb-Douglas utility function, (b) absence of bequest motives, and (c) exogenous retirement decision. In the more general case this may not be true. See Hu [1979].

5/ See also Samuelson [1975] for the two-period case.

6/ Of course, in order for the tax rate to be positive, it is required that

$$\theta_1(1-\frac{2+g}{1+g}\alpha_1) + \theta_2\alpha_3(2+g) > \Lambda(g)/\Omega(g) .$$

7/ Recall that the individual does not have any bequest motive.

8/ If $g>100\%$, the youngest generation would dominate the two older generations and, as a result, the desires of young voters would determine the outcome of the majority-voting process.

9/ To ensure that there is an intersection between XX and RR, it is required that the equilibrium market rate of interest in the absence of social security be less than $\Pi=1$. Otherwise, the outcome of the majority-voting equilibrium would be no social security.

10/ It is possible for the long-run equilibrium point (e) to lie below the horizontal axis. In this case, the young generation are the borrowers in the capital market.

11/ Let the discount rate be λ. Then α_n can be rewritten as

$$\alpha_n = \frac{(1+\lambda)^{3-n}}{1+(1+\lambda)+(1+\lambda)^2} , \quad n = 1, 2, 3.$$

Inequality (32) now is reduced to

$$\frac{(1+g)(2+g)}{(2+g)(2+\lambda)+1+(1+\lambda)+(1+\lambda)^2} < 1 \quad .$$

This inequality is satisfied for $\lambda=0$ and $g=1$. Note that the left-hand side is increasing with respect to g and decreasing with respect to λ. Therefore, the inequality is also satisfied for all $\lambda>0$ and $g<1$.

12/ It should be noted that this conclusion could be altered if, for example, the bequest motive and care for retired generation are incorporated into the utility function of the representative individual. Introduction of an imperfect capital market also reduces the desire for social security.

Although a similar conclusion was reached earlier by Browning [1975], his argument was based on the assumptions that (a) the indifference curves relating z (which determines what he called the annual rate of return on taxes paid over lifetime) and x are U shaped and identical for individuals of all ages, but the (government) budget line shifts with age, and (b) constant wage and interest rates. As can be seen from (10) and (16), the U shaped indifference curves cannot be derived from the life cycle model under constant wage and interest rates. Moreover, the indifference curves are not invariant to age.

13/ More explicitly

$$c_2^p = \frac{G + \sqrt{G^2 - 4H}}{2}$$

where

$$G = (\beta_2 + (1-p)\beta_3)y_2^a + (\beta_2 + p\beta_3)y_2^b$$

$$H = \beta_2 y_2^a y_2^b$$

14/ For further discussion of the importance of the previous tax rates in determining the outcome of the majority voting process, see Bridges [1978].

15/ This terminology follows Postlewaite and Schmeidler [1979]. It is interesting to note that the election of the British Parliament has some random elements in its timing.

16/ See also Bridges [1978].

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