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LABOR SUPPLY, THE PAYROLL TAX, AND INTERNAL RATES OF RETURN TO SOCIAL SECURITY

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LABOR SUPPLY, THE PAYROLL TAX, AND INTERNAL RATES OF RETURN TO SOCIAL SECURITY

There is empirical evidence that in the recent past, the Old Age Insurance (OAI) portion of the social security program has acted as a net wage subsidy. In addition, the program had significant intragenerational redistributive effects. Our purpose is to demonstrate how these findings alter conventional views of the labor supply effects of social security. Our method is the analysis of a labor supply model which is extended to include empirically significant operational components of the program. We show that the analyses of others are special cases of our more general approach.

We begin with a summary of empirical evidence on the redistributational character of the social security system. Then we develop a model of a worker's savings, consumption, and supply of hours of work decisions. This section describes the comparative statics results when OAI benefits are not dependent on the level of taxes paid while the third section deals with the case when they are.

I. Internal Rates of Return to Social Security

Our conclusion that the OAI program has acted as a net wage subsidy rests on estimates of expected real internal rates of return to OAI tax contributions.¹ The average expected real internal rate of return at

retirement for a random sample of worker-only beneficiaries retiring between 1967 and 1970 was 14.8 percent. This is substantially higher than real market interest rates. In addition, the relation between rates of return and a measure of lifetime taxable earnings was negative indicating that workers with low earnings had higher rates of return than workers with high earnings.

Table 1 contains the results of a regression analysis of 2,612

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ROFR</td>
<td>Expected real internal rate of return at retirement.</td>
</tr>
<tr>
<td>2. ACRN</td>
<td>Accumulated value of lifetime taxable real covered earnings. b.</td>
</tr>
<tr>
<td>3. ACTX</td>
<td>Accumulated value of lifetime real OAI taxes.</td>
</tr>
<tr>
<td>4. BENEFIT</td>
<td>Present value of expected real benefits at retirement.</td>
</tr>
<tr>
<td>5. AGEENT</td>
<td>Age at entry into covered employment: age first year of non-zero covered earnings.</td>
</tr>
<tr>
<td>6. SERLEN</td>
<td>Service length: number of years with non-zero covered earnings.</td>
</tr>
<tr>
<td>7. SE/TX</td>
<td>Self-employment taxable income as a proportion of total taxable earnings.</td>
</tr>
<tr>
<td>8. SEX</td>
<td>Dummy variable for sex: 0 for male, 1 for female.</td>
</tr>
<tr>
<td>9. RACE</td>
<td>Dummy variable for race: 0 for white, 1 for non-white.</td>
</tr>
<tr>
<td>10. AGE 62–64</td>
<td>Dummy variable for age at retirement: 1 for age 62 to 64, 0 otherwise.</td>
</tr>
<tr>
<td>11. AGE 66–71</td>
<td>Dummy variable for age at retirement: 1 for age 66 to 71, 0 otherwise.</td>
</tr>
<tr>
<td>12. AGE 72+</td>
<td>Dummy variable for age at retirement: 1 for age 72 and over, 0 otherwise.</td>
</tr>
</tbody>
</table>

a. See Freiden, et al. for complete descriptions of these variables.
b. Earnings, taxes and benefits are accumulated using a rough average of market real interest rates.
internal rates of return. If the market interest rate facing each worker is assumed constant, then the negative elasticity (-.276) of the internal rate of return with respect to accumulated earnings shows that the OAI

TABLE 1-2.---Regression Results (|t| in parentheses, N =2612).

<table>
<thead>
<tr>
<th>Variable</th>
<th>log ROFR</th>
<th>log ROFR</th>
<th>log BENEFIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. log ACRN......</td>
<td>-.276</td>
<td></td>
<td>.327</td>
</tr>
<tr>
<td></td>
<td>(65.5)</td>
<td></td>
<td>(70.0)</td>
</tr>
<tr>
<td>2. log ACTX......</td>
<td></td>
<td>-.202</td>
<td>.327</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(36.5)</td>
<td>(70.0)</td>
</tr>
<tr>
<td>3. AGENT.........</td>
<td>.024</td>
<td>.034</td>
<td>.0037</td>
</tr>
<tr>
<td></td>
<td>(53.4)</td>
<td>(54.1)</td>
<td>(6.92)</td>
</tr>
<tr>
<td>4. SERLEN.........</td>
<td>.0053</td>
<td>.0024</td>
<td>.0004</td>
</tr>
<tr>
<td></td>
<td>(9.55)</td>
<td>(2.89)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>5. SE/TX.........</td>
<td>.179</td>
<td>.165</td>
<td>.058</td>
</tr>
<tr>
<td></td>
<td>(16.6)</td>
<td>(11.5)</td>
<td>(4.83)</td>
</tr>
<tr>
<td>6. SEX...........</td>
<td>.087</td>
<td>.120</td>
<td>.187</td>
</tr>
<tr>
<td></td>
<td>(15.56)</td>
<td>(16.3)</td>
<td>(29.9)</td>
</tr>
<tr>
<td>7. RACE...........</td>
<td>-.018</td>
<td>.026</td>
<td>-.074</td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
<td>(2.31)</td>
<td>(7.92)</td>
</tr>
<tr>
<td>8. AGE 62-64.....</td>
<td>-.45</td>
<td>-.018</td>
<td>-.085</td>
</tr>
<tr>
<td></td>
<td>(7.42)</td>
<td>(2.31)</td>
<td>(12.5)</td>
</tr>
<tr>
<td>9. AGE 66-71.....</td>
<td>-.029</td>
<td>-.030</td>
<td>-.119</td>
</tr>
<tr>
<td></td>
<td>(2.18)</td>
<td>(1.67)</td>
<td>(7.94)</td>
</tr>
<tr>
<td>10. AGE 72+.......</td>
<td>-.436</td>
<td>-.473</td>
<td>-.184</td>
</tr>
<tr>
<td></td>
<td>(19.5)</td>
<td>(15.4)</td>
<td>(7.10)</td>
</tr>
<tr>
<td>CONSTANT.........</td>
<td>2.71</td>
<td>1.34</td>
<td>1.90</td>
</tr>
<tr>
<td>R²</td>
<td>.919</td>
<td>.858</td>
<td>.875</td>
</tr>
</tbody>
</table>

program had a large intragenerational redistribution component. Also, the regression of rates of return on accumulated taxes indicates that there was
was a close empirical link with taxes paid.\textsuperscript{2} The regression of the present value of expected benefits on accumulated taxes is particularly interesting. Here the relation between benefits and taxes is shown to be quite firm.\textsuperscript{3} This is contrary to the accepted view as expressed by Edgar Browning:

Most scholars agree that there is a very tenuous connection between the taxes a person pays under the U.S. Social Security system and the value of the benefits later received in retirement. ...(T)his fact has served as a primary justification to evaluate the tax structure separately from the benefit structure.\textsuperscript{4} The regression result is, however, consistent with Lawrence Thompson's study using a micro-simulation data base.\textsuperscript{5} Thus, recent empirical evidence supports a close link between taxes and benefits.

II. The Social Security System in a Life-Cycle Model of Work and Wealth Allocation

Having established the empirical relevance of the benefit-tax relation, we now incorporate it into a simple model of a worker's savings, consumption and work effort decisions during working life and retirement. The purpose of this model is the comparative statics analysis of alternative

\textsuperscript{2}The other independent variables were included to capture effects which were shown to be important in the complete analysis of rates of return.

\textsuperscript{3}The simple correlation between benefits and taxes was .90.


\textsuperscript{5}Although there are a number of notable modifications in the relationship between taxes paid and benefit received, when the program is viewed from the individual participant's perspective, it does bear a fairly strong resemblance to a private annuity or insurance scheme." Lawrence H. Thompson, "Intragrowth Redistribution in the Social Security Program," paper presented to the annual meetings of the American Statistical Association, Boston, Mass., August, 1976.
tax and benefit structures under OAI. Little of importance is lost and much clarity is gained by dividing the worker's lifetime into only two periods, work and retirement, and assuming that no labor earnings occur during retirement. Also, the worker leaves no bequest.

The following is a glossary of variables to be in the model:

- $U$ = Worker's lifetime utility;
- $C_1$ = Consumption during working life;
- $C_2$ = Consumption in retirement;
- $H$ = Lifetime work effort;
- $S$ = Private savings during working life;
- $Y_N$ = Non-wage income during working life;
- $Y$ = Present value of lifetime income;
- $w$ = Lifetime wage rate;
- $T$ = Payroll taxes paid under OAI;
- $t$ = Payroll tax rate;
- $E$ = Ceiling on earnings subject to tax;
- $B$ = Total OAI benefits;
- $B_C$ = Lump sum component of OAI benefit which is independent of taxes paid;
- $B_1$ = Marginal component of OAI benefit which is related to taxes paid;
- $r_s$ = Marginal rate of return to OAI tax payments;
- $r$ = Market interest rate at which the worker can borrow or lend.

Let

\[(1) \quad U = U(C_1, C_2, H), \quad U > 0, \quad U < 0\]

be the utility function. The lifetime income constraint is derived from the budget constraint in each period. First

\[(2) \quad C_1 + S + T = Y_N + wH\]
where the left-hand side is total expenditures during working life and the right-hand side is total income. For the retirement period consumption equals the OAI benefit plus the accumulated value of savings,

\[ C = B + (1+r)S. \]  

We assume that the capital market prevents borrowing against future OAI benefits so private saving is non-negative. A single lifetime income constraint is derived from (2) and (3) by solving for private savings. Thus

\[ C + \frac{C}{1+r} = Y + \frac{B}{1+r} - T + wh. \]

The right-hand side is \( Y \), the present value of lifetime income, including the difference between the present value of benefits received and taxes paid under the OAI program.

The net present value of the worker's OAI benefit is determined by the OAI tax and benefit structures. The tax structure is given by

\[ T = \begin{cases} 
  tWH & \text{for } WH \leq E \\
  tE & \text{for } WH > E 
\end{cases} \]

Thus, taxes are proportional to earnings up to some ceiling. The benefit structure is composed of a lump sum and a marginal component. That is,

\[ B = B + B \] where \( B = \bar{B} \) and \( B = (1+r)T \).

Here we take \( r = Y \) excluding the possibility that the marginal rate of return varies by worker; however, this will be relaxed in the next section. The net present value of benefits, NPVB, is, therefore, given by
\[
\text{NPVB} = \frac{B}{1+r} - T = \frac{B}{1+r} + \frac{(\tau_s - r)}{1+r} \text{twH for } \text{WH} \leq E
\]

(7) \[
\frac{B}{1+r} + \frac{(\tau_s - r)}{1+r} \text{twE for } \text{WH} > E.
\]

Four cases are of interest.

1) Let \( \tau_s = -1 \).

That is, each dollar in taxes reduces the \textbf{net} present value of benefits by a dollar. This case is considered by most students of payroll tax incidence and is the case referred to by Browning.

2) Let \( \tau_s < r \).

Here the marginal rate of return is less than the market rate. If \( \tau_s \) is positive but less than \( r \), the marginal component of the benefit structure acts as a marginal wage subsidy offsetting a portion of the wage tax. On net, however, OAI remains a marginal wage tax for workers with earnings below the ceiling although the net tax rate is lower than in the previous case.

3) Let \( \tau_s = r \).

Now the marginal component acts as a marginal wage subsidy which fully offsets the wage tax. The marginal wage rate is unaffected.

4) Let \( \tau_s > r \).

The worker receives a higher return on OAI taxes than on private savings and the marginal component more than offsets the marginal wage tax. The net effect of the tax and benefit structures is that of a marginal wage subsidy instead of a wage tax.

The relation between the present value of lifetime income, \( Y \), and labor supply is derived by substituting equation (7) into the right-hand side of equation (4).
(Y_N + \frac{B_0}{1+r} + [1+t \frac{Y_{S-R}}{1+r}] wH \text{ for } wH \leq E

Y(H) = \begin{cases} 
Y_N + \frac{B_0}{1+r} + \frac{tE}{1+r} & \text{for } wH > E
\end{cases}

This function is illustrated in Figure 1 for the four cases described above.

The labor supply curve is the relation between the wage rate and the utility maximizing value of work effort. We intend to show how this relation is altered by alternative rates of return to OAI tax payments. The labor supply curve is derived from the following maximization problem:

\[
\begin{align*}
\text{MAX } U &= U(C_1, C_2, H) \\
C_1, C_2, H
\end{align*}
\]

s.t. \( C_1 + \frac{C_2}{1+r} = Y(H), \)

\[
Y_N + [1+t \frac{Y_{S-R}}{1+r}] wH \text{ for } wH \leq E
\]

\[
Y(H) = \begin{cases} 
Y_N + \frac{Y_{S-R}}{1+r} tE + wH & \text{for } wH > E
\end{cases}
\]

and the restriction that private saving be non-negative. Note that the lump sum component of benefits, \( B_0 \), has been dropped since it contributes nothing to the analysis of marginal changes. In addition, we assume that the utility function has the properties necessary to permit dividing the maximization procedure into a two-stage process.\(^5\) In the first stage,

\(^5\)These conditions are developed for a general model such as ours by William A. Barnett in a series of Special Studies Papers of the Federal Reserve Board. These papers are "Household Consumption Allocation and Labor Supply" No. 51, "The Full-Employment Equivalent Price of Leisure" No. 52, and "Labor Supply and the Allocation of Consumption Expenditure" No. 53.
FIGURE 1—Present Value of Lifetime Income Function

Case 1: \( \bar{r}_s = -1 \)

Case 2: \( 0 < \bar{r}_s < r \)

Case 3: \( \bar{r}_s = r \), \( Y = Y_N + \frac{E_o}{1+r} + \frac{(1-t)w}{1+r} + \frac{\bar{r}_s - r}{1+r} tE \)

Case 4: \( \bar{r}_s > r \)
the worker chooses the optimal combination of income and work effort. This stage is our concern here. The second stage is the allocation of consumption over the lifetime (which determines the effect of OAI on private capital formation). Finally, since our only interest is the qualitative impact of OAI, it is useful to assume that labor supply, without the OAI program, is perfectly wage inelastic.

Figure 2 is an example of utility maximization with $r_s < r$ for a worker with optimal taxable earnings below the ceiling. Panel A shows the optimal level of income, $Y^*$, and work effort, $H^*$ at the tangency between the worker's income–work indifference curve and the income function $Y(H)$. The tax structure is in panel B. The total tax at $H^*$ given the worker's wage is $T^*$. The marginal benefit, $B^*$, associated with this level of tax payment is given by the benefit structure of panel C. The lifetime consumption decision is shown in panel D. The lifetime budget constraint is kinked at $Y_N + wH^* - T^*$ (the maximum consumption during working life) because of the restriction that the worker cannot borrow against his future OAI benefit.

Continuing with this example ($r_s < r$), we now indicate how the introduction of OAI and subsequent changes in the wage rate affect labor supply. $Y(H)$ and $Y_{1}(H)$ of Figure 3 are the lifetime income functions for a wage of $w_1$ before and after the introduction of OAI. $Y_{1}(H)$ is kinked at earnings of $Y = Y_N + [1+t\left(\frac{r_s - r}{1+r}\right)] E$ where $E$ is the ceiling on the taxable earnings. The income functions for a lower wage, $w_0$, and a higher wage, $w_2$, are also kinked at this same level of lifetime income as long as $E$ is constant as the wage rate changes.
FIGURE 2--Utility Maximization

(A)

(B)

(C)

(D)

\[ T = \frac{B_1}{1+T_g} \]
FIGURE 3—Lifetime Income Functions ($\bar{y}_s < r$)

\[ Y(H) \]

\[ Y_N + [1 + t(\bar{y}_s - r)]E \]

$Y(H)_0$

$Y(H)_1$

$Y(H)_2$

$Y(H)_3$

$Y(H)_4$

$Y(H)_5$

$Y_N$

$H$
This set of income functions can be imposed on a representative worker's indifference map to derive the labor supply curve.\footnote{Our result for this special case is equivalent to C. Duncan MacRae Elizabeth Chase MacRae, "Labor Supply and the Payroll Tax," \textit{AER} 66(3) (June, 1976) 408-9.} This is shown in Figure 4a. This worker (called A) has chosen to have OAI taxable earnings at wage \( w_2 \) below the ceiling. With no OAI tax, point F would be the worker's equilibrium. The imposition of the tax lowers the marginal wage to 
\[
\frac{\frac{w_2 - r}{1+r}}{1+r}
\]
and the equilibrium shifts to point G. Note the F and G are at the same optimal level of work effort \( H^*_A \). This results from the restriction that labor supply be perfectly wage inelastic. Lowering the wage to \( w_0 \) also leaves optimal work effort at \( H^*_A \). However, if the wage is increased, there will be some indifference curve (such as \( I_2 \)) which is tangent to the kinked income function at two places, \( H^*_A \) and \( H^{**}_A \). Further wage increases leave optimal labor supply at \( H^{**}_A \). Therefore, the OAI tax introduces a discontinuous rightward shift in the labor supply curve (the lower panel of Figure 4a) at some wage rate.

The case of a worker (B) choosing OAI taxable earnings at wage \( w_2 \) above the ceiling is shown in Figure 4b. Here, the imposition of the tax is an income effect shifting equilibrium from F to G with increased work effort \( H^{**}_B \). However, as the wage falls an indifference curve, \( I_1 \), will again be tangent to the income function at two points. The discontinuous labor supply function which results is in the lower panel of Figure 4b. The horizontal summation of such curves indicates that the imposition of the OAI program with \( r < r \) increases the elasticity of the labor supply.
FIGURE 4a—Labor Supply Function ($Y_s < r$)

\[ Y(H) = Y_N + \left[ 1 + t \left( \frac{Y_s - r}{1+r} \right) \right] E \]
FIGURE 4b—Labor Supply Function ($\bar{Y}_e < \bar{r}$)

\[ Y_N + [1 + t\left(\frac{\bar{Y}_e - \bar{r}}{1 + \bar{r}}\right)]E \]
curve. This is a general result depending only on the condition $\bar{T}_s > r$ and the existence of a ceiling on taxable earnings. The special cases discussed by Browning ($\bar{T}_s = r$) and MacRae and MacRae ($\bar{T}_s = -1$) are the limiting cases. If $\bar{T}_s = r$ the elasticity of the labor supply curve is unaffected by OAI while if $\bar{T}_s = -1$, the elasticity is at its greatest.

Case 4 ($\bar{T}_s > r$) remains. If the rate of return to OAI taxes exceeds the market rate, the system acts as a net marginal wage subsidy reducing the elasticity of labor supply. Figure 5 combines the analyses of Figures 3 and 4 to illustrate this. Two marked differences are evident. First, the income functions under OAI are kinked in the opposite direction as shown earlier (Figure 1). Because of this, the labor supply curves under OAI have no discontinuous increases but rather a continuously decreasing segment as wages rise. To see this consider point $A_3$. At $A_3$ worker A's indifference curve has a slope equal to that of the line segment $Y_A_3$. Further increases in $w$ result in corner solutions along the line $A_4A_3$. However, at $A_4$ the slope of the indifference curve has become equal to $w_4$. With perfectly wage inelastic labor supply, equilibria for wages higher than $w_4$ will be along $A_4A_5$ extended. The aggregate of individual labor supply curves (such as

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7. In a time series context the specification of the taxable earnings ceiling would be different. If E rises at the same rate as average wages, then the ratio $E/w$ remains constant. The kinks in the income function then lie along a vertical line at $E=E/w$. Imposition of the program has no effect on workers with earnings below E, but it would increase the labor supply of those above.

FIGURE 5—Labor Supply Function ($\pi_g > \pi$)
those of A and B) is less elastic (algebraically, not in absolute value) under OAI. \(^9\)

Case 4 \((\bar{F}_s > r)\) has been of little interest to economists studying the incidence of the payroll tax and other microeconomic questions. Even Browning, who recognized the relationship between taxes and benefits, ignored this possibility. Yet students of the aggregate effects of OAI (such as its implications for economic growth) have concentrated on this case exclusively. Paul Samuelson, for example, analyzed the implications of OAI for an economy with \(r=0\) but \(\bar{F}_s\) equal to the rate of growth of the tax base. Others, including David Cass and Menahem Yaari extended Samuelson's analysis but retained the assumption that \(\bar{F}_s > r\).\(^{10}\) These studies do not consider labor supply effects, but they do establish the importance of examining Case 4.

III: The Labor Supply Effect of OAI Under Intragenerational Redistribution

We now consider the effects of OAI when the system is used for intra as well as intergenerational transfers. The basic model is the same, but we make two important modifications. First, the lump sum benefit, \(B_0\), is reintroduced to capture an important operational aspect of the system, namely the minimum benefit provision. We also assume that marginal rates of return decline with the amount of taxes paid. Therefore, the benefit

\(^9\)Again, this qualitative result is unaltered by the alternative specification that \(E/w\) be constant as \(w\) changes.

and tax structures are now given by:

\[ B = B_0 + B_1, \]

\[ B_0 = \overline{B}_0, \]

\[ B_1 = (1 + r_s)T, \text{ where} \]

\[ r_s = \begin{cases} -1 & \text{for } T < T_{\text{min}}, \quad T_{\text{min}} = t \min \min \varepsilon(\tau_s, T) \text{ for } T_{\text{min}} \leq T \leq tE \quad \text{with } r_s(tE) = r_{\text{min}}, \text{ and} \varepsilon(\tau_s, T) < 0 \text{ and constant.} \end{cases} \]

The minimum work effort needed to achieve insured status under Social Security is \( H_{\text{min}} \). \(^{11}\) \( \varepsilon(\tau_s, T) \) is the elasticity of the internal rate of return with respect to OAI tax payments. In Table 1, \( \varepsilon(\tau_s, T) \) is shown to be about \(-2\). Figure 6 is an illustrative relation between \( r_s \) and \( T \). Note that while all insured workers have positive rates of return, only workers with \( T < T \) have rates of return above the market rate. It may be that \( r_{\text{min}} > r \). Then all insured workers would receive a net wage subsidy. There are, of course, other possibilities including the case where \( r_s(T) \) crosses the \( T \)-axis.

Under the tax structure specified earlier (5) and the benefit structure above, the net present value of a worker's benefit is:

\[ \text{NPVB} = \frac{B}{1 + r} - T = \begin{cases} \frac{B_0}{1 + r} + \frac{r_s(T) - r}{1 + r} t \min \quad \text{for } H_{\text{min}} < H < \frac{E}{w} \varepsilon(\tau_s, T) < \infty \varepsilon(\tau_s, T) \leq 0 \text{ and constant.} \end{cases} \]

\[ \frac{B_0}{1 + r} + \frac{r_{\text{min}} - r}{1 + r} tE \quad \text{for } H = \frac{E}{w} \]

FIGURE 6—Benefit Structure
This can be combined with (4) to get the present value of lifetime income function,

\[ Y_N + (1-t) \frac{wH}{1+r} \]  
for \( H < H_{\min} \)

\( Y(H) = \begin{cases} 
Y_N + \frac{H_0}{1+r} + \frac{r_s(t)-r}{1+r} wH & \text{for } H_{\min} \leq H < E_w \\
Y_N + \frac{H_0}{1+r} + \frac{\min \left\{ \frac{r_s(t)-r}{1+r} tE \right\}}{w} + wH & \text{for } H \geq E_w 
\end{cases} \)

The properties of this function for a useful special case \( (0 < r_{\min} < r < r_{\max}, \ H_0 = 0) \) are given below:

\[ Y(H) = Y_N + wH \text{ for } r_s(t) = r, \]

\( \frac{\partial Y(H)}{\partial H} = \frac{1-t}{1+r} \frac{r_s(t)[1+\varepsilon(r_s,t)]-r}{1+r} \)

If we choose \( |\varepsilon(r_s,t)| > 1 \), then:

\[ \frac{\partial Y(H)}{\partial H} \geq 0, \quad \frac{\partial Y(H)}{\partial H} > (1-t)w, \text{ and } \frac{\partial^2 Y(H)}{\partial H^2} < 0. \]

We illustrate this function in Figure 7. If \( r_{\min} > r \), which means that all

\[ \frac{\partial Y(H)}{\partial H} \geq 0 \text{ for } r_s(t)[1+\varepsilon(r_s,t)] \geq \frac{(1-t)r}{t}, \]

\[ \frac{\partial Y(H)}{\partial H} < 0 \text{ for } r_s(t)[1+\varepsilon(r_s,t)] > r, \]

\[ \frac{\partial Y(H)}{\partial H} < 0 \text{ for } r_s(t)[1+\varepsilon(r_s,t)] > -1; \]

\[ \frac{\partial^2 Y(H)}{\partial H^2} = (tw)^2 \frac{(1+\varepsilon(r_s,t))r_s}{l+r} \frac{\partial r_s}{\partial t} > 0 \text{ for } \varepsilon(r_s,t) < -1. \]
FIGURE 7—Present Value of Lifetime Earnings Function
workers receive an intergenerational transfer, then the entire income function to the right of $H_{\text{min}}$ $(Y(H))$ would be above the income function without OAI $(Y_N+\omega H)$. Note that workers with $r_s > r$ receive a net wage subsidy and that workers with $\frac{\partial Y(H)}{\partial H} > \omega$ receive a marginal wage subsidy.

Given the income functions of Figure 7, the labor supply effects of introducing the system will differ depending on worker's preferences.

Consider individual B of Figure 8. This worker is in equilibrium at points $B_0$ and $B_1$ with and without OAI. Note that point $B_1$ is the tangency between the income function and a ray from $Y_N$. Since we assume perfectly wage inelastic labor supply, points $B_0$ and $B_1$ indicate equal desired supply of effort. Individual B represents the dividing line. Workers in equilibrium to the right of B, such as C with desired work effort below $E/\omega$, will reduce labor effort. Worker D, however, who is above the ceiling will experience only a negative income effect and will increase labor supply. The opposite would be true if $\frac{\text{Min}}{s} > r$. For workers to the left of B, the incentive to increase labor supply may be substantial. Worker A's net marginal wage is much higher under OAI, and the magnitude of this increase depends on the magnitude of the intragenerational redistributive component. It is even possible that a worker in equilibrium at the corner solution $H=0$ will be attracted into the labor force.

IV. Conclusions and Implications of Intrigenerational Redistribution

Even with our simplifying assumptions, the impact of OAI on aggregate labor supply is unclear. We may speculate, however, that the majority
FIGURE 8—Labor Supply with Intragenерational Redistribution
of actual workers most closely resemble individual C of Figure 8. These workers would reduce labor supply under OAI. Workers in the upper tail of the earnings distribution would also reduce labor supply if $\min_r r$ as was the case in the recent past and which is likely to hold for the near future. The only group with a strong incentive to increase the supply of work effort consists of workers in the lower tail of the earnings distribution. Therefore OAI may very well be structured in a manner which counteracts known disincentive effects of other transfer programs.

Unfortunately, a substantial proportion of workers in the lower tail of the covered earnings distribution may not be in the lower tail of the total earnings distribution. Our model demonstrates the magnitude of the incentive for workers in uncovered employment (federal government workers, for example) to structure their supply of work effort to qualify for a minimal OAI benefit.

Our task here has been to establish the relevance of the operational character of the OAI program for labor supply studies. To accomplish this we have simplified leaving open a number of questions. However, we believe that the importance of the link between taxes and benefits under OAI has been substantiated, and we may consider how our analysis applies to policy issues. Note that we deal only with the old age program abstracting from the package of social insurance functions actually under the Social Security

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13Evidence from the earnings distribution of covered workers for a single year while not completely appropriate for our life cycle model is suggestive. From 1970 to 1974, the proportion of workers with annual earnings below the maximum ranged from 72.5 percent in 1971 to 85.9 percent in 1974. These figures are from U.S. Department of H.E.W., Social Security Administration, Social Security Bulletin Annual Statistical Supplement, 1974.
First, consider the expected behavior of workers with the option to leave the system. As shown on Figure 7, the net transfer is greatest at the point where

\[
\frac{\partial Y(H)}{\partial H} = w.
\]

Therefore, it is advantageous to attain this level of hours in covered employment by opting out as \( H \) begins to exceed this level. It appears that some state and local government employees are aware of such considerations.

We have shown that the combination of minimum benefit and work effort provisions result in large marginal wage subsidies for some workers. It is unlikely that this form of subsidy was the intent of these program provisions. Thus, the potential labor supply distortions they create must be considered undesirable. Since the maintenance of minimum standards is now met by the supplemental security income program, these provisions must be judged anachronistic.

We may speculate on the effects of eliminating the ceiling of taxable earnings and expanding the system's redistributional character. A specific policy meeting this end would be a progressive payroll tax with no ceiling. This would increase the wage subsidy for workers below the point where \( \frac{\partial Y(H)}{\partial H} = w \) and increase the wage tax for workers above this point. The labor supply distortion would therefore be greater.

Finally, consider the case of an old age insurance program which approximates a quid pro quo. That is, with adequacy concerns and other redistributional motives served through general revenues. The labor supply distortions of the system would be minimized while maintaining much of the system's social insurance function.
Selected References


